Revision

ch 1.

$\dot{x} + kx = u \Rightarrow x = e^{-kt} \cdot x_0 + e^{-kt} \int_0^t e^{kt_1} u \, dt_1$

$u = \text{const}$, $x_0 = 0$ $\quad x = \frac{u}{k}(1 - e^{-kt})$

$u \not\Rightarrow$ stability

$\downarrow$

s.s. $\quad x_{ss} = u/k \quad (k > 0)$

$u = 0$

$\dot{x} + kx = 0 \Rightarrow x = e^{-kt} \cdot x_0 \quad \begin{cases} k > 0 & \text{stable} \\ k < 0 & \text{unst.} \end{cases}$

Try $x = e^{rt}$

$r + k = 0$ C.E

$r \to$ eigenvalue

$\ddot{x} + A\dot{x} + Bx = u \xRightarrow{u=0} \ddot{x} + A\dot{x} + Bx = 0$

$x = e^{rt} \Rightarrow r^2 + Ar + B = 0$ C.E.

- $r_1 \neq r_2 \in \mathbb{R}$, $x_1 = e^{r_1 t}$, $x_2 = e^{r_2 t}$
- $r_1 = r_2 = r \in \mathbb{R}$, $x_1 = e^{rt}$, $x_2 = e^{rt} \cdot t$
- $r_1 = a + bi$, $r_2 = \bar{r}_1$, $x_1 = e^{r_1 t}$, $x_2 = e^{r_2 t}$

  $x_1 = e^{at} \cdot \cos bt$

  $x_2 = e^{at} \sin bt$

$\Bigg\}$ $x = c_1 x_1 + c_2 \cdot x_2$

$x_0 = \ldots \quad \dot{x}_0 = \ldots$

$c_1 = \ldots$

$c_2 = \ldots$

$$\dot{x} = Ax + Bu$$
$$y = C\cdot x + D\cdot u$$

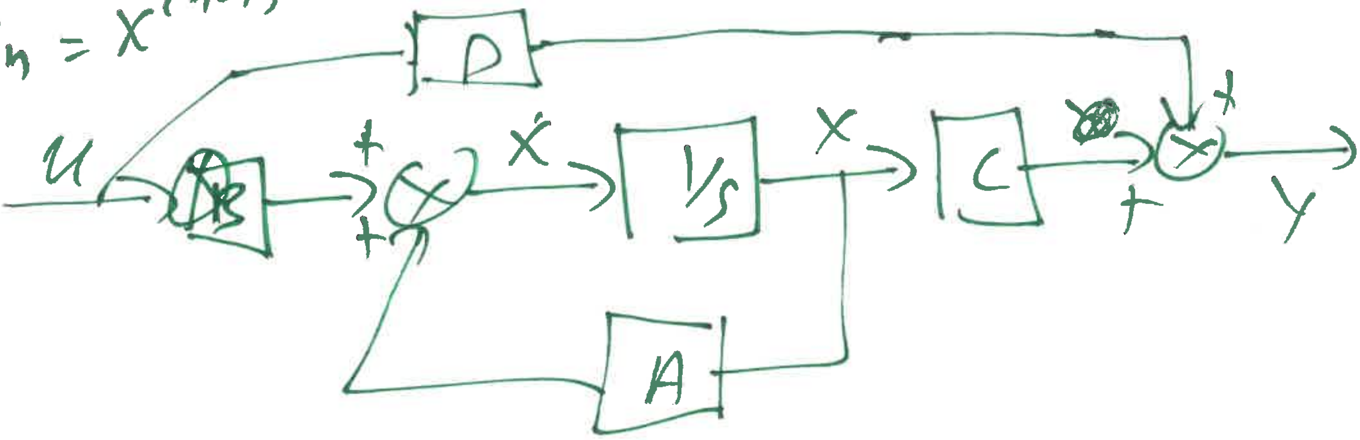
ODE → S.S.

$x^{(n)} = \dots$

$x_1 = x$
$x_2 = \dot{x}$
x_3
$\vdots \qquad \vdots$
$x_n = x^{(n-1)}$

$\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \dfrac{d}{dt} \Rightarrow \dot{x} = Ax + Bu$

$\qquad y = Cx + Du$



S. S. → T.F.

$$G(s) = C\cdot(sI - A)^{-1}\cdot B + D$$

matrix.

$$|sI - A| = 0$$

CTRB	OBSV
$u \to x$	$y \to x$
control inf.	obs understand

$\dot{x} = Ax + Bu$, $u = 0$, $x = e\, e^{\lambda t}$

eigen vector.

