Revision

$\dot{x} = Ax + B \cdot u$

$y = C \cdot x + \cancel{D \cdot u}^{\to 0}$

$|A - \lambda I| = 0$

- $Re(\lambda) < 0 \to$ stable
- $Re(\lambda) > 0 \to$ unstable

$\to$ eigenvectors

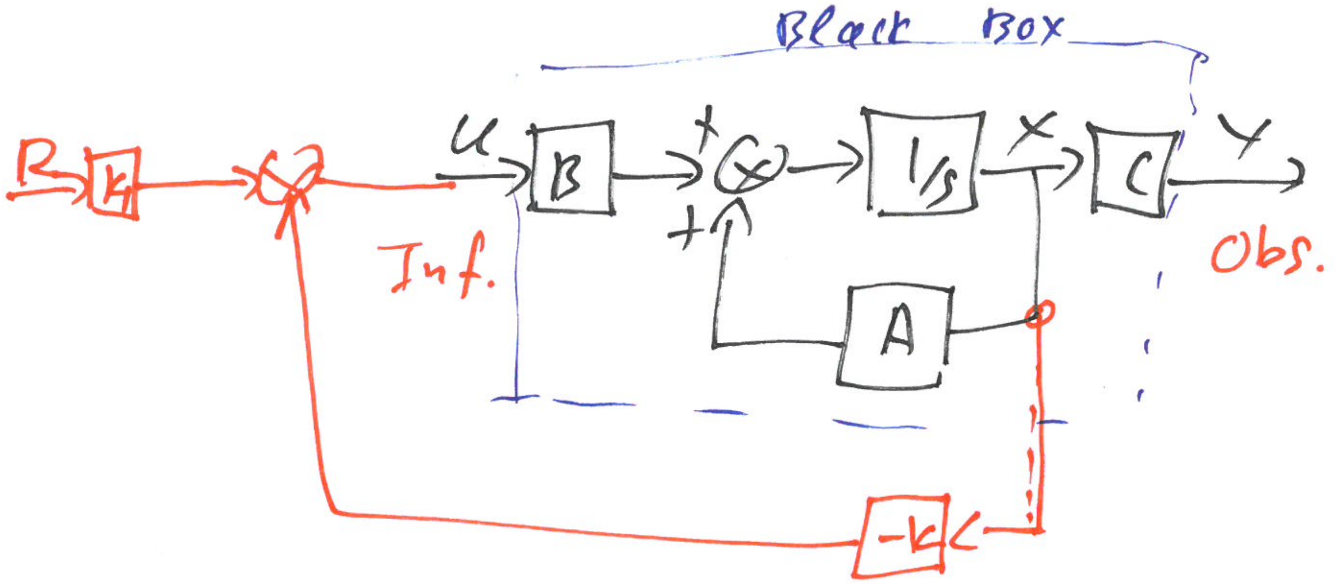
- $x = c_1 e_1 e^{\lambda_1 t} + c_2 e_2 e^{\lambda_2 t}$
-
-

$X(t) = [x_1 \quad x_2]$ Soln Matrix

$x(t) = \underbrace{X(t) \cdot X^{-1}(0)}_{\text{S.T.M.}} \cdot x(0)$

L.T.I. S.T.M. $(\Phi(t, t_0))$

$= e^{At} \Rightarrow x(t) = e^{At} \cdot x_0$



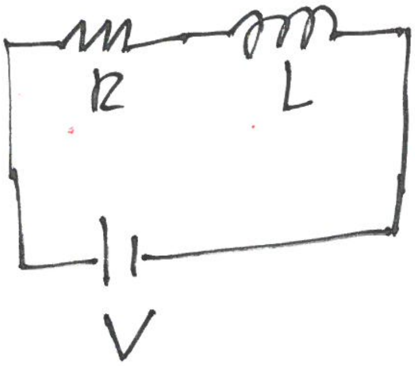
$$u = R \cdot K_1 - K \cdot x \quad , \quad u \in \mathbb{R}^{p \times 1}$$

$$\dot{x} = \underbrace{A}_{\text{O.L.S.M}} x + B \cdot u.$$

$$\dot{x} = Ax + B \cdot (R \cdot K_1 - Kx)$$

$$= Ax - BK \cdot x + BR \cdot K_1$$

$$= \underbrace{(A - BK)}_{\text{C.L.S.M.}} \cdot x$$



$$\frac{di}{dt} = \frac{1}{L}(V - iR).$$

Steady state: $i = V/R$

$x = i$, $u = V$, $A = -R/L$; $B = 1/L$.

Time constant.
eigenvalue.

$u = ?$: $i \to 0$ (very fast).

