Chapter #1

EEE8013

Linear Controller Design and State Space Analysis

Using Matlab to Solve ODE

1. First order ODEs

Introduction: In this part of the module, we will extensively use Matlab/Simulink to enhance the material that we covered during the normal lectures. As this part of the module is addressed only to MSc/MEng students, it is anticipated that you will be prepared, you will have studied and reproduced this material before you come to the lab, and hence these sessions will only address problems that you faced, and cover extra material/examples. I want to emphasize that these sessions in the computing lab are **NOT** normal lectures, but are here only to help with your self-driven studies.

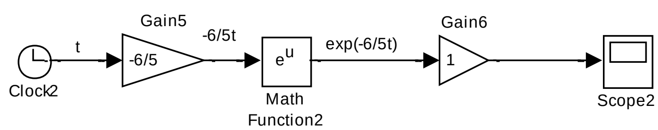
In the first chapter, we will see how we can find the numerical solution of ODEs and how to simulate analytical solutions of ODEs. It is crucial to understand the difference between numerical solution and simulation of the analytical solution. Obviously both solutions must give you the same result.

Example:

$$\frac{dx}{dt} = -\frac{6}{5}x$$

Hence $k = -\frac{6}{5}$ and $u = 0$

Since k is negative the system is ***stable*** and the solution will converge to zero (unforced system) starting from the initial condition $x(0) = 1$. The **analytical solution** can be written as: $x(t) = e^{kt}x(0) = e^{\frac{-6}{5}t}$

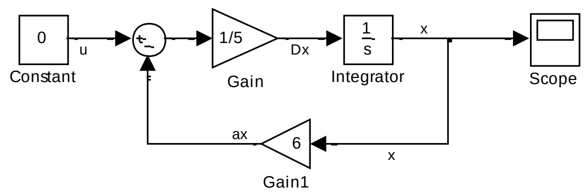


Now, we will numerically solve the given ODE:

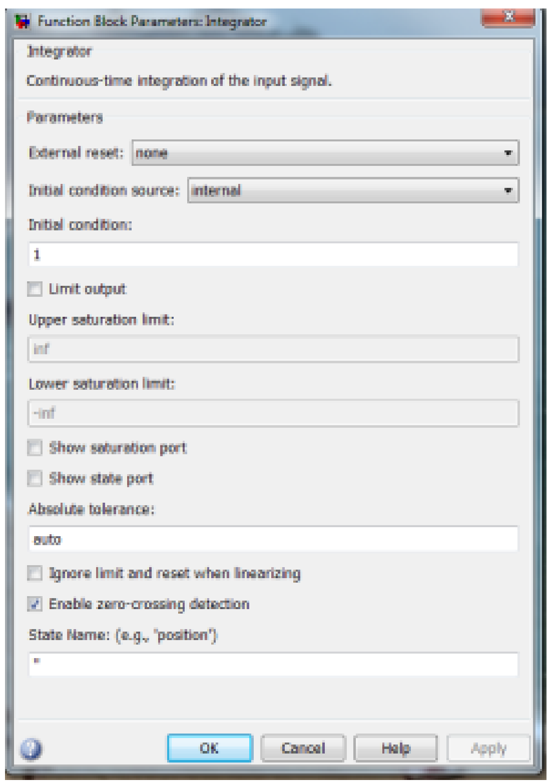
$$5\frac{dx}{dt} + 6x = 0, \qquad x(0) = 1$$

This ODE can be written as:

