

$$\dot{x}(t) = x'(t) = \frac{dy(t)}{dt} = f(x(t), t) \quad (1)$$

$$\dot{x} = f(x, t)$$

An. Soln $x(t)$

eg

$$\dot{x} = -3x$$

$$x_1 = e^{-3t}$$

$$x_2 = 10 \cdot e^{-3t}$$

$$\dot{x}_1 = -3 \cdot e^{-3t}$$

$$\dot{x}_2 = -30 \cdot e^{-3t}$$

$$-3 \cdot e^{-3t} = -3 \cdot e^{-3t}$$

$$1 = 1$$

$$-30 \cdot e^{-3t} = -3(10 \cdot e^{-3t})$$

$$1 = 1$$

x_1, x_2 are general solns of ODE

ODE + I.C. \rightarrow I.V.P

$$\dot{x} = -3x, \quad x_0 = 1$$

$$x_1 = e^{-3t}$$

$$x_1(0) = e^{-3 \cdot 0}$$

$$= 1$$

~~$$x_2 = 10 \cdot e^{-3t}$$~~

~~$$x_2(0) = 10 \cdot 1$$~~

~~$$= 10$$~~

$$\varphi(t, t_0, x_0) = e^{-3t}$$

Linear F.O. I.V.P.

(2)

$$a(t) \cdot \dot{x} + b(t) \cdot x = c(t).$$



$$a \cdot \dot{x} + b \cdot x = c$$



$$\boxed{\dot{x} + kx = u.} \quad \left(\begin{array}{l} k = b/a \\ u = c/a \end{array} \right)$$

↓ Int. Factor

$$x(t) = \underbrace{e^{-kt} \cdot x_0}_{\text{I.C.}} + \underbrace{e^{-kt} \int_0^t e^{kt_1} u(t_1) \cdot dt_1}_{\text{input}}$$

• $u = \cos t$.

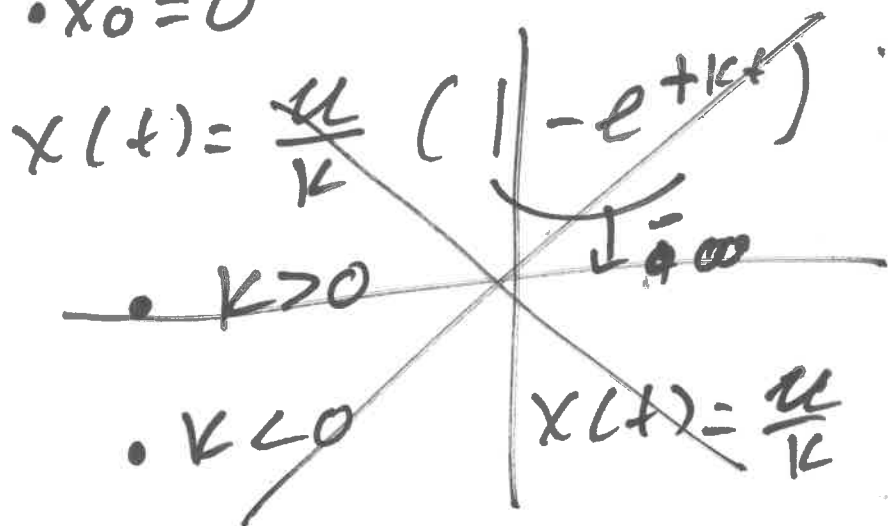
$$x(t) = e^{-kt} \cdot x_0 + \frac{u}{k} (1 - e^{-kt}).$$

