Final Revision

$$\dot{X} = AX \qquad A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \quad \begin{matrix} \nearrow \lambda_1 = -1 \rightarrow e_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ \searrow \lambda_2 = -2 \rightarrow e_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \end{matrix}$$

$$x_1 = e^{-t} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad x_2 = e^{-2t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\text{F.S.M.} \quad X(t) = \begin{bmatrix} e^{-t} & e^{-2t} \\ -e^{-t} & -2e^{-2t} \end{bmatrix}$$

$$X(0) = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}, \quad |X(0)| = -1$$

$$X^{-1}(0) = \frac{1}{-1} \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\text{S.T.M.} = \Phi(t) = X(t) \cdot X^{-1}(0)$$

$$= \begin{bmatrix} e^{-t} & e^{-2t} \\ -e^{-t} & -2e^{-2t} \end{bmatrix} \cdot \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix} \cdot (-1)$$

$$= -\begin{bmatrix} -2e^{-t} + e^{-2t} & -e^{-t} + e^{-2t} \\ 2e^{-t} - 2e^{-2t} & e^{-t} - 2e^{-2t} \end{bmatrix}$$

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$$STM = \begin{bmatrix} 2e^{-t} - e^{-2t} & 2e^{-2t} - e^{-t} \\ 2e^{-2t} - 2e^{-t} & 2e^{-2t} - e^{-t} \end{bmatrix}$$

- Find S.T.M using expm

$$e^{At} = I + At + \frac{(At)^2}{2!} + \dots$$

$$e^{At} = T \cdot e^{\Lambda t} \cdot T^{-1}$$

$$e^{\Lambda t} = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$$

$$T = [e_1 \quad e_2]$$

I know this

$$\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} e^{-t} & e^{-2t} \\ -e^{-t} & -2e^{-2t} \end{bmatrix}, \dots\dots$$

$$\dot{x} = Ax + Bu$$
$$y = C\cdot x + D\cdot u$$

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$B = \cancel{\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}}$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \checkmark$$

$$G(s) = C\cdot(sI - A)^{-1}\cdot B$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}$$

$$|sI - A| = s\cdot(s+2) + 1 = s^2 + 2s + 1$$

$$(sI - A) = \frac{1}{s^2 + 2s + 1} \begin{bmatrix} s+2 & +1 \\ -1 & s \end{bmatrix}$$

$$G(s) = \underbrace{\underset{1\times 2}{\begin{bmatrix} 1 & 1 \end{bmatrix}} \cdot \underset{2\times 2}{\begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}} \cdot \underset{2\times 2}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}}_{1\times 2} \cdot \frac{1}{(\;)}$$

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$$G(s) = \frac{1}{(\ )}\begin{bmatrix} s+1 & s+1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{(\ )}\begin{bmatrix} s+1 & 2s+2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cancel{s+1}}{(\cancel{s+1})(s+2)} & \frac{2(\cancel{s+1})}{(\cancel{s+1})(s+2)} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{s+2} & \frac{2}{s+2} \end{bmatrix}$$

$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \rightarrow \lambda_1 = -1 \quad \lambda_2 = -2$ $\quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

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$K = \dots \quad A = \quad B = \dots$

eig. $\quad J = \int (x^T \cdot Q x + R^T U R)\, dt$

$G_3 : -1+i, -1+i$

$G_4 :$ $-10+i, -10-i$

$M_c = \begin{bmatrix} B & AB \end{bmatrix}$

$= \left[\begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix} \quad \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right]$

$= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -2 & -5 \end{bmatrix}$ $\quad$ 2 L.I. row vectors

rank = 2

$$A_{CL} = A - BK. \longrightarrow 2\times 2$$

$A_{CL} \downarrow 2\times2$, $A \downarrow 2\times2$, $K \searrow 2\times2$.

$$K = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad M_c = \begin{bmatrix} B & AB \end{bmatrix}$$

$$= \left[\begin{matrix} 1 \\ 0 \end{matrix} \quad \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$|M_c| = -2 \neq 0$ rank $= 2$ CTRB.

place the poles at $-2, -4$

Find O.L. eigenvalues

$K = ?$: C.L. sys is 2 times faster

$$A - BK = A_{CL}$$

$A \to 2\times 2$, $B \to 2\times 1$, $K \to 1\times 2$ $\qquad K = [K_1 \quad K_2]$

