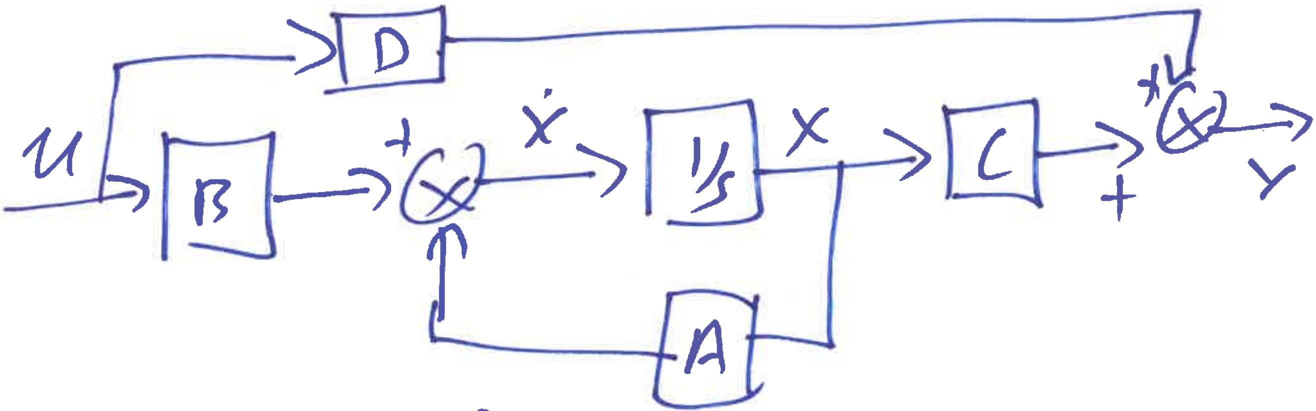
# Revision

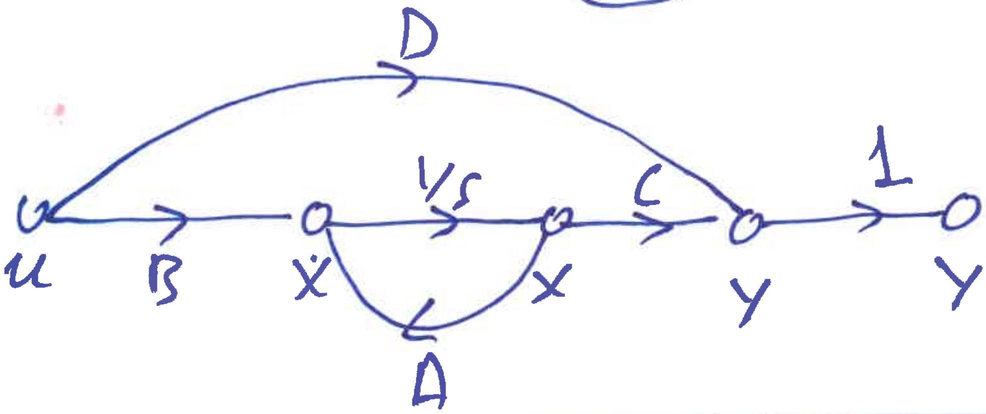
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$x \in \mathbb{R}^{n\times 1}$, $A \in \mathbb{R}^{n\times n}$

$u \in \mathbb{R}^{m\times 1}$ $\quad B \in \mathbb{R}^{n\times m}$ $\quad D \in \mathbb{R}^{p\times m}$

$y \in \mathbb{R}^{p\times 1}$ $\quad C \in \mathbb{R}^{p\times n}$

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$$\begin{array}{ccc} \text{ODE} & \longrightarrow & \text{S.S.} \\ \downarrow & \swarrow & \\ \text{T.F.} & & \end{array}$$

