$$\dot{x}(t) = x'(t) = \frac{dx(t)}{dt} = f(x(t), t)$$

$$\dot{x} = f(x, t)$$

An. Soln $x(t)$

eg $\dot{x} = -3 \cdot x$

$x_1 = e^{-3t}$ $\quad x_2 = -10 e^{-3t}$

$x_1 = -3 e^{-3t}$ $\quad \dot{x}_2 = 30 e^{-3t}$

$-3 e^{-3t} = -3 \cdot e^{-3t}$ ✓

$30 \cdot e^{-3t} = -3 \cdot (-10 e^{-3t}) = 30 e^{-3t}$ ✓

$x_1, x_2 \rightarrow$ Gen Solns.

I.V.P. $\dot{x} = f(x, t)$ + I.C. $= x(t_0) = x(0) = x_0$

eg $\dot{x} = -3 \cdot x$, $x_0 = 1$

$x_1(0) = e^{-3 \cdot 0} = e^0 = 1$

$x_2(0) = -10 e^{-3 \cdot 0} = -10 \neq 1$

$\boxed{\varphi(t, t_0, x_0)}$

Linear 1st ODE

$$a(t)\cdot\dot{x} + b(t)\cdot x = c(t)$$

$$a\cdot\dot{x} + b\cdot x = c(t)$$

$$\boxed{\dot{x} + k\cdot x = u(t)}$$

Int. Factor

$$x(t) = e^{-kt}\cdot x_0 + e^{-kt}\int_0^t e^{kt_1}u(t_1)\,dt_1$$

- if u = const.

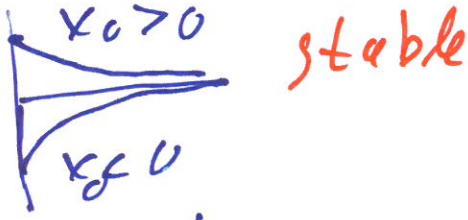
$$x(t) = \underbrace{e^{-kt}\cdot x_0}_{\text{trans}} + \underbrace{\frac{u}{k}\left(1-e^{kt}\right)}_{\text{input}}$$

$\dot{x} + kx = u$

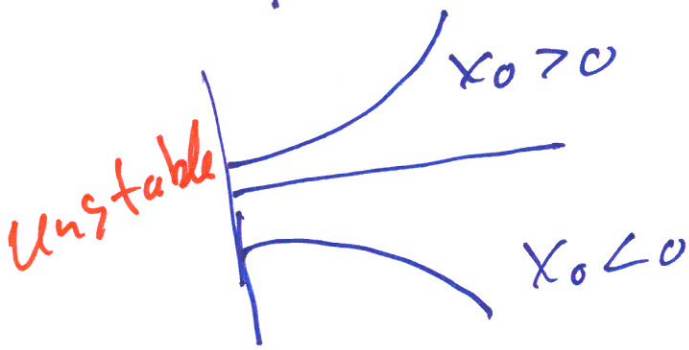
$x = e^{-kt} \cdot x_0 + \frac{u}{k}(1 - e^{-kt})$

- $u = 0 \qquad x = e^{-kt} \cdot x_0$

$k > 0$



$k < 0$



eg. $k > 0$ • $k_1 = 10$

$k_2 = 100$

---

eg.

$\dot{x} + 5x = 3$

$$\ddot{x} = f(\dot{x}, x, t).$$

↓ Linear

$$\ddot{x} + A\dot{x} + Bx = 0$$

$$\ddot{x} - 2\dot{x} - 3x = 0 \qquad x_1 = e^{3t} \qquad x_2 = e^{-t}$$

$$\downarrow$$

$$\dot{x}_1 = 3e^{3t}$$

$$\downarrow$$

$$\ddot{x}_1 = 9e^{3t}$$

$$9e^{3t} - 2(3e^{3t}) - 3e^{3t} = 0$$

$$0 = 0$$

$$\ddot{x} + A\dot{x} + Bx = 0 \qquad x_1 = \text{soln}$$

$$x_2 = \text{soln.}$$

is (L.C.) x_1, x_2 also a soln.