$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot [K_1 \quad K_2]$$

$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} \boxed{K_1} & \boxed{K_2} \\ \boxed{0} & \boxed{0} \end{bmatrix}$$

$$= \begin{bmatrix} -K_1 & 1-K_2 \\ -2 & -3 \end{bmatrix} = A_{CL}.$$

$$|A_{CL} - \lambda I| = 0 \Rightarrow \begin{vmatrix} -K_1-\lambda & 1-K_2 \\ -2 & -3-\lambda \end{vmatrix} = 0$$

$$(-K_1-\lambda)\cdot(-3-\lambda) + 2\,(1-K_2) = 0$$

$\lambda = -4$ $\qquad (-K_1+4)\,\overset{\to 1}{(-3+4)} + 2(1-K_2) = 0$

$\lambda = -2$ $\qquad (-K_1+2)\,\overset{\to -1}{(-3+2)} + 2(1-K_2) = 0$

$$K_1 = 3 \qquad K_2 = 1.5$$

$A = \begin{cases} -1 \\ -2 \end{cases}$ $\quad K_{PP} = [3 \quad 1.5]$

$A_{CL} = \begin{cases} -2 \\ -4 \end{cases}$

- Designed L.Q.R. $\quad K_{LQR} = [-4 \quad -2]$

  C.L. eigs = ?

  Compares ~~Kpp~~ and KLQR.

  ~~faster.~~

  C.L. = -2, 3

- Designed L.Q.R: $\quad K_{LQR_2} = [57 \quad -313.5]$

  C.L. eigs = ?

  Faster = ?

  C.L. = $[-20 \quad -40]$ $\quad$ Faster

- Use previous LQR, $R \rightarrow 100 \cdot R_{OLD}$

  $|\lambda| = ?$

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place dyn. of estimator $= -100$

-200

$\dot{e} = (A - G\,C) \cdot e$

~~place dyn of est. at~~ -0.1

-0.2

10 times est.

Cont $\to -2$, $\to -4$ est. $\to -20$, $\to -40$

$$\dot{e} = (A - G\,C) \cdot e$$

2×1 2×2 1×2 2×1

2×1

$G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} G_1 & 0 \\ G_2 & 0 \end{bmatrix} = A_e = \begin{bmatrix} -G_1 & 1 \\ -2-G_2 & -3 \end{bmatrix}$$

(140)

$$|A_e - \lambda I| = 0$$

$$\begin{vmatrix} -G_1 - \lambda & 1 \\ -2 - G_2 & -3 - \lambda \end{vmatrix} = 0$$

$$(-G_1 - \lambda)\cdot(-3-\lambda) - 1\cdot(-2-G_2) = 0$$

- $\lambda = -20 \quad -17G_1 + G_2 + 342 = 0$
- $\lambda = -40 \quad -37G_1 + G_2 + 1482 = 0$

$\Rightarrow$

$$G_1 = 57$$
$$G_2 = 627$$

OBSV = ???????