C.E $|A - rI| = 0$

$\Downarrow$

$r = \ldots$

$(A - rI)\cdot e = 0$

$n = 2$

- $r_1 \neq r_2$ $\Rightarrow$ e_1, e_2 L.I. $\left\{ \begin{array}{l} x_1 = e_1 e^{r_1 t} \\ x_2 = e_2 e^{r_2 t} \end{array} \right.$

- $r_1 = r_2 = r$ $\Rightarrow$ $e \rightarrow b$ $\left\{ \begin{array}{l} x_1 = e\, e^{rt} \\ x_2 = (et + b)\, e^{rt} \end{array} \right.$

- $r_1 = a + bi$ $\Rightarrow e$, $r_2 = \bar{r}_1$ $\Rightarrow \bar{e}$ $\left\{ \begin{array}{l} x_1 = e_1 e^{r_1 t} \\ x_2 = e_2 e^{r_2 t} \\ \text{or } x_1 = \mathrm{Re}(e\, e^{\lambda t}) \\ \quad x_2 = \mathrm{Im}(e\, e^{\lambda t}) \end{array} \right.$

$x = c_1 x_1 + c_2 x_2$

$x_0 = \ldots$

$c_1 = \ldots$

$c_2 = \ldots$

$x = c_1 x_1 + c_2 X \Longrightarrow x(t) = X(t)\cdot C_i$

$\Downarrow$ FSM

$X = [x_1 \;\; x_2]$, $C_i = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$x = \underbrace{X(t)\cdot X^{-1}(0)}_{\text{STM}}\cdot x_0$

$A = \text{const.}$

$$e^{At} = I + At + \frac{(At)^2}{2!} + \ldots$$

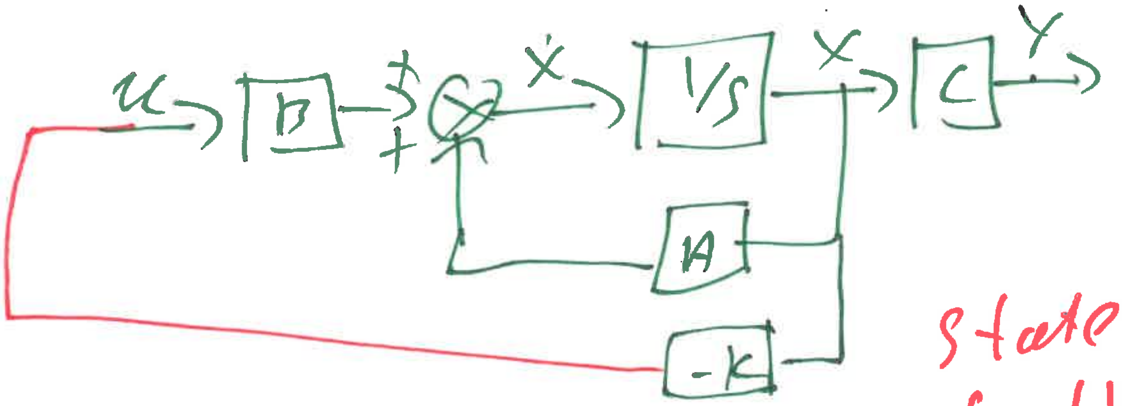
$n=2 \quad \lambda_1 \neq \lambda_2$

$$e^{At} = T \cdot e^{\Lambda t} \cdot T^{-1}$$

eigenmatrix

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$$



state feedback

$\dot{x} = Ax + B \cdot u$, $u = -K \cdot x$

$\dot{x} = Ax + B(-Kx)$

$= (A - BK) \cdot x$

$\dot{x} = A_{CL} \cdot x$, $A_{CL} = A - BK$

$K = ?$: $eig(A_{CL}) = \_\_\_ \quad \_\_\_$

Target specific eigs

pole placement

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 161

1) stable?