• $u = 0$

$$x(t) = e^{-kt} \cdot x_0$$

$$x(t) = e^{-kt} \cdot x_0 + \frac{u}{k} (1 - e^{-kt}). \quad (4)$$

• $x_0 = 0$

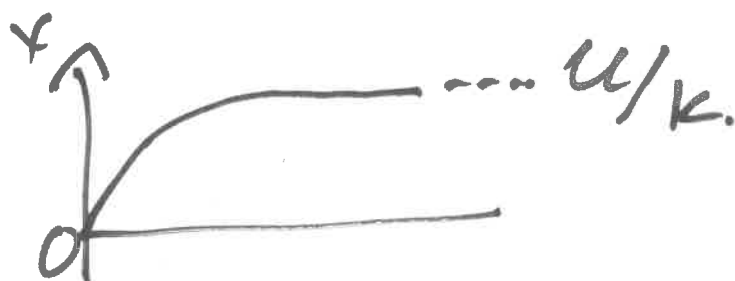


$$x(t) = e^{-kt} \cdot x_0 + \frac{u}{k} (1 - e^{-kt})$$

• $x_0 = 0 \Rightarrow x(t) = \frac{u}{k} (1 - e^{-kt})$



$$\frac{u}{k} (1 - e^{-kt})$$



$$x(t) = e^{-kt} \cdot x_0 + \frac{u}{k} (1 - e^{-kt}) \quad (5)$$

• $k > 0$

$$e^{-kt} \rightarrow 0$$

$$\downarrow$$
$$u/k$$

$$\ddot{x} = f(\dot{x}, x, t)$$

⑥



$$\ddot{x} + A\dot{x} + Bx = u$$

↓ $u=0$

$$\ddot{x} + A\dot{x} + Bx = 0$$

$$\ddot{x} - 2\dot{x} - 3x = 0$$

$$\begin{aligned} x_1 &= e^{3t} \\ \downarrow \\ \dot{x}_1 &= 3e^{3t} \\ \ddot{x}_1 &= 9e^{3t} \end{aligned}$$

$$x_2 = e^{-t}$$

$$9e^{3t} - 2 \cdot (3e^{3t}) - 3(e^{3t}) = 0 \quad (=)$$

$$(9 - 6 - 3)e^{3t} = 0 \quad (=)$$

$$0 \cdot e^{3t} = 0 \quad (=)$$

$$0 = 0$$

is $x_3 = 10 \cdot x_1$ a soln.

$$= 10e^{3t}$$

$$\dot{x}_3 = 30e^{3t}, \quad \ddot{x}_3 = 90e^{3t}$$

$$(90 - 60 - 30)e^{3t} = 0 \quad \checkmark$$

$$\ddot{x} - 2\dot{x} - 3x = 0 \quad x_1 = e^{3t}, \quad x_2 = e^{-t} \quad (7)$$

$$x_4 = x_1 + 2 \cdot x_2 \quad \text{a soln?}$$

$$\dot{x}_4 = e^{3t} + 2e^{-t}$$

$$\dot{x}_4 = 3e^{3t} - 2e^{-t}$$

$$\ddot{x}_4 = 9e^{3t} + 2e^{-t}$$

if x_1, x_2 are 2 solns

then any L.C. $x = C_1 x_1 + C_2 x_2$

$$\ddot{x} + A\dot{x} + Bx = 0 \quad \left. \begin{array}{l} x_1 = \text{soln} \\ x_2 = \text{soln} \end{array} \right\} \Rightarrow$$

$$C_1 \ddot{x}_1 + A C_1 \dot{x}_1 + B C_1 x_1 = 0$$

$$C_2 \ddot{x}_2 + A C_2 \dot{x}_2 + B C_2 x_2 = 0$$

$$C_1 \cdot \ddot{x}_1 + C_2 \cdot \ddot{x}_2 + C_1 \cdot A \dot{x}_1 + C_2 \cdot A \dot{x}_2 + C_1 B x_1 + C_2 B x_2 = 0$$

$$\left. \begin{array}{l} (C_1 x_1 + C_2 x_2)'' + A(C_1 x_1 + C_2 x_2)' \\ + B(C_1 x_1 + C_2 x_2) = 0 \end{array} \right\} \Rightarrow$$

$$y'' + A \cdot y' + B \cdot y = 0$$

$$\ddot{x} - 3\dot{x} - 2x = 0$$

$$x_7 = 10 \cdot e^{3t}$$

$$x_1 = e^{3t} \quad x_2 = e^{-t} \quad (8)$$

$$x_3 = e^{3t} + e^{-t}$$

$$x_4 = \sqrt{11} e^{3t} - 10 \cdot e^{-t}$$

$$x_5 = 3 e^{3t} + 10 \cdot 001 e^{-t}$$

$$x_6 = 4 e^{3t} + 2 e^{-t}$$

Test $x_6 \begin{cases} \rightarrow x_1 \\ \rightarrow x_2 \end{cases}$

$$4e^{3t} + 2e^{-t} = c_1 e^{3t} + c_2 \cdot e^{-t}$$

$$\Rightarrow c_1 = 4, \quad c_2 = 2$$

Test $x_6 \begin{cases} \rightarrow x_1 \\ \rightarrow x_7 \end{cases}$

$$\begin{aligned} 4 \cdot e^{3t} + 2 e^{-t} &= c_1 e^{3t} + c_2 10 \cdot e^{3t} \\ &= (c_1 + 10c_2) e^{3t} \end{aligned}$$

x_1 is a mult of x_7