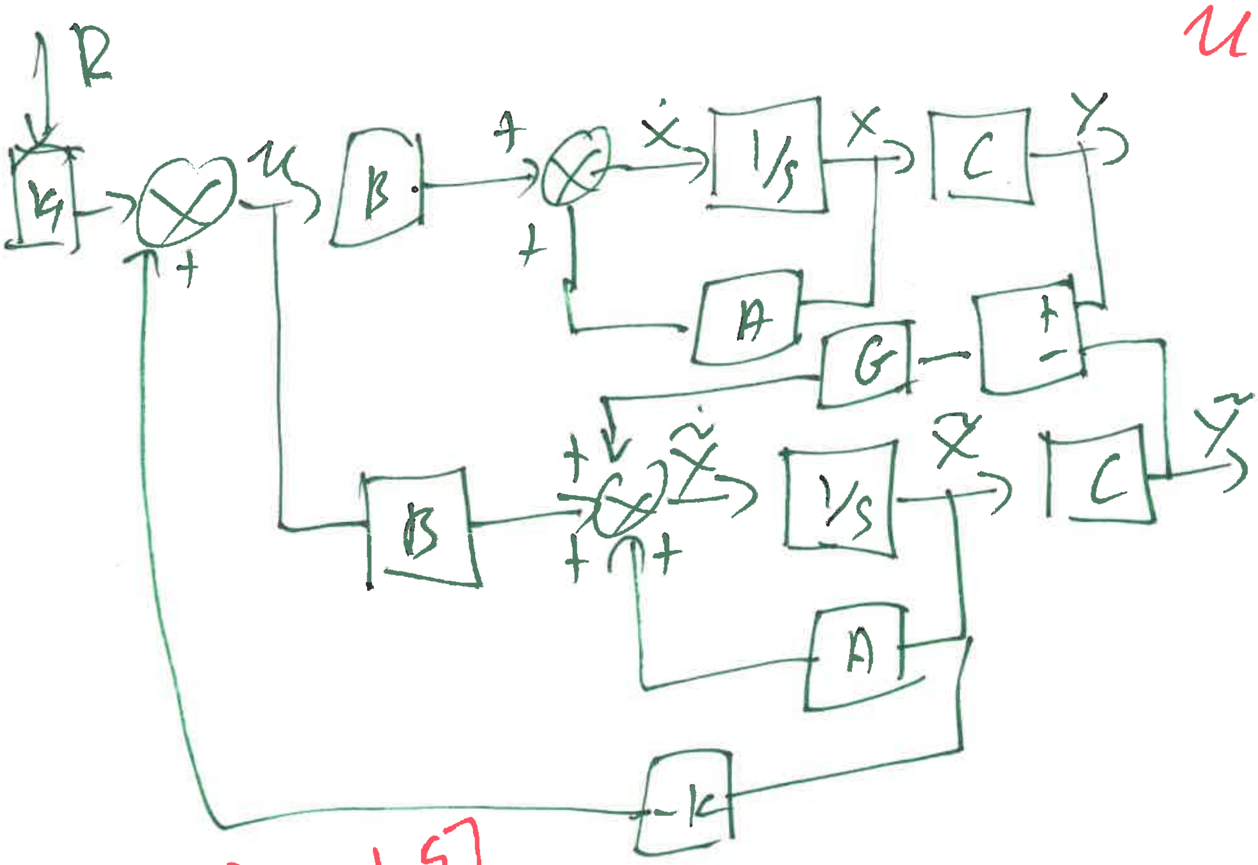
$$M_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} [1 \quad 0] \\ [1 \quad 0]\cdot\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad |M_o| = 1 \neq 0$$

rank = 2

- B.D.
- S.F.
- est.
- tracking mech. $y_{ss} = 5$

$u = ?$



$$u = -\overset{[3\ \ 1.5]}{\cancel{k}}x + \underbrace{k_1}_{N_u + k\cdot N_x} \cdot \overset{5}{\cancel{r_{ss}}}$$

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 1 & 1 \\ -2 & -3 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1/3 & -2/3 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

I am a 2x1 vector.

Given.

I am a 1x1 vector.

$$= \begin{bmatrix} 1 \\ -2/3 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ -2/3 \end{bmatrix} \to N_x$