2) If <u>NOT</u>, $K = ?$ at $-10, -11$

1) $|A - \lambda I| = 0 \Rightarrow$

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0 \Rightarrow \dots \quad \lambda \begin{cases} 5.37 \\ -0.37 \end{cases} \text{saddle}$$

$A - \cancel{BK} \cancel{=} \dots$

$$M_c = [B \quad A \cdot B]$$

$$= \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right]$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}, \quad |M_c| = 3 \neq 0$$

$$A - BK = A_{CL}$$

A: 2×2, B: 2×1, K: 1×2

$K = [K_1 \quad K_2]$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot [K_1 \quad K_2] \rightarrow \begin{bmatrix} K_1 & K_2 \\ 0 & 0 \end{bmatrix}$$

$$A_{CL} = \begin{bmatrix} 1-K_1 & 2-K_2 \\ 3 & 4 \end{bmatrix}$$

$$|A_{CL} - \lambda I| = 0$$

$$\begin{vmatrix} 1-K_1-\lambda & 2-K_2 \\ 3 & 4-\lambda \end{vmatrix} = 0 \Rightarrow \dots$$

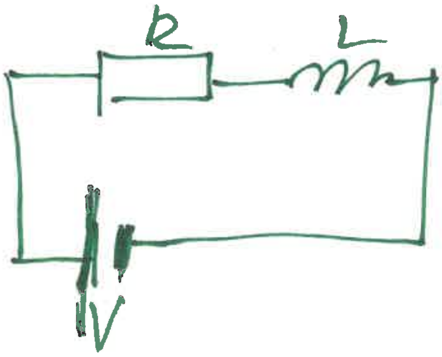
$$(1-K_1-\lambda)(4-\lambda) - 3\cdot(2-K_2) = 0 = \dots$$

- $\lambda = -10: -14\boxed{K_1} + 3 K_2 + 148 = 0$ crosscheck
- $\lambda = -11: -15\boxed{K_1} + 3 K_2 + 174 = 0$

$\Big\} \Rightarrow \dots$ $K_1 = 26$, $K_2 = 75$ $\quad K = [26 \quad 75]$

$$A - BK = \begin{bmatrix} -25 & -70 \\ 3 & 4 \end{bmatrix}$$

$$|A - BK - \lambda I| = 0 \Rightarrow \dots \quad \lambda_1 = -10, \ \lambda_2 = -11$$



$$\frac{di}{dt} = \frac{1}{L}(V - iR)$$

$$i_{ss} = \frac{V}{R}$$

$x = i$, $u = V$, $A = -R/L$, $B = 1/L$

$|A - \lambda I| = 0$

$|-R/L - \lambda \cdot 1| = 0 \Leftrightarrow -\frac{R}{L} - \lambda = 0$

$e^{-R/L\, t}$ $\lambda = -R/L$ O.L. eigenvalue

$u = ?$: $i(0) \neq 0$ $i \to 0$ very fast

$u = -K \cdot x = -K \cdot i$

$A_{CL} = A - BK = -\frac{R}{L} - \frac{1}{L} \cdot K$

eigs of A_{CL} $\lambda = -\frac{R}{L} - \frac{1}{L}K$ C.L. eigenvalue

desired C.L. eigen $= -6\frac{R}{L}$

$\cancel{-}\frac{R+K}{\cancel{L}} = \cancel{-}6\frac{R}{\cancel{L}} \Rightarrow K = 5 \cdot R$

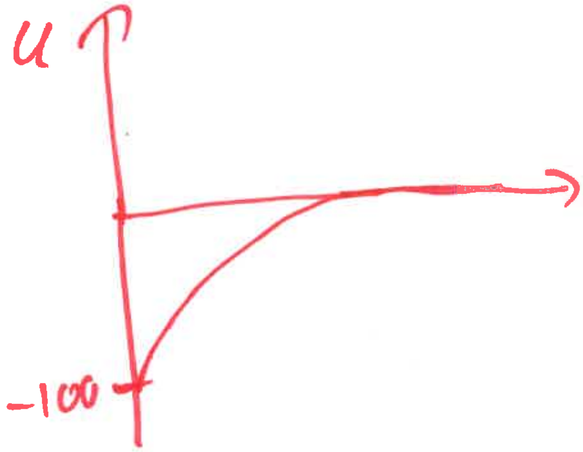
$u = -K\cdot x = -K\cdot i = -5\cdot R\cdot i$

assume $i(0) = 10$ ~~ohms~~ A

$R = 2$ ohms.

