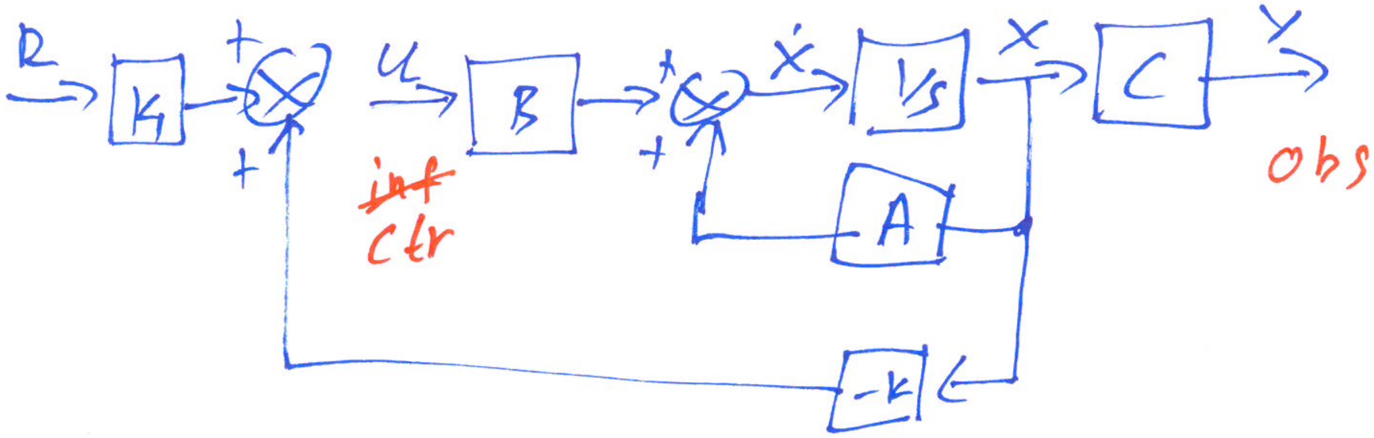
Revision



$\dot{x} = Ax + B\cdot u$

$\Downarrow$

O.L.S.M.

$u = R\cdot K_1 - K\cdot x$

- $R = 0$ Regulator $u = -K\cdot x$

$\dot{x} = \underbrace{(A - BK)}_{\Downarrow} \cdot x$

C. L. S. M.

K: → stable

→ Fast

$\boxed{\text{CTRB}}$ Do not forget me

$K \uparrow$ $\to$ more eigs $\to -\infty$ (faster)

$\to u = -K \cdot x \uparrow$

pole placement $\to$ L.Q.R.

$$I = \int (X^T \cdot Q \cdot X + U^T \cdot R U) \, dA.$$

$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$

R.R.E.

$P = \ldots\ldots \qquad K = R^{-1} \cdot B^T P$

$K_1 = ?$

- SISO, $B \in \mathbb{R}^{n \times 1}$, $u, y \in \mathbb{R}$
- $R = \text{const.} = r_{ss}$ ~~r_{ss}~~

given

$y \to y_{ss} = r_{ss}$ des out

$x \to x_{ss}$ des state

$u \to u_{ss}$ des ctr signal

NOT KNOWN.

$$\boxed{u = K_1 \cdot r_{ss} - K \cdot x}$$ I have

$$\Big\downarrow K_1$$

$u \to u_{ss}$

$$\boxed{u = u_{ss} - K(x - x_{ss})}$$ I want

$K_1 = ?$: $K_1 r_{ss} - Kx \to u_{ss} - K(x - x_{ss})$

u_{ss}, x_{ss}

NOT KNOWN

$\dot{x} = Ax + Bu.$

$y = C \cdot x$

$x \to x_{ss}$

$\dot{x}_{ss} = A \cdot x_{ss} + B \cdot u_{ss}$ $\qquad \dot{x}_{ss} = 0$

$y_{ss} = C \cdot x_{ss}$

$0 = A \cdot x_{ss} + B \cdot u_{ss}$

$y_{ss} = C \cdot x_{ss} = r_{ss}$

$$\underset{n\times 1}{x_{ss}} = \underset{n\times 1}{N_x} \cdot \underset{1\times 1}{r_{ss}} \qquad \underset{1\times 1}{u_{ss}} = \underset{1\times 1}{N_u} \cdot \underset{1\times 1}{r_{ss}}$$

