State Space Analysis and Controller Design

EEE3001 8013
Tutorial Exercise IV Solutions

1. A system is described by $A=\begin{bmatrix}2 & 2\\ 2 & 5\end{bmatrix}, B=\begin{bmatrix}1\\ 1\end{bmatrix}, C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

   - Is that system stable? [NO, eigenvalues 1,6]
   - Check the system controllability and observability. [Controllable and Observable]
   - Stabilise the system using a pole placement controller. Assume that the poles of the closed loop system are -2,-4. $K=\begin{bmatrix}-0.667 & 13.667\end{bmatrix}$

2. Repeat question1 for $A=\begin{bmatrix}-2 & 2\\ 2 & 5\end{bmatrix}, B=\begin{bmatrix}1\\ 1\end{bmatrix}, C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$.

   - [Unstable, eigenvalues -2.5,5.53]
   - Controllable and Observable
   - To place the poles at -2,-4 $K=\begin{bmatrix}2 & 7\end{bmatrix}$

3. Design a closed loop estimator for the following state space system described by:

   $A=\begin{bmatrix}-2 & 2\\ 2 & 5\end{bmatrix}, B=\begin{bmatrix}1\\ 1\end{bmatrix}, C=\begin{bmatrix}2 & 0\end{bmatrix}$. Assume that the poles of the closed loop estimator are placed at -10,-15. [$G=\begin{bmatrix}14\\ 76\end{bmatrix}$]

4. Repeat question 3 for $A=\begin{bmatrix}-2 & 2\\ 2 & 5\end{bmatrix}, B=\begin{bmatrix}1\\ 1\end{bmatrix}, C=\begin{bmatrix}2 & 4\end{bmatrix}$.

   To locate the poles at -10,-15 $G=\begin{bmatrix}-10\\ 12\end{bmatrix}$

5. A state system is described by: $A=\begin{bmatrix} 0 & 1 \\ -5 & 2 \end{bmatrix}, B=\begin{bmatrix} 1 \\ 0 \end{bmatrix}, C=\begin{bmatrix} 1 & 4 \end{bmatrix}$

- Is that system stable? [NO, eigenvalues $1\pm j2$]
- Check the system controllability and observability. [Controllable and observable]
- Design a closed loop estimator for this system assuming its poles located at -40,-50. [$G=\begin{bmatrix} -88.62 \\ 45.15 \end{bmatrix}$]
- Create a state feedback controller that will stabilise the system. Suggest a proper location for the closed loop poles of the control system.

  To place the poles at -4,-5 $K=\begin{bmatrix} 11 & -7.4 \end{bmatrix}$
- Write down the characteristic equation of the overall system including the controller and the estimator.