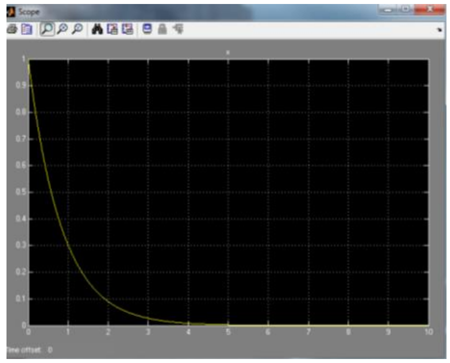
$$5\frac{dx}{dt} = -6x + 0$$



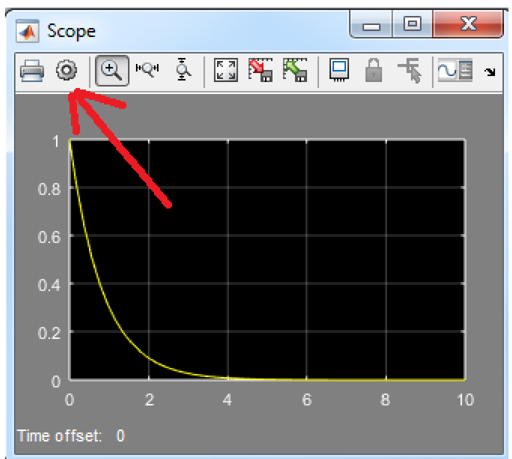
Remember to include the initial condition $x(0) = 1$ by double clicking on the integrator block and put 1 in "Initial condition":



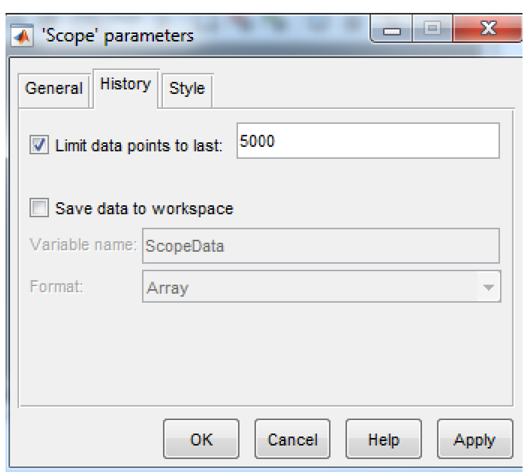
To see the output *x*(*t*) double click on Scope:



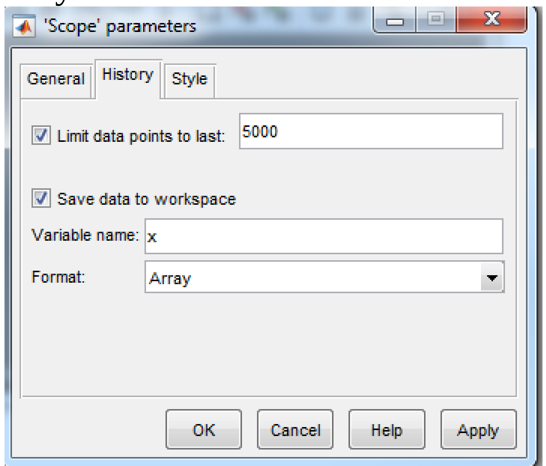
Now you need to check that all scopes are showing the same result for x(t). If I want to plot x(t) versus time in Matlab then select the Scope's parameters:



Select "History":



And give the name that you want:



Now if you check, the workspace has a variable called "*x*" and has 2 columns. The first column is the time and the second your signal. Hence you can plot your simulink response by typing:

plot(x(:,1),x(:,2))

Example: Simulate both analytical and numerical solutions for the following ODE:

$$5\frac{dx}{dt} - 6x = 0, \qquad x(0) = 1$$

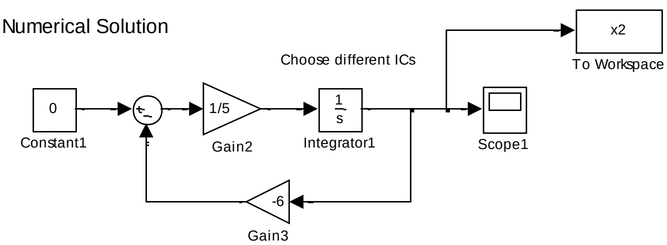
Rea-arranging the DE to put it on the general form $\frac{dx}{dt} = ax(t) + bu(t)$

