$$\dot{x}(t) = x'(t) = \frac{dx(t)}{dt} = f(x(t), t)$$

$$\dot{x} = f(x, t)$$

~~Ana~~ An. Soln of ODE $x(t)$

e.g. $\dot{x} = -3 \cdot x$ $\qquad x_1 = e^{-3t}$

$$\dot{x}_1 = -3e^{-3t}$$

$$-3e^{-3t} = -3e^{-3t}$$

$$1 = 1$$

$$x_2 = 10 \cdot e^{-3t} \Rightarrow \dot{x}_2 = -30e^{-3t}$$

$$-30e^{-3t} = -3 \cdot 10e^{-3t}$$

- x_1, x_2 are solns of $\dot{x} = -3x$
- Inf. # of soln.

ODE + I.C. = I.V.P.

$\dot{x} = -3x$, $x_0 = 1$

$x_1 = e^{-3t} \Rightarrow x_1(0) = e^0 = 1. \Rightarrow$ soln to IVP

$x_2 = 10\, e^{-3t} \Rightarrow x_2(0) = 10 \neq 1 \Rightarrow$ Not a soln

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1st order <u>Linear</u> ODE

$a(t)\,\dot{x} + b\,x(t) = c(t)$

$a \cdot \dot{x} + bx = c$

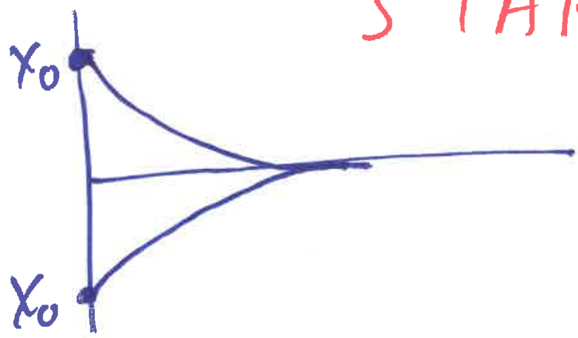
$\dot{x} + kx = u \qquad k = b/a, \quad u = c/a.$

$\downarrow$ I.F

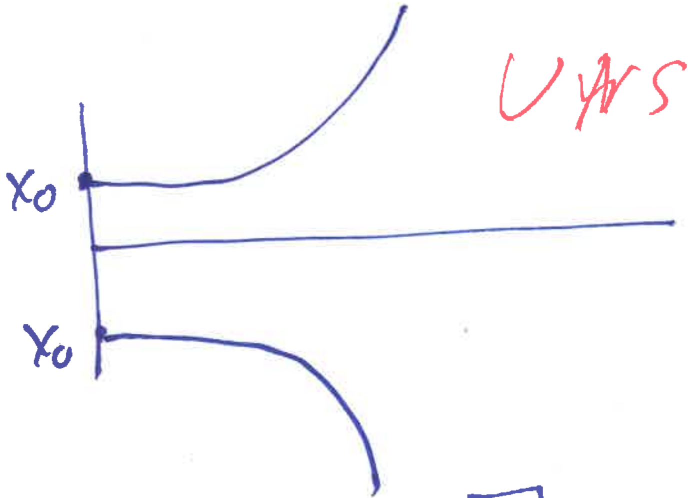
$$x(t) = e^{-kt} \cdot x_0 + e^{-kt} \cdot \int_0^t e^{kt_1} u(t_1)\, dt_1$$

$u=0 \quad x = e^{-k\cdot t}\cdot x_0$

- $k>0$

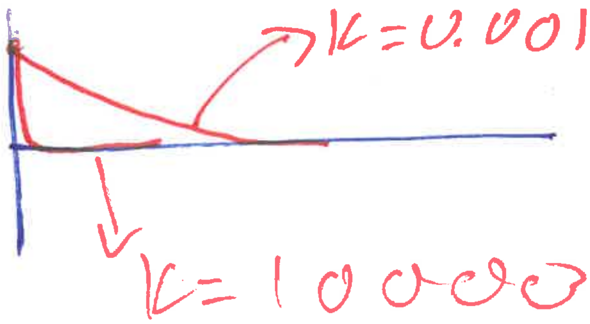


- $k<0$



- $k>0$ $\quad k_1 = 0.001 \qquad e^{-t\,\square}$

$\quad k_2 = 1000000 \qquad e^{t\,\square}$



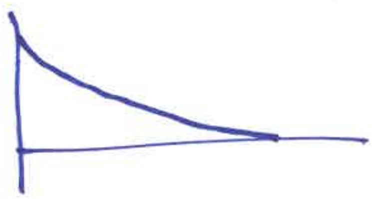
$k \to$ stability, $\to$ speed	$x_0 \to$ I.C.
	$u \to$ S.S.

$$x = e^{-kt} \cdot x_0 + e^{kt} \cdot \int e^{kt_1} u(t_1)\, dt_1,$$

$u = const.$

$$x = \underbrace{e^{-kt} \cdot x_0}_{} + \frac{u}{k}\left(1 - e^{-kt}\right)$$

e^{kt}, $k > 0$



$1 - e^{-kt}$

$\frac{u}{k}\left(1 - e^{-kt}\right)$