Chapter #1

EEE8013

Linear Controller Design and State Space Analysis

Using Matlab to Solve ODE

1. First order ODEs

Introduction: In this part of the module, we will extensively use Matlab/Simulink to enhance the material that we covered during the normal lectures. As this part of the module is addressed only to MSc/MEng students, it is anticipated that you will be prepared, you will have studied and reproduced this material before you come to the lab, and hence these sessions will only address problems that you faced, and cover extra material/examples. I want to emphasize that these sessions in the computing lab are <u>NOT</u> normal lectures, but are here only to help with your self-driven studies.

In the first chapter, we will see how we can find the numerical solution of ODEs and how to simulate analytical solutions of ODEs. <u>It is crucial to understand the difference between numerical solution and simulation of the analytical solution.</u> Obviously both solutions must give you the same result.

Example:

$$\frac{dx}{dt} = -\frac{6}{5}x$$

Hence $k = -\frac{6}{5}$ and u = 0

Since *k* is negative the system is *stable* and the solution will converge to zero (unforced system) starting from the initial condition x(0) = 1. The **analytical solution** can be written as: $x(t) = e^{kt}x(0) = e^{\frac{-6}{5}t}$



Now, we will numerically solve the given ODE:

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$$5\frac{dx}{dt} + 6x = 0, \qquad \qquad x(0) = 1$$

This ODE can be written as:

$$5\frac{dx}{dt} = -6x + 0$$



Remember to include the initial condition x(0) = 1 by double clicking on the integrator block and put 1 in "Initial condition":

Function Block Parameters: Integrator		
Integrator		
Continuous-time integration of the input signal.		
Parameters		
External reset: none 🔹		
Initial condition source: internal 🔹		
Initial condition:		
1		
🖂 Limit output		
Upper saturation limit:		
inf		
Lower saturation limit:		
-inf		
Show saturation port		
E Show state port		
Absolute tolerance:		
auto		
Ignore limit and reset when linearizing		
Enable zero-crossing detection		
State Name: (e.g., 'position')		
•		
OK Cancel Help Apply		

To see the output x(t) double click on Scope:



Now you need to check that all scopes are showing the same result for x(t). If I want to plot x(t) versus time in Matlab then select the Scope's

parameters:

	Scope
	1 0.8 0.6 0.4 0.2 0 0 2 4 6 8 10 Time offset: 0
Select "History":	
	Scope parameters
	General History Style
	✓ Limit data points to last: 5000
	Save data to workspace Variable name: ScopeData Format: Array
	OK Cancel Help Apply

And give the name that you want:

承 'Scope' para	meters X			
General History Style				
✓ Limit data points to last: 5000				
Save data to workspace				
Variable name:	x			
Format:	Array			
	OK Cancel Help Apply			

Now if you check, the workspace has a variable called "x" and has 2 columns. The first column is the time and the second your signal. Hence you can plot your simulink response by typing:

plot(x(:,1),x(:,2))

Example: Simulate both analytical and numerical solutions for the following ODE:

$$5\frac{dx}{dt} - 6x = 0, \qquad x(0) = 1$$

Rea-arranging the DE to put it on the general form $\frac{dx}{dt} = ax(t) + bu(t)$

$$\frac{dx}{dt} = \frac{6}{5}x$$

Hence $a = \frac{6}{5}$ and u = 0

Since *a* is positive the system is *unstable* and the solution will converge to infinity (unforced system) starting from the initial condition x(0) = 1. The solution can be written as: $x(t) = e^{at}x(0) = e^{\frac{6}{5}t}$



Analytical Solution







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Example: $5\frac{dx}{dt} + 6x = 1$, x(0) = 1k=6/5, u=1/5

As described in the lecture notes:

$$x(t) = e^{\frac{-6}{5}t} + \frac{1}{5}e^{\frac{-6}{5}t} \int_0^t e^{\frac{6}{5}\tau} d\tau = \frac{4}{5}e^{\frac{-6}{5}t} + \frac{1}{5}$$



Exercise: Repeat the previous steps for the system described by:

$$5\frac{dx}{dt} + 6x = 15,$$
 $x(0) = 1$

Exercise:
$$5\frac{dx}{dt} - 6x = 1$$
, $x(0) = 1$

Example: x'' = -4x' - 3x

Again don't forget to include the initial conditions!!

Numerical Solution



Analytical Solution





Example: x'' = -2x' - x

Numerical Solution



Analytical Solution





Example: x''+x'+x=0

A=1, B=1, x(0)=1, $x'(0)=0 \Rightarrow c_1=1$, $c_2=1/\sqrt{3}$

$$r = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

The solution can be written as:

$$x = e^{at} \left(c_1 \cos(bt) + c_2 \sin(bt) \right) = e^{-\frac{1}{2}t} \left(\cos(\frac{\sqrt{3}}{2}t) + \frac{1}{\sqrt{3}} \sin(\frac{\sqrt{3}}{2}t) \right)$$

If you select a part of your Simulink model, you can create a subsystem:



And hence in that case you can have:



With





You can also use these 2 blocks:

🖬 🖬 Simulink Library Browser					
💠 🖒 Enter search term 🕞 🗞 👻 🔯	▼ 🔄 🕂 🕐				
Simulink/Signal Routing					
 Simulink Commonly Used Blocks Continuous Dashboard Discrete Logic and Bit Operations Lookup Tables Math Operations Model Verification Model Verification Model Verification Model-Wide Utilities Ports & Subsystems Signal Attributes Signal Routing Sinks Sources User-Defined Functions Additional Math & Discrete Communications System Toolbox Computer Vision System Toolbox Computer Vision System Toolbox DSP System Toolbox DSP System Toolbox HDL Coder Image Acquisition Toolbox Neural Network Toolbox Neural Network Toolbox Neural Network Toolbox SimEvents SimEvents	Bus Bus Assignment Data Store Data Store Demux Merge Manual Switch Merge Selector Switch Merge Selector Switch	A Data Store Memory Sim Out Coder Environment Controller Index Vector Mux			