$$\frac{dx}{dt} = \frac{6}{5}x$$

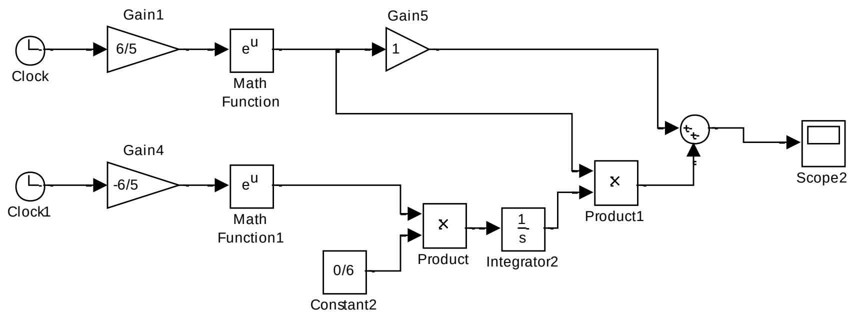
Hence $a = \frac{6}{5}$ and $u = 0$

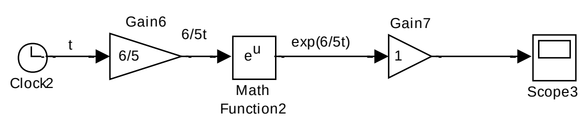
Since a is positive the system is ***unstable*** and the solution will converge to infinity (unforced system) starting from the initial condition $x(0) = 1$. The solution can be written as: $x(t) = e^{at}x(0) = e^{\frac{6}{5}t}$

