Revision

ODE → S.S.

$$x^{(n)} + a_{n-1} x^{(n-1)} + \dots + a_0 x = b_1 u_1 + b_2 u_2 + \dots$$

$$y_1 = c_{1,1} x + c_{1,2} \dot{x} + \dots$$

$$y_2 = \dots$$

$\vdots$

- $x^{(n)} = -a_{n-1} x^{(n-1)} - \dots - a_0 x_0 + b_1 u_1 + \dots$

- $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \ddot{x}$, $\vdots$, $x_n = x^{(n-1)}$ $\overset{d/dt}{\Longrightarrow}$

$$\dot{x}_1 = \dot{x} = x_2 \Leftrightarrow \dot{x}_1 = x_2$$

$$\dot{x}_2 = \ddot{x} = x_3 \Rightarrow \dot{x}_2 = x_3$$

$$\vdots$$

$$\dot{x}_n = x^{(n)} = -a_{n-1} x_n - a_{n-2} x_{n-1} \dots - a_0 x_1 + b_1 u_1 + \dots$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots \\ - & - & - & - & - \\ -a_0 & -a_1 & \dots & & -a_{n-1} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 & 0 & \dots \\ \vdots & \vdots & \vdots \\ b_1 & b_2 & \dots \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ \vdots \end{bmatrix}$$

• ODE $\quad x^{(n)} + \dots\dots \longrightarrow$ C.E.

$r^n + \dots\dots = 0$

L.T. $\downarrow \uparrow$ I.L.T.

• TF. $\quad \dfrac{OUT(s)}{In(s)} = \dfrac{P(s)}{\boxed{Q(s)}} = 0 \quad s^n$ C.E.

cha

• S.S.

$u(t) \rightarrow U(s)$

$x(t) \rightarrow \underline{X}(s)$

$\dot{x} = Ax + Bu \qquad \dot{x} \rightarrow s\underline{X}(s)$

$\downarrow$ L.T.

$sX(s) = A \cdot X(s) + BU(s)$

$sX(s) - AX(s) = B \cdot U(s)$

~~(s - A) X(s)~~

$sI \cdot \underline{X(s)} - A \cdot \underline{X(s)} = BU(s)$

$(sI - A) \cdot X(s) = BU(s).$

$X(s) = (sI - A)^{-1} \cdot B \cdot U(s)$

$Y(t) = C \cdot x(t) + D.u. \xrightarrow{L.T.}$

$Y(s) = C \cdot X(s) + D \cdot U(s)$

$Y(s) = C \cdot (sI - A)^{-1} \cdot B \cdot U(s) + D \cdot U(s)$

$$Y(s) = \Big(\underbrace{C \cdot (sI - A)^{-1} B + D}_{G(s) = T.F.} \Big) \cdot U(s)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u.$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

1 in
1 out

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & -0.5 \end{bmatrix} =$$

$$= \begin{bmatrix} s & -1 \\ 1 & s+0.5 \end{bmatrix}$$

$$|sI - A| = s(s+0.5) - (-1) \cdot 1.$$

$$= s^2 + 0.5s + 1$$

$$(sI - A)^{-1} = \frac{1}{s^2 + 0.5 \cdot s + 1} \cdot \begin{bmatrix} s+0.5 & 1 \\ -1 & s \end{bmatrix}$$

$C \cdot (sI - A)^{-1} \cdot B$

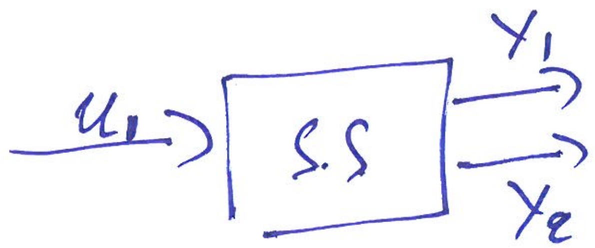
$$\underset{1\times 2}{\begin{bmatrix} 1 & 0 \end{bmatrix}} \cdot \frac{1}{(\quad)} \underset{2\times 2}{\begin{bmatrix} s+0.5 & 1 \\ -1 & s \end{bmatrix}} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\frac{1}{(\quad)} \underset{1\times 2}{\begin{bmatrix} s+0.5 & 1 \end{bmatrix}} \cdot \underset{2\times 1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} =$$

$$\frac{1}{s^2 + 0.5 \cdot s + 1} \cdot 1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 \\ -1 & -0.5 \end{bmatrix}} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underset{B}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} u$$

$$Y = \underset{C}{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}} \cdot X$$



