OBSV

$$\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \cdot x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u, \quad y = [1 \quad 1] \cdot x$$

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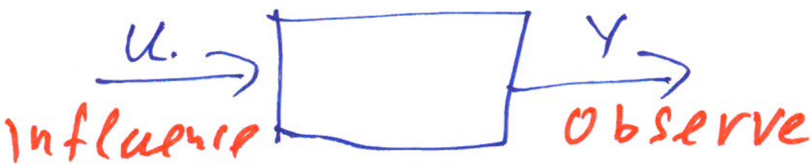


$\dot{x}_2 = x_2 + u$

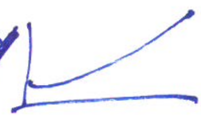
1) $\dot{x}_1 = -2x_1 + 2 \cdot u$

x_1 

$y = x_1 + x_2 \Rightarrow$ 

x_2 

2) $y = x_1$, BUT $\dot{x}_1 = -2x_1 + x_2 + 2 \cdot u$

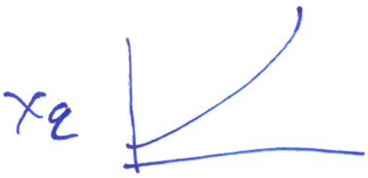
x_1  $\Rightarrow \cancel{y}$ 

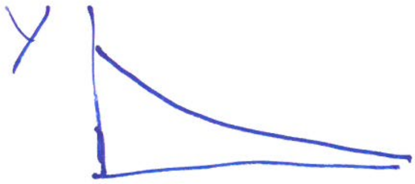
3) $\dot{x}_1 = -2x_1 + 2\cdot u$

$\dot{x}_2 = x_2 + u$

$y = [1 \quad 0]\cdot x$

$= x_1$





3) $A = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$, $C = [1 \quad 0]$; $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$G(s) = [1 \quad 0]\cdot (sI - A)^{-1} \cdot B$

$= [1 \quad 0] \cdot \begin{bmatrix} s+2 & 0 \\ 0 & s-1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\begin{vmatrix} s+2 & 0 \\ 0 & s-1 \end{vmatrix} = (s+2)(s-1). \longrightarrow$ C.E $(s+2)(s-1) = 0$

$G(s) = [1 \quad 0] \dfrac{1}{(s+2)(s-1)} \cdot \begin{bmatrix} s-1 & 0 \\ 0 & s+2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$= \dfrac{1}{(\quad)} [s-1 \quad 0] \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$= \dfrac{2\cancel{(s-1)}}{(s+2)\cancel{(s-1)}} = \dfrac{2}{s+2}$

$$M_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \qquad n \to \text{order of sys}$$

if $\quad \text{Rank}(M_o) = n \longrightarrow$ obsv

$\qquad\qquad\qquad\quad < n \longrightarrow$ NOT obsv

if $\quad |M_o| = 0 \implies$ NOT obsv

$\qquad\qquad\quad \neq 0 \implies$ obsv

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \qquad C_1 = [1 \quad 1]$$

$$M_o = \begin{bmatrix} C \\ CA \end{bmatrix}, \quad CA = [1 \quad 1] \cdot \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= [-2 \quad 0]$$

$$M_o = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \qquad |M_o| = 2 \neq 0$$

$$A_2 = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} \qquad C_2 = [1 \quad 0].$$

$$C_2 \cdot A_2 = [1 \quad 0] \cdot \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} = [-2 \quad 1].$$

$$M_0 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \Rightarrow |M_0| \neq 0 \rightarrow \text{OBSV}$$