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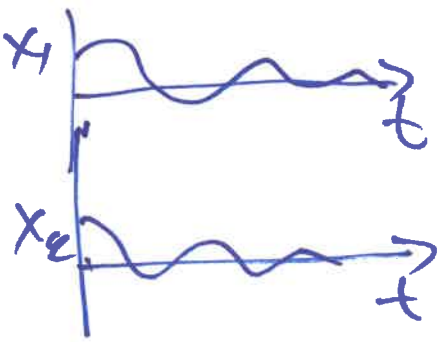
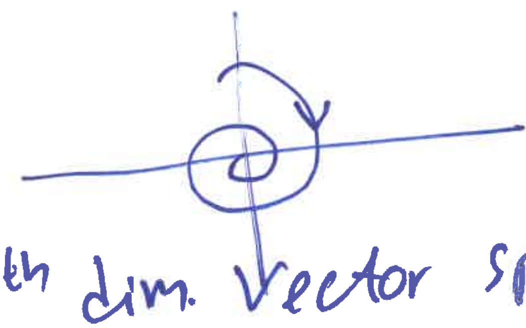
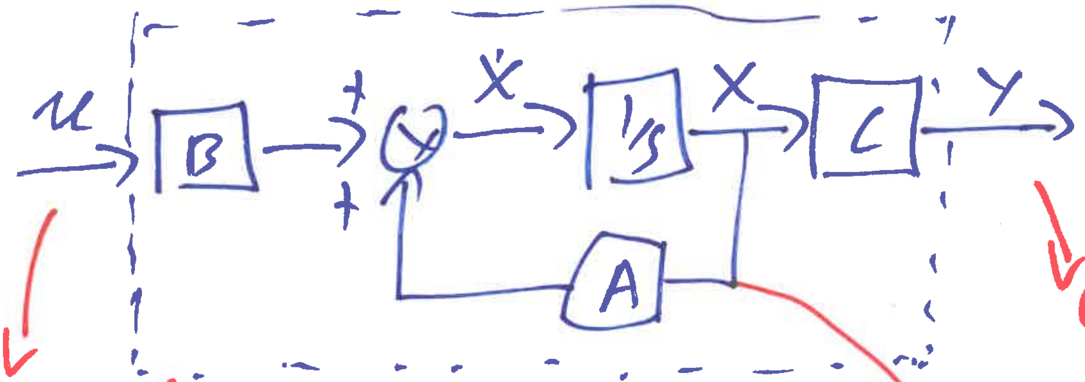
$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$ ← State Space

$$x = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



$n^{th}$ dim. vector space




control

observe

BLACK BOX

# THE MOST IMPORTANT

- I want to control $x$ using $u$.
- I want to see $x$ behaves through $y$

CTRB

OBSV

$$\dot{x} = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} \cdot x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot u \quad , \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$ 50

$$\dot{x}_1 = -2 \cdot x_1 + x_2 + 2 \cdot u$$
$$\dot{x}_2 = x_1 - x_2 + u$$

CTRB

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$$\dot{x} = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$

CTRB

$$\dot{x}_1 = -2 x_1 + x_2 + 2u$$
$$\dot{x}_2 = x_1 \pm x_2$$

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$$\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$$

CTRB

$$\dot{x}_1 = -2 \cdot x_1 + 2 \cdot u$$
$$\dot{x}_2 = -x_2 + u$$

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$$\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$

$$\dot{x}_1 = -2 \cdot x_1 + 2 \cdot u$$
$$\dot{x}_2 = -x_2 \; ?$$

NOT CTRB

$C = [3 \quad 2]$ (51)

$$G(s) = \frac{8s+17}{s^2+3s+1}$$

$$G(s) = \frac{6s+10}{s^2+3s+1} \qquad \frac{1}{|sI-A|}$$

$$G(s) = \frac{8s+10}{s^2+3s+2}$$

$$G(s) = \frac{6s+6}{s^2+3s+2} = \frac{6\cancel{(s+1)}}{\cancel{(s+1)}(s+2)}$$

(52)

$$M_c = [B \quad A\cdot B \quad A^2 B \; \dots \; A^{n-1}\cdot B]$$

$$\text{Rank}(M_c) = n \longrightarrow \text{CTRB}$$
$$< n \longrightarrow \text{unCTRB}$$

$\downarrow$

# of L.I. coluns / ~~rows~~

$$|M_c| \neq 0 \longrightarrow \text{CTRB}$$
$$= 0 \longrightarrow \text{un CTRB}$$

