Summary

$\dot{x} = f(x, t)$ → Analytical soln.

→ Gen. Soln

→ $\underbrace{\text{Spec. soln}}_{\text{I.V.P}}$ x_0, t_0

$\dot{x} + kx = u$. $\xrightarrow{C.E}$ $r + k = 0 \Rightarrow r = -k$.

$\longrightarrow x = e^{-kt}\left(x_0 + \int_0^t e^{kt_1} u(t_1)\, dt_1\right)$

- $k > 0$ → Stable
- $k < 0$ → unstable.

☒ x_0, u → Shape

→ NOT: → speed, → stability

$\ddot{x} + A\dot{x} + Bx = 0$

$\left.\begin{matrix} x_1 \\ x_2 \end{matrix}\right\} \to$ soln. $\Rightarrow$ ANY L.C. of $x_1, x_2 \to$ soln

If x_1, x_2 are L.I. $\Rightarrow$

ANY other soln φ : $\varphi = c_1 x_1 + c_2 x_2$

Find x_1, x_2

$x = e^{rt} \Rightarrow r^2 + Ar + B = 0$

- $A^2 - 4B > 0 \to r_1 \neq r_2 \in \mathbb{R}.$

  $x_1 = e^{r_1 t}, \quad x_2 = e^{r_2 t}$

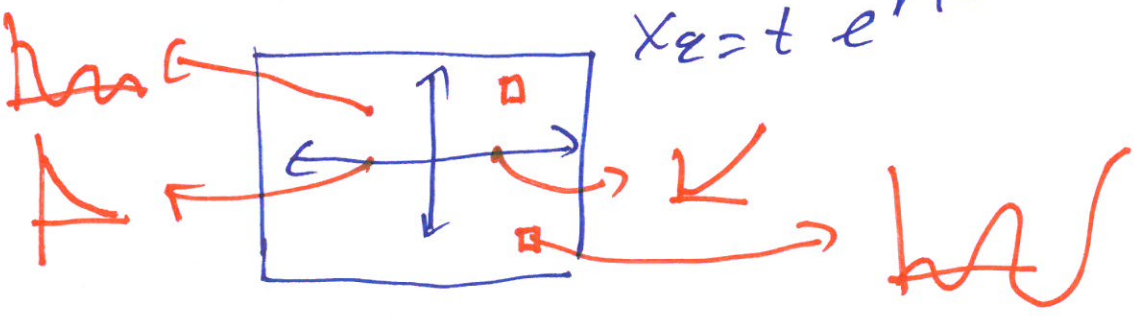
  $\varphi = c_1 x_1 + c_2 \cdot x_2.$

- $A^2 - 4B < 0$ $\quad r_1 = a + bj$, $r_2 = a - bj$ | $a < 0 \to$ stable

  $x = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

  $x = e^{at} \cdot (c_1 \cos bt + c_2 \cdot \sin bt)$

- $A^2 - 4B = 0$ $\quad r \in \mathbb{R}.$ $\quad x_1 = e^{rt}$, $x_2 = t\, e^{rt}$



$\ddot{x} + 3\dot{x} + 2x = u \qquad y = 4 \cdot x$ 26

1) $\ddot{x} = -3\dot{x} - 2x + u.$

2) $\left.\begin{array}{l} x_1 = x \\ x_2 = \dot{x} \end{array}\right\} \Rightarrow \begin{array}{l} \dot{x}_1 = \dot{x} = x_2 \Rightarrow \dot{x}_1 = x_2 \\ \dot{x}_2 = \ddot{x} = -3\dot{x} - 2x + u \\ \qquad = -3x_2 - 2x_1 + u. \end{array}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad 1$$

$$\dot{x} = Ax + Bu.$$

3) $y = 4 \cdot x = 4 \cdot x_1 = 4 \cdot x_1 + 0 \cdot x_2$

$$y = \begin{bmatrix} 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad 2$$

$$\dddot{x} - 3\ddot{x} - 2\dot{x} + 2x = u_1 - 6\cdot u_2.$$

$$y_1 = \ddot{x} + u_2 \qquad y_2 = \ddot{x} + 3\cdot x + 5u_1$$

→ 2 dots

$$y_3 = -3\ddot{x} + x + 5\cdot u_2.$$

1) $\dddot{x} = 3\ddot{x} + 2\dot{x} - 2x + u_1 - 6\cdot u_2$

2) $$\left.\begin{array}{l} x_1 = x \\ x_2 = \dot{x} \\ x_3 = \ddot{x} \end{array}\right\} \xRightarrow{d/dt} \begin{array}{l} \dot{x}_1 = \dot{x} = x_2 \Rightarrow \dot{x}_1 = x_2 \\ \dot{x}_2 = \ddot{x} = x_3 \Rightarrow \dot{x}_2 = x_3 \\ \dot{x}_3 = 3x_3 + 2x_2 - 2x_1 + u_1 - 6u_2 \end{array}$$

$$\underset{3\times 1}{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 2 & 3 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} + \underset{3\times 2}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -6 \end{bmatrix}} \cdot \underset{2\times 1}{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}$$

$$\dot{x} = Ax + Bu$$

3) $y_1 = x_3 + u_2.$, $y_2 = x_3 + 3\cdot x_1 + 5\cdot u_1$

$$y_3 = -3x_3 + x_1 + 5\cdot u_2.$$

$$\underset{3\times 1}{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix} 0 & 0 & 1 \\ 3 & 0 & 1 \\ 1 & 0 & -3 \end{bmatrix}} \cdot \underset{3\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} + \underset{3\times 2}{\begin{bmatrix} 0 & 1 \\ 5 & 0 \\ 0 & 5 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}$$