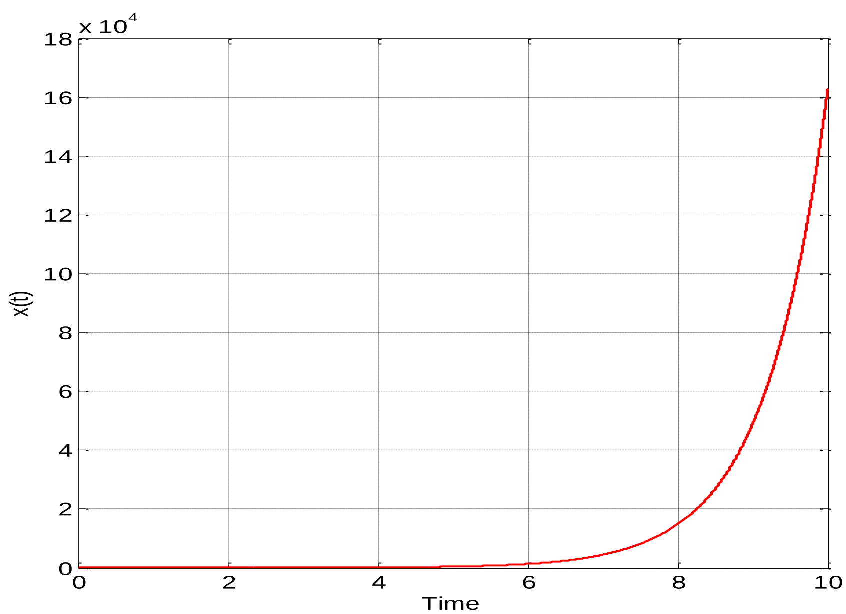
Numerical Solution



Analytical Solution





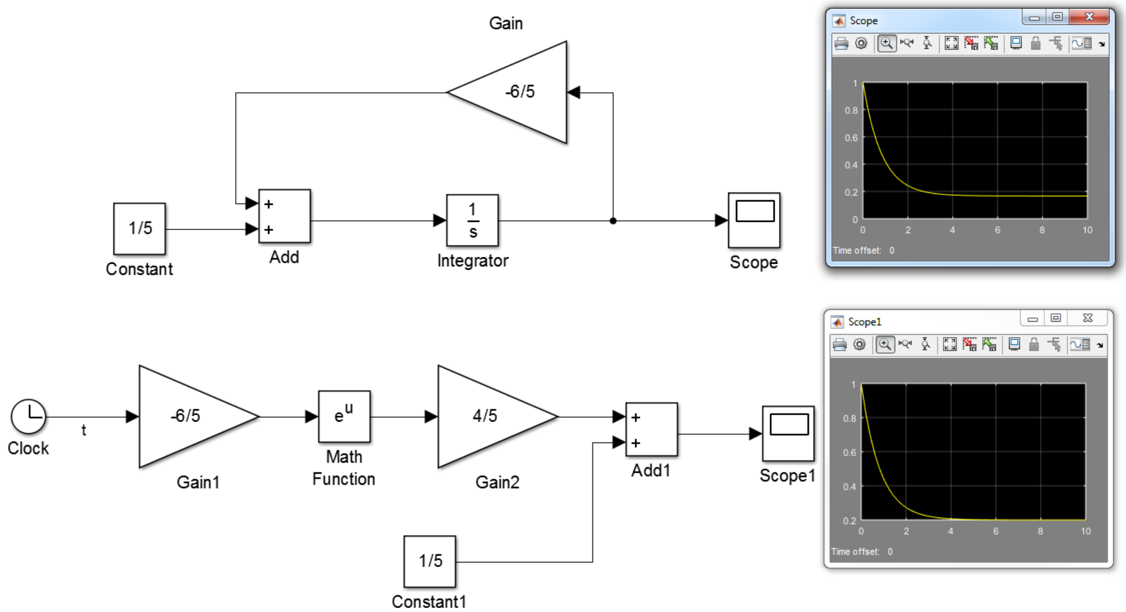


Example: $5\frac{dx}{dt} + 6x = 1, \qquad x(0) = 1$

k=6/5, u=1/5

As described in the lecture notes:

$$x(t) = e^{\frac{-6}{5}t} + \frac{1}{5}e^{\frac{-6}{5}t}\int_0^t e^{\frac{6}{5}\tau}\,d\tau = \frac{4}{5}e^{\frac{-6}{5}t} + 1/5$$



Exercise: Repeat the previous steps for the system described by:

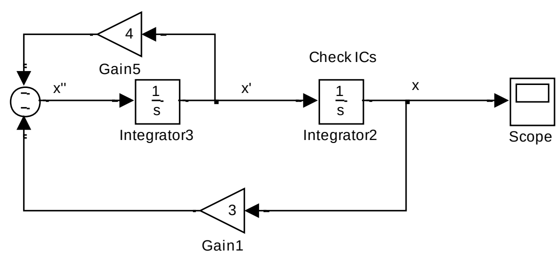
$$5\frac{dx}{dt} + 6x = 15, \qquad x(0) = 1$$

Exercise: $5\frac{dx}{dt} - 6x = 1, \qquad x(0) = 1$

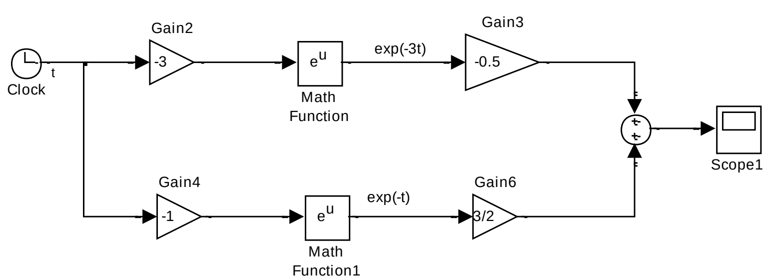
Example: $x'' = -4x' - 3x$

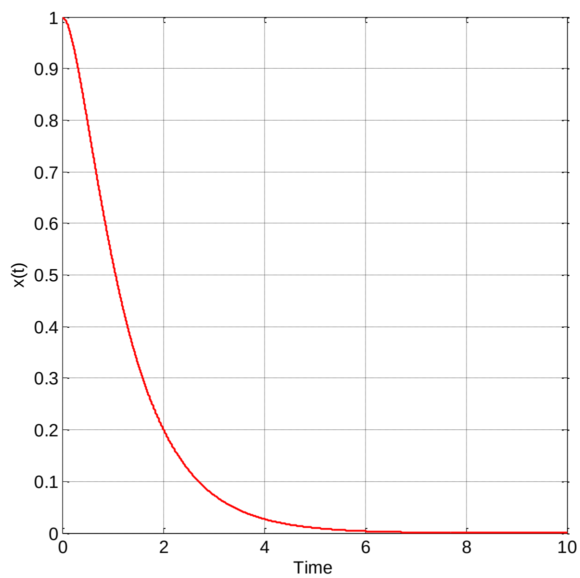
Again don't forget to include the initial conditions!!