$N_x = ?$ $\qquad N_u = ?$

$$\left.\begin{aligned} 0 &= A \cdot N_x \cdot \cancel{r_{ss}} + B\, N_u \cdot \cancel{r_{ss}} \\ \cancel{r_{ss}} &= C \cdot N_x \cdot \cancel{r_{ss}} \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} 0 &= A \cdot N_x + B \cdot N_u \\ 1 &= C \cdot N_x \end{aligned}\right\} \quad \begin{aligned} N_x &\in \mathbb{R}^{n\times 1} \\ N_u &\in \mathbb{R} \end{aligned}$$

$n+1$ unknowns

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$N_x = \begin{bmatrix} N_{x1} \\ N_{x2} \end{bmatrix} \qquad N_u$$

$$\left.\begin{aligned} 0 &= A \cdot N_x + B\, N_u \\ 1 &= C \cdot N_x \end{aligned}\right\} \Rightarrow$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} N_{x1} \\ N_{x2} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot N_u.$$

$$= \begin{bmatrix} N_{x1} + 2 N_{x2} \\ 3 \cdot N_{x1} + 4 \cdot N_{x2} \end{bmatrix} + \begin{bmatrix} N_u \\ N_u \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$N_{x1} + 2 \cdot N_{x2} + N_u = 0$$
$$3 \cdot N_{x1} + 4 N_{x2} + N_u = 0$$

$$1 = \begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} N_{x1} \\ N_{x2} \end{bmatrix} \Leftrightarrow 1 = N_{x1} + N_{x2}\, 2$$

$$0 = A \cdot N_x + B N_u$$

$$1 = C \cdot N_x$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \cdot \begin{bmatrix} N_x \\ N_u \end{bmatrix}$$

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$u = u_{ss} - K(x - x_{ss}).$$

$$\cancel{u = N_u \cdot r_{ss} - K(x - x_{ss})}.$$

$$u = N_u \cdot r_{ss} - K(x - N_x \cdot r_{ss}).$$

$$= N_u r_{ss} - Kx + K N_x \cdot r_{ss}$$

$$u = \underbrace{(N_u + K \cdot N_x)}_{K_1} \cdot r_{ss} - K \cdot x$$

$$u = K_1 \cdot r_{ss} - K \cdot x$$

$$u = K_1 \cdot r_{ss} - Kx$$

$$K_1 = N_u + K \cdot N_x$$

$$u = (N_u + K N_x)\, r_{ss} - Kx$$

$$= \underbrace{N_u \cdot r_{ss}}_{u_{ss}} + K \underbrace{N_x r_{ss}}_{x_{ss}} - Kx$$

$$u = u_{ss} + K \cdot x_{ss} - Kx$$

$$u = u_{ss} - K(x - x_{ss}).$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \overset{n\times 1}{N_x} \\ \underset{1\times 1}{N_u} \end{bmatrix} = \begin{bmatrix} \overset{n\times n}{A} & \overset{n\times 1}{B} \\ \underset{1\times n}{C} & \underset{1\times 1}{0} \end{bmatrix}^{-1} \cdot \underbrace{\begin{bmatrix} 0 \;{\scriptstyle n\times 1} \\ 1 \;{\scriptstyle 1\times 1} \end{bmatrix}}_{n\times 1}$$

$$= \begin{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \end{bmatrix} & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.25 & -0.75 \\ 1 & -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 \\ -0.75 \\ 0.5 \end{bmatrix}$$

$$N_x = \begin{bmatrix} 1 \\ -0.75 \end{bmatrix} \qquad N_u = 0.5$$

$[26 \quad 72]$ → poles at -10, -11

$$K_1 = N_u + K N_x$$

$$= 0.5 + [26 \quad 72] \cdot \begin{bmatrix} 1 \\ -0.75 \end{bmatrix}$$

$$= 0.5 + 26 - 72 \cdot 0.75 = -27.5$$

$\dot{x} = Ax + Bu$, $A \to$ unstable

$Y = C \cdot x$ controller $R \to r_{ss}$

$u = K_1 r_{ss} - Kx$

- CTRB $M_c = [B \;\; AB \;\dots]$.