$$\dddot{x} + 3\ddot{x} + 2\dot{x} - 5x = u_1 + u_2 + u_3 + u_4$$

$$y_1 = \ddot{x} + x + u_1 - 3u_2$$

$$y_2 = 5\ddot{x} - x + u_2 - u_1$$

1) $\dddot{x} = -3\ddot{x} - 2\dot{x} + 5x + u_1 + u_2 + u_3 + u_4.$

2) $$\left.\begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \\ x_3 &= \ddot{x} \end{aligned}\right\} \xrightarrow{d/dt} \begin{aligned} &\dot{x}_1 = \dot{x} = x_2 \Rightarrow \dot{x}_1 = x_2 \\ &\dot{x}_2 = \ddot{x} = x_3 \Rightarrow \dot{x}_2 = x_3 \\ &\dot{x}_3 = \dddot{x} = -3\cdot x_3 - 2x_2 + 5x_1 \\ &\qquad + u_1 + u_2 + u_3 + u_4 \end{aligned}$$

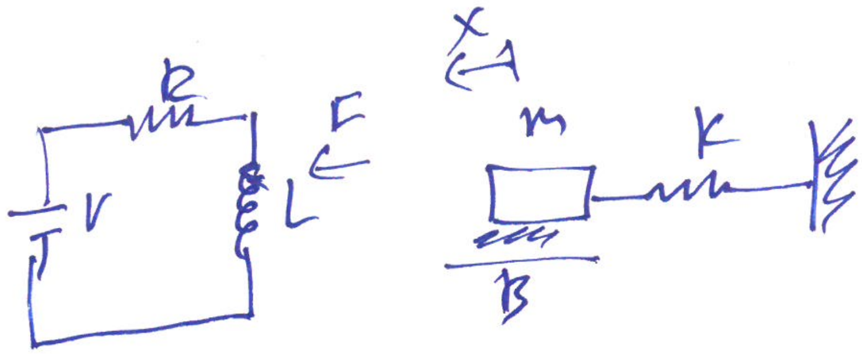
$$\underset{3\times1}{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}} = \underset{3\times3}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & -3 \end{bmatrix}} \cdot \underset{3\times1.}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} + \underset{3\times4}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}} \cdot \underset{4\times1}{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}}$$

$$y_1 = x_3 + x_1 + u_1 - 3u_2$$

$$y_2 = 5\cdot x_3 - x_1 - u_1 + u_2$$

$$\underset{2\times1}{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}} = \underset{2\times3}{\begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 5 \end{bmatrix}} \cdot \underset{3\times1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} + \underset{2\times4}{\begin{bmatrix} 1 & -3 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix}} \cdot \underset{4\times1}{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}}$$

T.F.



$x = ?$ given $V = \dots$

$V = iR + L\, di/dt$ $\quad (u = kx + \dot{x})$

$\mathcal{L}(i(t)) = \dots$

$-f - B\dot{x} - k\cdot x = m\cdot \ddot{x}$

$\ddot{x} + \frac{B}{m}\dot{x} + \frac{k}{m}x = f/m$ $\quad (\ddot{x} + A\dot{x} + Bx = 0).$

?

L.T. $\quad f(t) \longrightarrow F(s)$

$$F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)\, dt \qquad s = a + bi = \sigma + j\omega$$

$f(t) \longrightarrow F(s)$

$f(t) = 1 \longrightarrow F(s) = 1/s$

$f(t) = t \longrightarrow F(s) = 1/s^2$

$\dot{f}(t) \longrightarrow s\,F(s)$

$f'' \longrightarrow s^2 \cdot F(s)$

$\vdots \qquad \vdots$

$\dot{x} + 5x = u. \overset{C.E}{\Rightarrow} r + 5 = 0 \Rightarrow$

$r = -5 < 0 \rightarrow$ stable

$x(t) = ? \longrightarrow X(s)$

$\dot{x} \longrightarrow sX(s).$

$u \longrightarrow U(s)$

L.T.

$sX(s) + 5X(s) = U(s).$

$$\frac{X(s)}{U(s)} = \frac{1}{s+5}$$

$$v = iR + L\,\frac{di}{dt}$$

$$f = K_A \cdot i$$

$$f - B\dot{x} - Kx = m \cdot \ddot{x}$$

$$i(t) \rightarrow I(s)$$
$$v \rightarrow V(s)$$
$$x \rightarrow X(s)$$
$$\dot{x} \rightarrow s\,X(s)$$
$$\ddot{x} \rightarrow s^2 X(s)$$

$$V(s) = I(s) \cdot R + Ls\,I(s)$$

$$F(s) - B \cdot s\,X(s) - K\,X(s) = m s^2 X(s)$$

$$I(s) = V(s)\,\frac{1}{Ls+R}$$

$$F(s) = K_A I(s) = K_A \cdot V(s) \cdot \frac{1}{Ls+R}$$

$$F(s) = X(s) \cdot (m s^2 + Bs + K)$$

$$\frac{K\,V(s)}{Ls+R} = X(s) \cdot (m s^2 + Bs + K)$$

$$\frac{X(s)}{V(s)} = \frac{K_A}{(Ls+R)(ms^2+Bs+K)}$$

C.E. $(Ls+R)(ms^2+Bs+K)=0$

- $Ls+R=0 \Rightarrow \ldots \quad s=\ldots$

or

- $ms^2+Bs+K=0 \Rightarrow \ldots \quad s_1 =$
  $s_2 = \ldots$