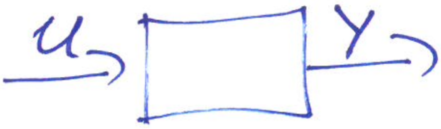
$$A_3 = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \qquad C_3 = [1 \quad 0]$$

$$C_3 \cdot A_3 = [1 \quad 0] \cdot \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$= [-2 \quad 0].$$

$$M_0 = \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} \Rightarrow |M_0| = 0 \rightarrow \text{NOT OBSV.}$$

$$\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdot u, \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot x$$



$$M_c = \begin{bmatrix} B & A \cdot B & \ldots & A^{n-1} B \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad A_1 = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$A \cdot B = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \end{bmatrix}$$

$$M_c = \begin{bmatrix} 2 & -4 \\ 0 & 0 \end{bmatrix}, \quad |M_c| = 0 \rightarrow \text{NOT CTRB}$$

$$A_2 = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\cancel{B_2 \cdot A_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 \end{bmatrix}}$$

$$A_2 \cdot B_2 = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -2 & 0 \\ 1 & 1 \end{bmatrix} \qquad B_3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$A_3 \cdot B_3 = \begin{bmatrix} -2 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$M_C = \begin{bmatrix} 2 & -4 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow |M_C| \neq 0 \rightarrow \text{CTRB}$$

$$\ddot{x} + A\dot{x} + Bx = 0 \xRightarrow[e^{rt}]{} r^2 + Ar + B = 0$$

$$r_{1,2} = \frac{-A \pm \sqrt{D}}{2}$$

- $D < 0$
- $D = 0$
- $D > 0$

$$\dot{x} = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix} x \qquad x = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} e^{\lambda t}$$

$$\dot{x} = \lambda \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot \cancel{e^{\lambda t}} = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix} \cancel{e^{\lambda t}} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\left.\begin{aligned} \lambda \cdot a_1 &= -2a_1 + 2 \cdot a_2 \\ \lambda \cdot a_2 &= 2a_1 - 5 \cdot a_2 \end{aligned}\right\} \Rightarrow$$

$$\begin{aligned} (-2-\lambda) a_1 + 2 \cdot a_2 &= 0 \\ 2 \cdot a_1 + (-5-\lambda) \cdot a_2 &= 0 \end{aligned} \qquad 2 \times 3 \text{ N.L.S.}$$

$$\lambda = \text{GIVEN} \longrightarrow 2 \times 2 \text{ L.S.}$$

$$\Rightarrow \begin{vmatrix} -2-\lambda & 2 \\ 2 & -5-\lambda \end{vmatrix} = 0 \Rightarrow$$

$$(-2-\lambda)(-5-\lambda) - 4 = 0 \Rightarrow$$

$$(\lambda+2)(\lambda+5) - 4 = 0 \Rightarrow$$

$$\lambda^2 + 7\lambda + 10 - 4 = 0$$

$$\lambda^2 + 7\lambda + 6 = 0$$

$\lambda^2 + 7\cdot\lambda + 6 = 0 \Rightarrow$ C.E.

$\lambda_1 = -1$

$\lambda_2 = -6$ eigenvalues

$a_1 = ?$ $\quad a_2 = ?$

• $\lambda = -1$

$$\left.\begin{aligned} (-2+1)\,a_1 + 2\cdot a_2 &= 0 \\ 2\cdot a_1 + (-5+1)\cdot a_2 &= 0 \end{aligned}\right\} \Rightarrow$$

$$\boxed{\left.\begin{aligned} -a_1 + 2\cdot a_2 &= 0 \\ 2\cdot a_1 - 4\cdot a_2 &= 0 \end{aligned}\right\} \Rightarrow}$$

$-a_1 + 2\cdot a_2 = 0$

Assume $a_2 = 1 \Rightarrow a_1 = 2$

$\lambda = -1 \Rightarrow$ eigenvector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\lambda_2 = -6$

$$\left.\begin{aligned} 4\cdot a_1 + 2\cdot a_2 &= 0 \\ 2\cdot a_1 + a_2 &= 0 \end{aligned}\right\} \Rightarrow 2\cdot a_1 + a_2 = 0$$