$1 \to N_u$

$$K_r = N_u + K \cdot N_x$$

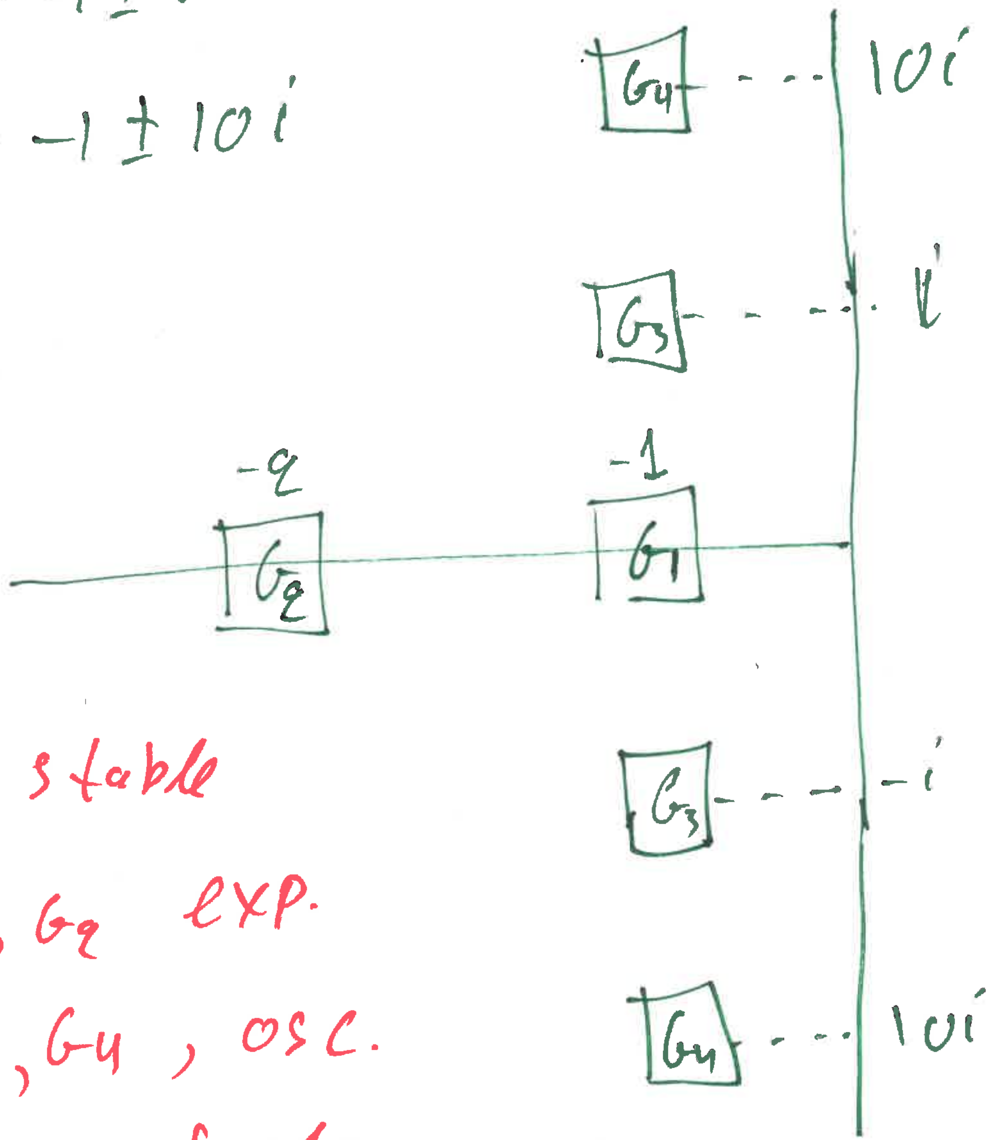
$$u = -Kx + K_1 \cdot 5$$

$G_1 = -1$,

$G_2 = -2$

$G_3 = -1 \pm i$

$G_4 = -1 \pm 10i$



All stable

G_1 , G_2 exp.

G_3 , G_4 , osc.

G_2 is faster than G_1, G_3, G_4

G_4 more osc. than G_3

$\dot{x} + 5x = 2$

$x_0 = 1$

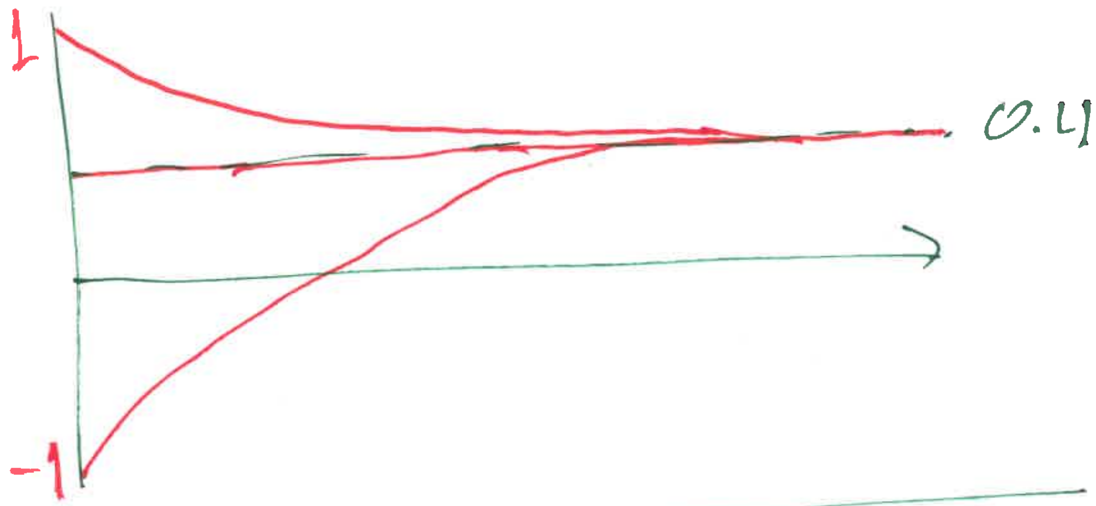
$x_0 = -1$

$x_0 = 0.4$

$K = 5 > 0$

$u = 2.$

$x_{ss} = \frac{u}{k} = \frac{2}{5} = 0.4.$



$\underset{q\times q}{A} = \ldots$ $B = \ldots$ $C = \ldots$

$L\,Q\,R.$

$R_1 = 1$
$Q_1 = 10 \cdot I_q$

$R_2 = 10$
$Q_2 = I_q$

$R_3 = 10$
$Q_3 = 10 \cdot I_q$