$\frac{Y_1}{u_1} = ?$ G_{11}

$\frac{Y_2}{u_1} = ?$ G_{21}

$$C \cdot (sI - A)^{-1} B$$

$$(sI - A)^{-1} = \frac{1}{s^2 + 0.5s + 1} \cdot \begin{bmatrix} s+0.5 & 1 \\ -1 & s \end{bmatrix}$$

$$\underset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}} \cdot \frac{1}{(\quad)} \underset{2\times 2}{\begin{bmatrix} s+0.5 & 1 \\ -1 & s \end{bmatrix}} \cdot \underset{2\times 1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \frac{1}{(\quad)} \underset{2\times 1}{\begin{bmatrix} 1 \\ s \end{bmatrix}}$$

$$\frac{1}{(\quad)} \underset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}} \cdot \underset{2\times 1}{\begin{bmatrix} 1 \\ s \end{bmatrix}} = \frac{1}{(\quad)} \cdot \begin{bmatrix} 1 \\ 2s \end{bmatrix} \rightarrow$$

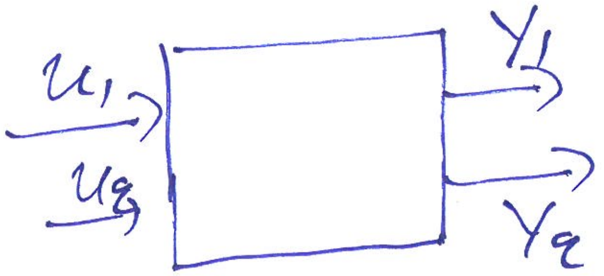
$$G(s) = \frac{1}{s^2+0.5s+1} \cdot \begin{bmatrix} 1 \\ 2s \end{bmatrix}$$

$$G_{11} = \frac{1}{s^2+0.5s+1}$$

$$G_{21} = \frac{2s}{s^2+0.5\cdot s+1}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -0.5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} X$$



$$\frac{y_1}{u_1} = \quad \frac{y_1}{u_2} =$$

$$\frac{y_2}{u_1} = \quad \frac{y_2}{u_2} =$$

$$(sI-A)^{-1} = \frac{1}{(\quad)} \cdot \begin{bmatrix} s+0.5 & 1 \\ -1 & s \end{bmatrix}$$

$$G = \frac{1}{(\quad)} \cdot \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}_{2\times 2} \cdot \underbrace{\begin{bmatrix} s+0.5 & 1 \\ -1 & s \end{bmatrix}}_{2\times 2} \cdot \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{2\times 2}$$

$$G = \frac{1}{(\ \ )}\begin{bmatrix} s+0.5 & 1 \\ -2 & 2s \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

2x2 2x2

$$= \frac{1}{(\ \ )}\begin{bmatrix} s+0.5 & s+0.5+1 \\ -2 & -2+2s \end{bmatrix}$$

$$G_{1,1} = \frac{s+0.5}{s^2+0.5s+1} \qquad G_{2,1} = \frac{-2}{s^2+0.5s+1}$$

$$G_{1,2} = \frac{s+1.5}{s^2+0.5s+1} \qquad G_{2,2} = \frac{2s-2}{s^2+0.5s+1}$$

$$G_{i,j} = \frac{\begin{vmatrix} sI-A & -B_i \\ C_j & D \end{vmatrix}}{|sI-A|.}$$

$B_i \rightarrow i^{th}$ coln. of B

$C_j \rightarrow j^{th}$ row of C.