$a_1 = 1$

$a_2 = -2.$

$\Rightarrow$ eigenvector $= \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$$\dot{x} = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix} \qquad x_1 = e^{-t} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$x_2 = e^{-6t} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\Rightarrow x(t) = c_1 e^{-t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \cdot e^{-6t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow$$

$$x(0) = c_1 \cdot 1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} 2 \cdot c_1 + c_2 &= 1 \\ c_1 - 2c_2 &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} c_1 &= 0.4 \\ c_2 &= 0.2 \end{aligned}$$

$$\dot{x} = Ax \qquad x = e\, e^{\lambda t}$$

$e \in \mathbb{R}^{n\times 1}$

$\lambda \in \mathbb{R}.$

$A \in \mathbb{R}^{n\times n} \qquad x \in \mathbb{R}^{n\times 1}$

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$$\lambda e \cancel{e^{\lambda t}} = A \cdot e \cancel{e^{\lambda t}}$$

$$\lambda e = A \cdot e \Rightarrow (\lambda I - A)\cdot e = 0$$

$$|\lambda I - A| = 0 \Rightarrow \text{eigvalue}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -6 & -7 \end{bmatrix} \cdot x$$

$$|\lambda I - A| = 0 \Leftrightarrow |A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ -6 & -7-\lambda \end{vmatrix} = 0$$

$$-\lambda(-7-\lambda) - 1\cdot(-6) = 0$$

$$\lambda(\lambda+7) + 6 = 0 \Rightarrow$$

$$\lambda^2 + 7\lambda + 6 = 0 \Rightarrow$$

$$\lambda_{1,2} = \frac{-7 \pm \sqrt{49 - 4\cdot 6}}{2} \rightarrow \lambda_1 = -1, \quad \lambda_2 = -6$$

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• $\cancel{\lambda=1}$: $\begin{bmatrix} \lambda & -1 \\ 6 & \lambda+7 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0$

• $\lambda = -1 \Rightarrow \begin{bmatrix} -1 & -1 \\ 6 & 6 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0 \Rightarrow$

$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$-a_1 - a_2 = 0$ $\qquad a_1 = 1$

$a_2 = -1.$

• $\lambda = -6$ $\quad \begin{bmatrix} -6 & -1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \Rightarrow$

$\begin{bmatrix} 1 \\ -6 \end{bmatrix}$

$-6 \cdot a_1 - a_2 = 0 \Rightarrow a_1 = 1$

$\Rightarrow a_2 = -6$

$$X = c_1 e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 \cdot e^{-6t} \begin{bmatrix} 1 \\ -6 \end{bmatrix}$$

$X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow$$

$c_1 = 6/5 \qquad c_2 = -1/5$

$\dot{x} = Ax \quad , \quad A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} \quad , \quad x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 56

$\cdot |A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & -1 \\ 1 & 3-\lambda \end{vmatrix} = 0 \Rightarrow$

$$(1-\lambda)(3-\lambda) + 1 = 0$$

$$(\lambda - 1)(\lambda - 3) + 1 = 0$$

$$\lambda^2 - 4\lambda + 3 + 1 = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$(\lambda - 2)^2 = 0 \Rightarrow \lambda_1 = \lambda_2 = 2$$

$(A - \lambda I) \cdot e = 0 \Rightarrow \ldots \Rightarrow e = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$e = (A - \lambda I) \cdot b.$ $b \rightarrow$ gen. eigenvector.

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} = (A - 2I)\, b$$

$$= \begin{bmatrix} 1-\lambda & -1 \\ 1 & 3-\lambda \end{bmatrix} \cdot b =$$

$$= \begin{bmatrix} 1-2 & -1 \\ 1 & 3-2 \end{bmatrix} \cdot b \Rightarrow$$

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$$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow$$

$-b_1 - b_2 = 1 \quad b_1 = 0 \Rightarrow b_2 = -1.$

$$x(t) = c_1 (e t + b) e^{\lambda t} + c_2 e\, e^{\lambda t}$$

$$= c_1 \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} . t + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right) e^{2t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t}$$