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$$A = \underbrace{\begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}}_{2\times 2} \qquad B = \underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{2\times 1} \qquad n = 2$$

$$M_c = [B \quad A\cdot B]$$

$$\begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$$

2 L.I

$$|M_c| = 2 + 3 = 5 \neq 0 \qquad \text{CTRB}$$

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$M_c = [B \quad AB] \qquad A \cdot B =$$

$$\begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$M_c = \begin{bmatrix} 2 & -4 \\ 0 & 2 \end{bmatrix}$$

2 L.I. CTRB

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdot c = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$|M_c| = 4 \neq 0$$

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$$A = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

CTRB

$$M_c = \begin{bmatrix} 2 & -4 \\ 1 & 1 \end{bmatrix}$$

2 L.I.

$$|M_c| = 2 + 4 = 6 \neq 0$$

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} -4 \\ 0 \end{bmatrix} \qquad M_c = \begin{bmatrix} 2 & -4 \\ 0 & 0 \end{bmatrix}$$

1 L.I. Not CTRB

$$|M_c| = 0 + 0 = 0$$

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H.W. $B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

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$$A = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{aligned} \dot{x}_1 &= -2x_1 + 2\cdot u \\ \dot{x}_2 &= x_2 \end{aligned} \quad \Bigg| \quad \begin{aligned} y_1 &= x_1 \\ y_2 &= x_2 \end{aligned}$$

OBSV


$$C = [3 \quad 2] \qquad y = 3\underbrace{\cancel{x_1}}_{\to 0} + 2\overbrace{\cancel{x_2}}^{\to +\infty}$$

OBSV

$$C = [3 \quad 0] \qquad y = 3\cdot x_1$$ NOT OBSV

$$A = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} \qquad C = [3 \quad 0]$$

OBSV

$$\begin{aligned} \dot{x}_1 &= -2x_1 + x_2 + 2\cdot u \\ \dot{x}_2 &= x_2 \end{aligned} \quad \Bigg| \quad y = 3x_1$$

$$A = \begin{bmatrix} -2 & 0 \\ 1 & 1 \end{bmatrix} \qquad C = [3 \quad 0]$$

$$\begin{aligned} \dot{x}_1 &= -2x_1 + 2\cdot u \\ \dot{x}_2 &= x_1 + x_2 \end{aligned} \quad \Bigg| \quad y = 3\cdot x_1$$

Not OBSV.

$M_o = [C \quad CA \quad CA^2 \quad \dots \quad CA^{n-1}]^T$ 56

$\downarrow$ Rank

$\mathrm{Rank}(M_o) = n \longrightarrow$ OBSV

$< n \longrightarrow$ Not OBSV.

$|M_o| \neq 0$ ~~STEB~~ OBSV

$= 0$ Not OBSV.

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$A = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 3 & 0 \end{bmatrix}$

$2\times2$ $1\times2$

$\underset{1\times2}{C} \cdot \underset{2\times2}{A} = \underset{1\times2}{\begin{bmatrix} 3 & 0 \end{bmatrix}} \cdot \underset{2\times2}{\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} -6 & 0 \end{bmatrix}$

$1\times2$

$M_o = \begin{bmatrix} 3 & 0 \\ -6 & 0 \end{bmatrix}$ $|M_o| = 3 \cdot 0 + 6 \cdot 0 = 0$

NOT OBSV.

$\dot{x} = a \cdot x \qquad x = e^{rt}$

C.E

$r \cancel{e^{rt}} = a \cancel{e^{rt}} \Rightarrow r = a \qquad x = C \cdot e^{at}$

$\ddot{x} + A\dot{x} + Bx = 0 \qquad x = e^{rt}$

C.E.

$r^2 + Ar + B = 0$

$r_1 \qquad r_2 \qquad \Delta =$

$x = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

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$\dot{x} = A \cdot x \qquad n = 2$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$

Assume $x = e^{rt}$

$x = e \, e^{rt}$

$\in \mathbb{R}^{2\times 1} \qquad e = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$