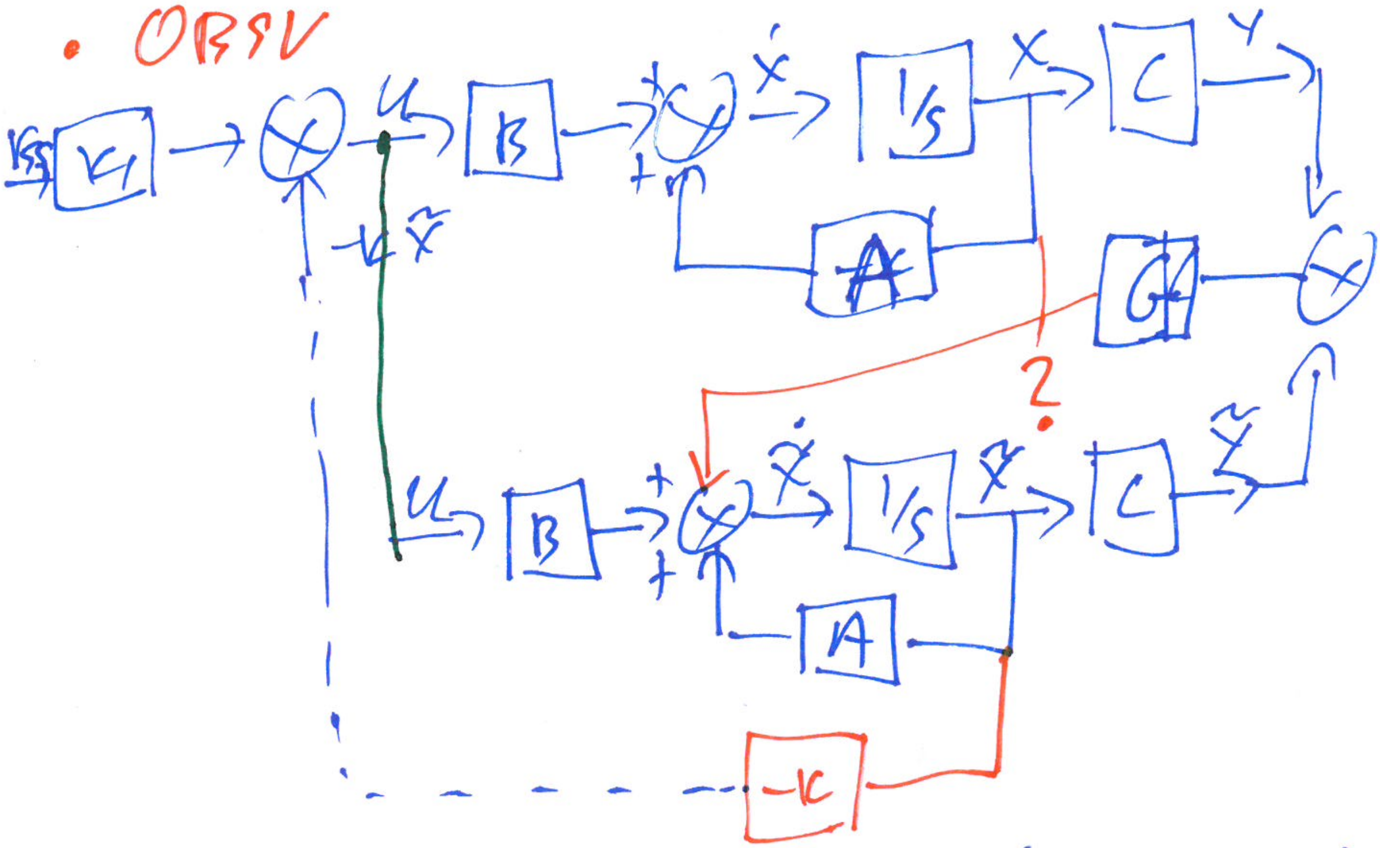
$\text{rank}(M_c) = n \to$ CTRB

$Zn \to Un$ CTRB

$A - BK$

- $K = ?$
  - $\to$ pole placement $\to$ pole location
  - $\to$ LQR $\to$ Q, R

- OBSV



- $G = ?$ pole placemet $\dot{e} = (A - GC)e$

- $K_1 = \dots$

$$\dot{x} = (A - BK) \cdot x$$

$\hookrightarrow$ eigs $\to -10$, $\to -11$

$$\dot{e} = (A - G \cdot C) \cdot e$$

$\hookrightarrow$ eigs $\to -100$, $\to -110$