Numerical Solution

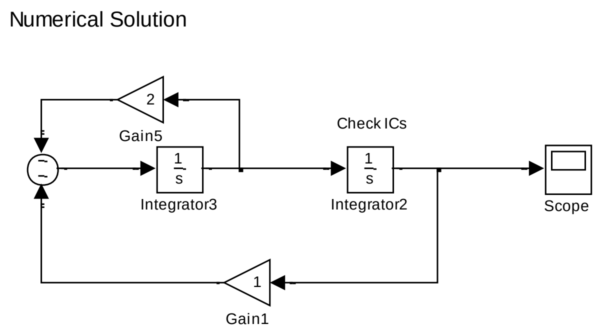


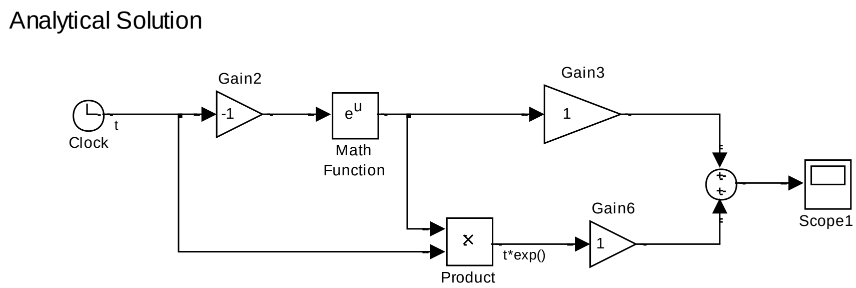
Analytical Solution

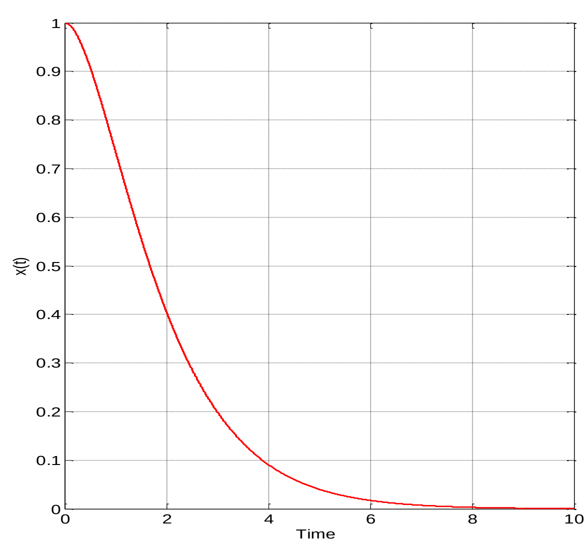




Example: $x'' = -2x' - x$







Example: x’’+x’+x=0

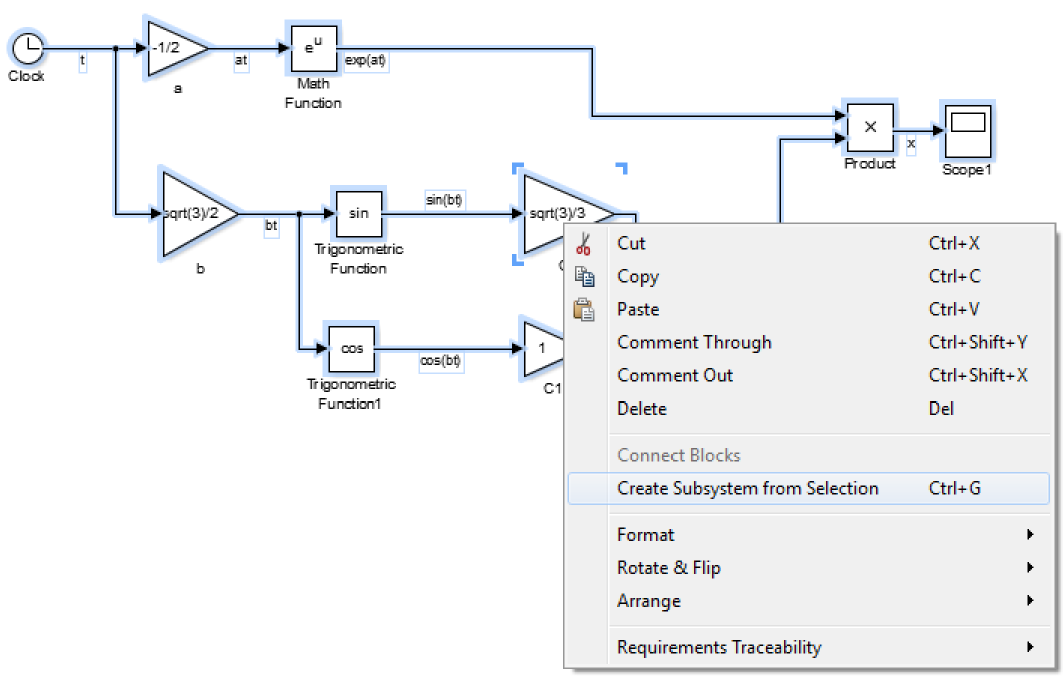
A=1, B=1, $x(0)=1$, $x'(0)=0$ => $c_1=1$, $c_2=1/\sqrt{3}$

$$r=-\frac{1}{2}+j\frac{\sqrt{3}}{2}$$

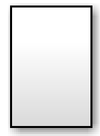
The solution can be written as:

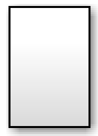
$$x=e^{at}\left(c_1\cos(bt)+c_2\sin(bt)\right)=e^{-\frac{1}{2}t}\left(\cos(\frac{\sqrt{3}}{2}t)+\frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}t)\right)$$

If you select a part of your Simulink model, you can create a subsystem:

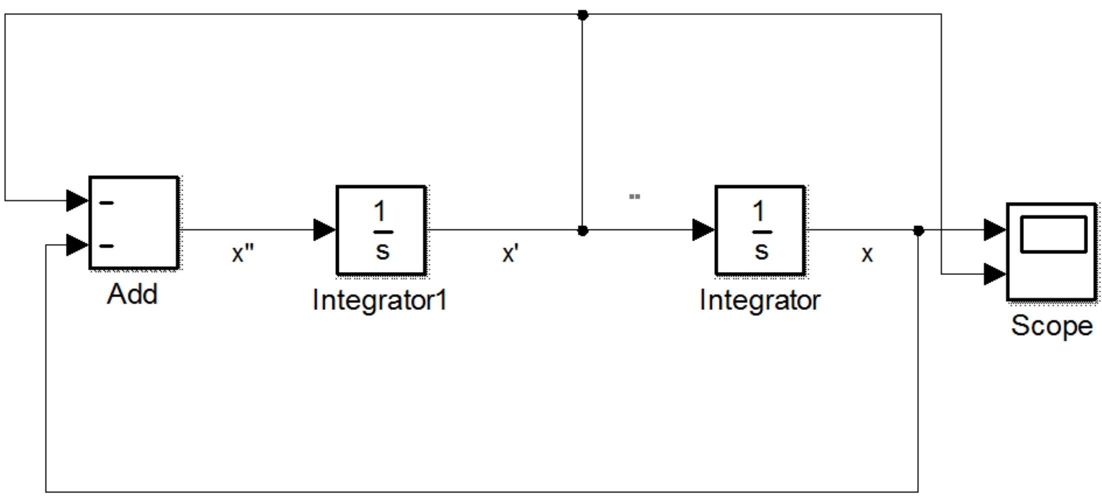


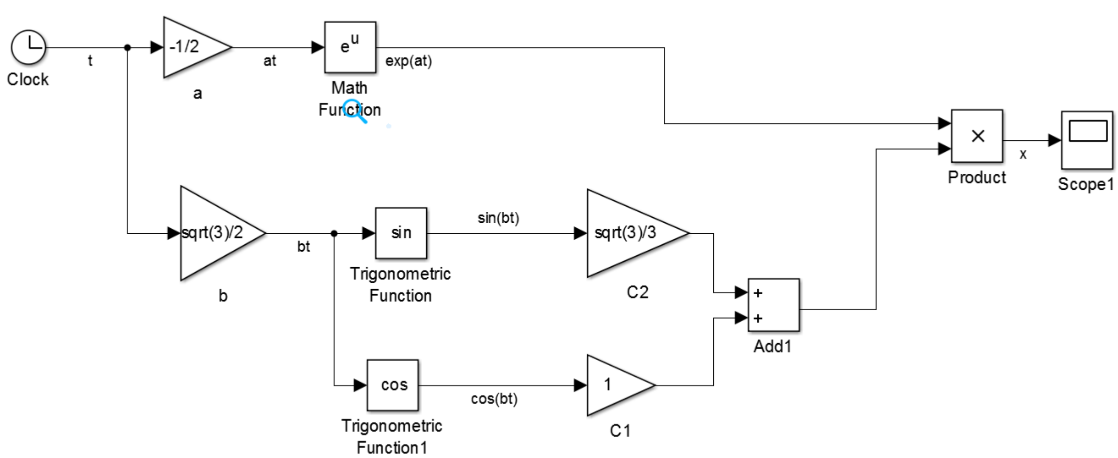
And hence in that case you can have:


Numerical solution ODE

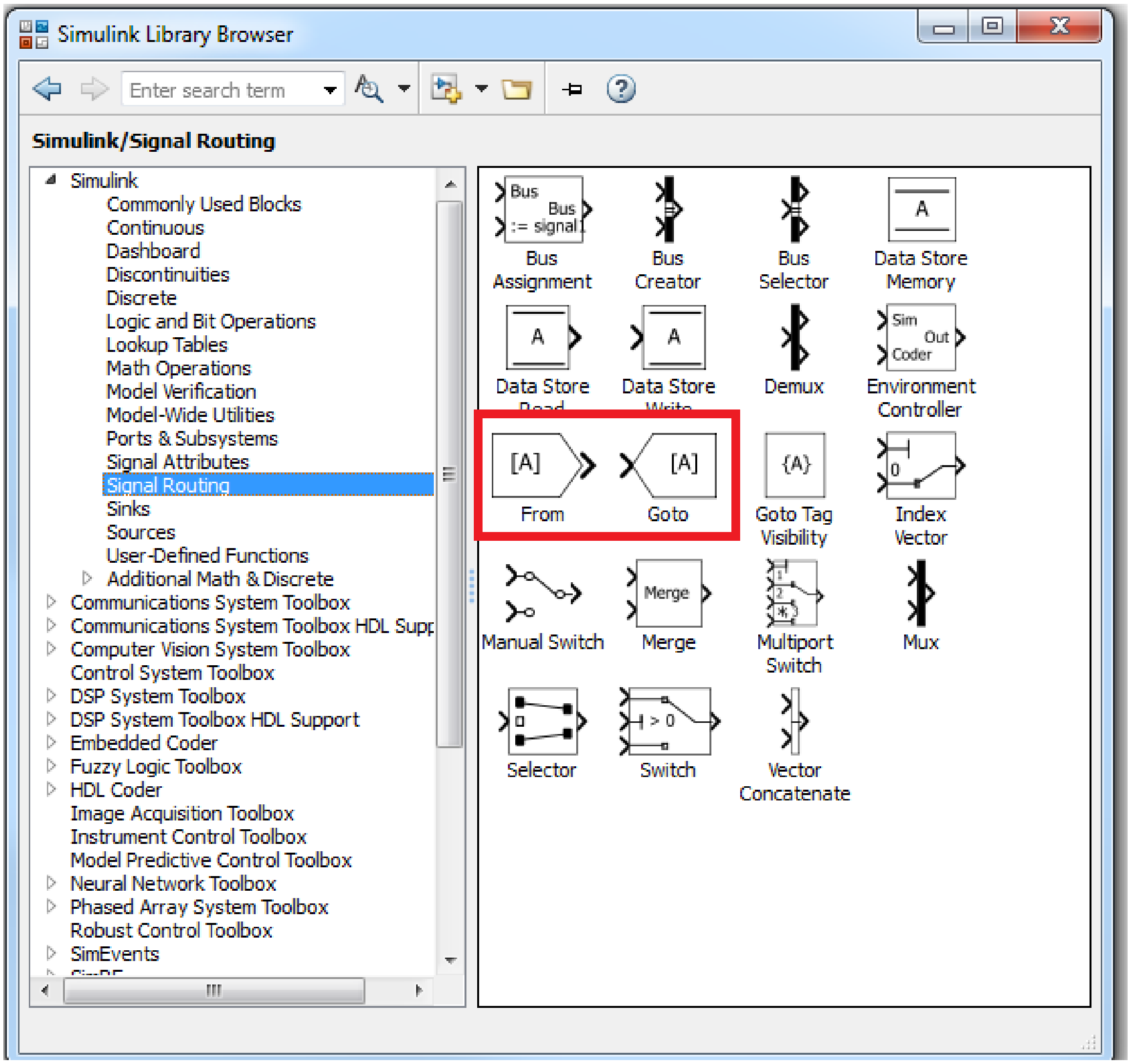

Analytical solution

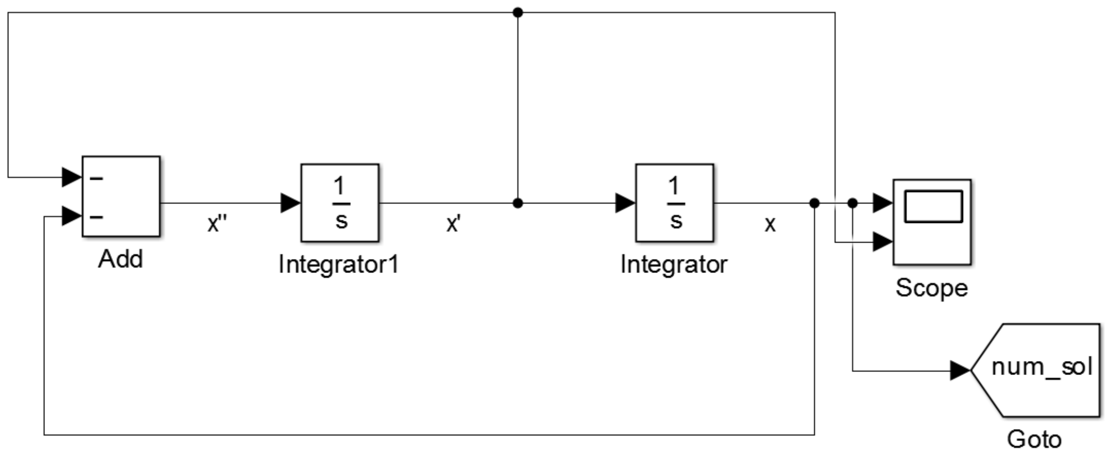
With

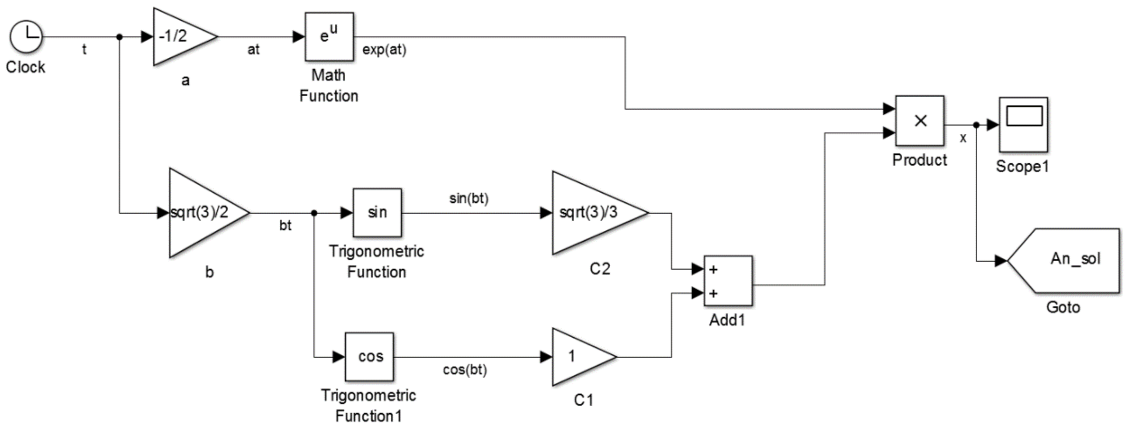

Add
x''
1
s
Integrator1
x'
1
s
Integrator
x
Scope


Clock
t
-1/2
a
at
e^u
Math Function
exp(at)
Product
x
Scope1
sqrt(3)/2
b
bt
sin
Trigonometric Function
sin(bt)
sqrt(3)/3
C2
+
+
Add1
cos
Trigonometric Function1
cos(bt)
1
C1

You can also use these 2 blocks:




Add
x''
1
s
Integrator1
x'
1
s
Integrator
x
Scope
num_sol
Goto


Clock
t
-1/2
a
at
e^u
Math Function
exp(at)
Product
x
Scope1
sqrt(3)/2
b
bt
sin
Trigonometric Function
sin(bt)
sqrt(3)/3
C2
+
+
Add1
An_sol
Goto
cos
Trigonometric Function1
cos(bt)
1
C1


Numerical solution ODE
Analytical solution
[num_sol]
From
[An_sol]
From1
Scope1