$u(0) = -5\cdot R\cdot i(0)$

$= -5\cdot 2\cdot 10 = -100v. = V(0)$



$e^{-\frac{R}{L}t}$

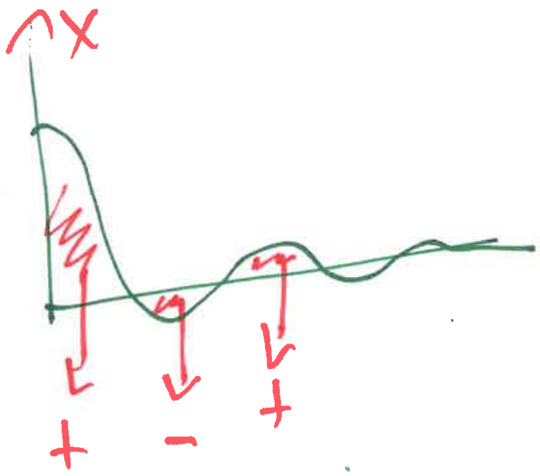
$e^{-\frac{6R}{L}t}$

$I_{x_1} = \int e^{-\frac{R}{L}t}\, dA$

$\int e^{-\frac{R}{L}\cdot 6t}\, dt = I_{x_2}$

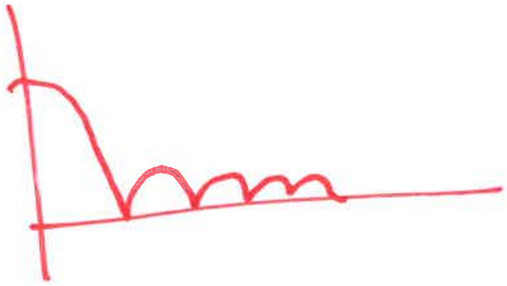
$I_{x_1} = \dots = \frac{L}{R}$

$I_{x_2} = \frac{L}{R}\frac{1}{6}$



$$I_x = \int_0^\infty x(t)\, dt$$

$$I_x = \int_0^\infty |x(t)|\, dt.$$



$$I_x = \int_0^\infty x^2(t)\, dt.$$

I want $-6000 \frac{R}{L}$ = des. c.L. eigs.

$$\frac{-R+K}{L} = -6000 \frac{R}{L} \Rightarrow K = 5999R$$

$$u = -5\cancel{000}999 \cdot R \cdot i$$

at $t=0$ $\quad u(0) = -5999 \cdot 2 \cdot 10 \simeq 120kW$

spend too much energy

$x \Downarrow \quad u = low$

(106)

$$I_{x_1} = \int_0^\infty 10 x_1^2(t)\, dt$$

$$I_u = \int_0^\infty u^2(t)\, dt$$

$$I_{x_2} = \int x_2^2(t)\, dt$$

$$I_{x_3} = \int 5 x_3^2(t)\, dt$$

$\Rightarrow$

$$I = \int \left(10 x_1^2 + x_2^2 + 5 x_3^2 + u^2\right) dt$$

$$= \int \left(X^T \cdot Q X + u^2\right) dt$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_4 \end{bmatrix} \qquad Q = \begin{bmatrix} 10 & 0 & 0 \cdots \\ 0 & 1 & 0 \cdots \\ 0 & 0 & 5 \cdots \\ \cdots & & \end{bmatrix}$$

$$I = \int \left(X^T \cdot Q \cdot X + U^T \cdot R \cdot U\right) dt$$

$K = ?$: $I =$ low.

$K = ?$ given R, Q : $J \to$ low.

$$A^T \cdot P + PA - PBR^{-1}B^T \cdot P + Q = 0 \Rightarrow P = ..$$

$$\boxed{K = -R^{-1} \cdot B P}$$

LQR

$$J = \int (X^T \cdot Q X + U^T \cdot R U) dx$$

Exam Q.1: Given $P = \ldots$ find $K = ..$

$R = 1$	$R = 1$	$R = 10$
$Q = 10$	$Q = 1$	$Q = 1$

$Q \to$ Importance of speed

$R \to$ Importance of energy