$$B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad \text{1st. } c = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \text{2nd } c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \qquad \text{1st } r = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad \text{2nd } r = \begin{bmatrix} 0 & 2 \end{bmatrix}$$

$$\begin{vmatrix} sI-A & -B_i \\ C_j & D \end{vmatrix}$$

$i=1 \quad , j=1$

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$$B=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} s & -1 & -1 \\ 1 & s+0.5 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$$C=\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$D_1 = \begin{vmatrix} sI - A & -B_i \\ C_j & 0 \end{vmatrix}$$

$B_i \rightarrow$ i^{th} col. of B

$C_j \rightarrow$ j^{th} row of C

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$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & -0.5 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 \\ 1 & s+0.5 \end{bmatrix} \rightarrow 2\times 2$$

In = 1

Out = 1

$$D_1 = \begin{vmatrix} s & -1 & -1 \\ 1 & s+0.5 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$$s \begin{vmatrix} s+0.5 & 0 \\ 0 & 0 \end{vmatrix} - 1 \cdot (-1) \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} + (-1) \cdot \begin{vmatrix} 1 & s+0.5 \\ 1 & 0 \end{vmatrix}$$

$= \ldots$

$$\begin{vmatrix} sI - A & -B_2 \\ C_1 & 0 \end{vmatrix}$$

2nd Inp
1st out.

$$B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

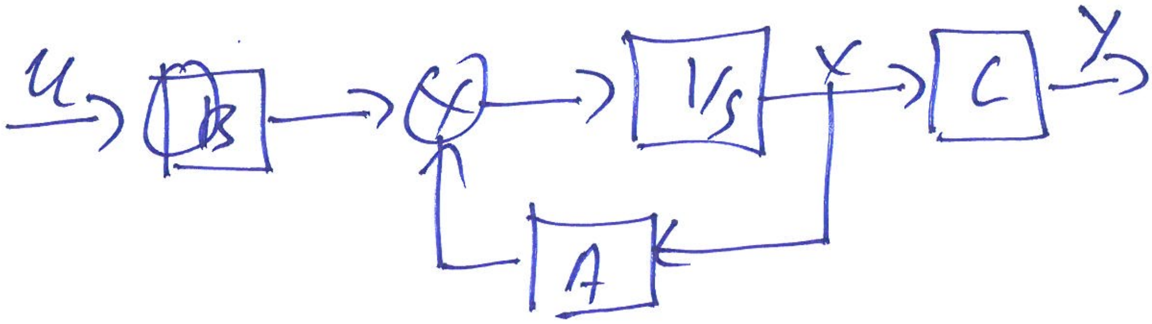
$$\begin{vmatrix} s & -1 & -1 \\ 1 & s+0.5 & -1 \\ 1 & 0 & 0 \end{vmatrix} = \ldots$$

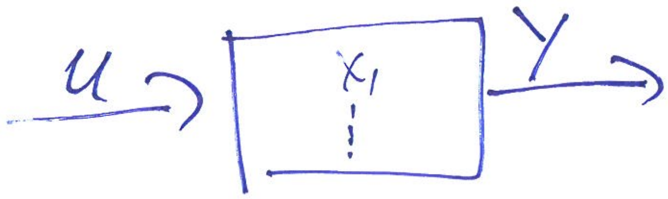
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

1st In
2nd out.

$$\begin{vmatrix} s & -1 & -1 \\ 1 & s+0.5 & 0 \\ 0 & 2 & 0 \end{vmatrix} = \ldots$$

H2





$$\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} \quad \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x}_1 = -2x_1 + x_2 + \dots$$

$$\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \ldots$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$M_0 = \begin{bmatrix} C \\ CA \\ \end{bmatrix} \qquad \operatorname{rank}(M_0) = n$$

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \qquad C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} \qquad C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \qquad C_3 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} C_1 \\ C_1 A_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$C_1 \cdot A_1 = \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \end{bmatrix}$$

$$|M_1| = \begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = +1-(-2) = +3 \neq 0$$

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$$M_3 = \begin{bmatrix} c_3 \\ c_3 A_2 \end{bmatrix}$$

$$c_3 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$c_3 \cdot A_3 = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$$