↳ $c_1 \cdot x_1 + c_2 \cdot x_2 = ?$

$$\ddot{x}_1 + A\dot{x}_1 + Bx_1 = 0$$

$$\ddot{x}_2 + A\dot{x}_2 + Bx_2 = 0$$

$$c_1\ddot{x}_1 + c_1 A\dot{x}_1 + c_1 Bx_1 = 0$$

$$c_2\ddot{x}_2 + c_2 A\dot{x}_2 + c_2 Bx_2 = 0$$

$+$

$$c_1\ddot{x}_1 + c_1 A\dot{x}_1 + c_1 Bx_1 = 0$$

$$+ c_2\ddot{x}_2 + c_2 A\dot{x}_2 + c_2 Bx_2 = 0$$

$$(c_1x_1 + c_2x_2)'' + A(c_1x_1 + c_2x_2)' + B(c_1x_1 + c_2x_2) = 0$$

$c_1x_1 + c_2x_2 = \varphi$

$$\varphi'' + A\cdot\varphi' + B\cdot\varphi = 0$$

$$\square'' + A\cdot\square' + B\square = 0$$

φ = soln

$= c_1\cdot x_1 + c_2 x_2$

If x_1, x_2 ~~is~~ are soln's $\Rightarrow c_1x_1 + c_2x_2$ soln

eg 18. $\ddot{x} - 2\dot{x} - 3x = 0$ (6)

$x_1 = e^{3t}$, $x_2 = e^{-t}$ are solns

prove $x = e^{3t} + 2e^{-t}$ is also a soln

↓

I am two

$(e^{3t} + 2e^{-t})''$ $-2\cdot(e^{3t} + 2e^{-t})'$

$-3\cdot(e^{3t} + 2e^{-t}) = 0$

↓

$9e^{3t} + 2e^{-t} - 2(3e^{3t} - 2e^{-t})$

$-3(e^{3t} + 2e^{-t}) = 0$

$0\cdot e^{3t} + 0\, e^{-t} = 0$

$\ddot{x} + A\dot{x} + Bx = 0$ $\quad x_1 \quad x_2$

ARE SOLNS

Any other soln $\varphi = c_1 x_1 + c_2 x_2$

$\dot{\varphi} = c_1 \dot{x}_1 + c_2 \dot{x}_2$

$$\begin{bmatrix} x_1 & x_2 \\ \dot{x}_1 & \dot{x}_2 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \varphi \\ \dot{\varphi} \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ \dot{x}_1 & x_2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \varphi \\ \dot{\varphi} \end{bmatrix}$$

if $\det(A) = 0$ A^{-1} ~~DNE~~

$$\begin{vmatrix} x_1 & x_2 \\ \dot{x}_1 & \dot{x}_2 \end{vmatrix} \neq 0$$

$$\begin{bmatrix} x_1 & x_2 \\ \dot{x}_1 & \dot{x}_2 \end{bmatrix} = \text{Wronskian of } x_1, x_2$$

$$\begin{bmatrix} x_1 & x_2 \\ \dot{x}_1 & \dot{x}_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} \neq P \cdot \begin{bmatrix} x_2 \\ \dot{x}_2 \end{bmatrix} \Rightarrow \text{I can find } c_1, c_2$$

e.g. $x'' - 2x' - 3x = 0$

$x_1 = e^{3t}$, $x_2 = e^{-t}$ 2 soln

$\varphi_1 = e^{3t} + 2e^{-t}$ is also a soln

$\varphi_2 = 10 \cdot e^{3t} + 5e^{-t}$

↳ also a soln.

$c_1 x_1 + c_2 x_2$

$c_1 = 10$

$c_2 = 5$

φ_2 can be written as a L.C. of x_1, x_2

or I can find c_1, c_2 :

$\varphi_2 = c_1 \cdot x_1 + c_2 \cdot x_2$

$x'' - 2x' - 3x = 0$

$x_1 = e^{3t}$ $\quad x_2 = 3 \cdot e^{3t}$

$x = e^{3t} + 2e^{-t}$ is also a soln.

$x_2 = 3 \cdot x_1$

$\dot{x} + kx = 0 \qquad x = e^{-kt} x_0.$

$\ddot{x} + A\dot{x} + Bx = 0 \qquad x = ?$

I hope

$x = e^{rt}$

$\dot{x} = r e^{rt}$

$\ddot{x} = r^2 e^{rt}$

$r^2 e^{rt} + A \cdot r e^{rt} + B e^{rt} = 0$

$r^2 + A \cdot r + B = 0$

$r_{1,2} = \frac{-A \pm \sqrt{?}}{2}$