$u = -K \cdot x \implies A_{CL} = A - B \cdot K.$

$$= -\frac{R}{L} - \frac{1}{L} \cdot K \quad \Bigg\} \Rightarrow$$

Place C.L. eigs at $-6\frac{R}{L}$

$$\cancel{-}\frac{R+K}{\cancel{L}} = \cancel{-}6\frac{R}{\cancel{L}}$$

$$K = 5R$$

$$u = -K \cdot x = -5 \cdot R \cdot i$$

$\dot{x} = Ax + B \cdot u$ $\quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$|A - \lambda I| = 0 \Rightarrow$

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0 \Rightarrow$$

$$(1-\lambda)(4-\lambda) - 6 = 0 \Rightarrow$$

$$\lambda^2 - 5\lambda - 2 = 0$$

$$\Delta = 25 - 4 \cdot (-2) = 33$$

$$\lambda = \frac{+5 \pm 5.74}{2} \rightarrow \lambda_1 = 5.37, \quad \lambda_2 = -0.37$$

CTRB

$$M_c = [B \quad A \cdot B] = \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right]$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$$

$|M_c| = 1 \cdot 3 - 0 \cdot 1 = 3 \neq 0$ $\quad$ C.T.R.B. ✓

$u = -K \cdot x$ $\quad$ $\underbrace{A_{CL}}_{2\times 2} = \underbrace{A}_{2\times 2} - \underbrace{B}_{2\times 1} \cdot K \;) \longrightarrow 1\times 2$ $\quad$ 86

$K = [k_1 \quad k_2]$

$$A_{CL} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot [k_1 \quad k_2]$$

$$= \begin{bmatrix} 1-k_1 & 2-k_2 \\ 3 & 4 \end{bmatrix}.$$

• Place the poles at $-10, -11$

$|A_{CL} - \lambda I| = 0 \Rightarrow$

$$\begin{vmatrix} 1-k_1-\lambda & 2-k_2 \\ 3 & 4-\lambda \end{vmatrix} = 0$$

• $\lambda = -10$ $\quad$ $\begin{vmatrix} 1-k_1+10 & 2-k_2 \\ 3 & 4+10 \end{vmatrix} = 0$

$-14 \cdot k_1 + 3 \cdot k_2 + 148 = 0$ (1)

• $\lambda = -11 \Rightarrow$ $\quad$ $-15 k_1 + 3 \cdot k_2 + 174 = 0$ (2) $\Big\} \Rightarrow$

$k_1 = 26 \qquad k_2 = 72.$

$$A_{CL} = A - BK$$

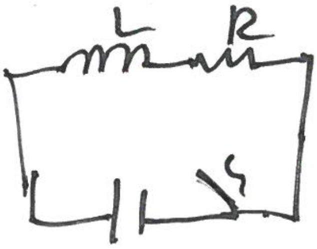
$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 26 & 72 \end{bmatrix}$$

$$= \begin{bmatrix} -25 & -70 \\ 3 & 4 \end{bmatrix}$$

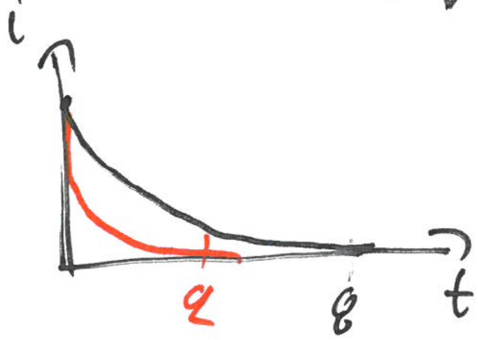
$$|A_{CL} - \lambda I| = 0 \Rightarrow \begin{cases} \lambda_1 = -10 \\ \lambda_2 = -11 \end{cases}$$

Pole Placement

Opt. Control



$$\frac{di}{dt} = \frac{1}{L}(V - iR)$$

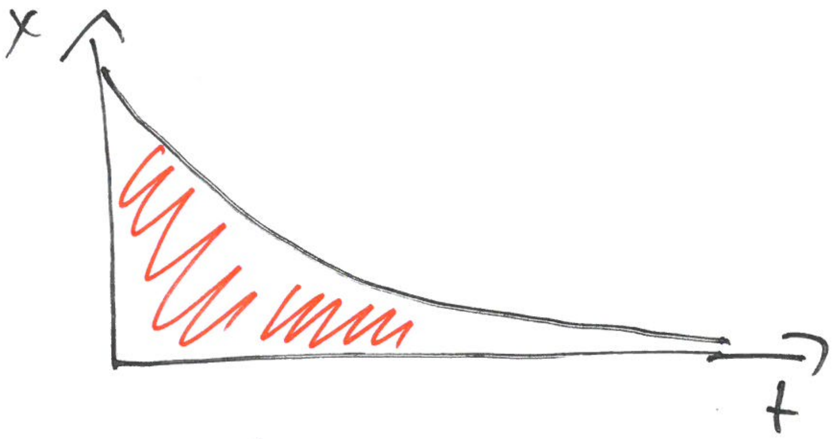


- $V = 0$ $\quad \frac{di}{dt} = -R/L \cdot i$

$$i(t) = e^{-R/L \cdot t} \cdot i(0).$$

- $V = -5 \cdot R \cdot i$ $\quad \frac{di}{dt} = -6 \frac{R}{L} \cdot i$

$$i(t) = e^{-6 \cdot \frac{R}{L} t} \cdot i(0).$$



$$I_x = \int_0^\infty x(t)\,dt$$

• $\frac{di}{dt} = -\frac{R}{L}\cdot i \Rightarrow i(t) = e^{-R/L\cdot t}\cdot i(0)$

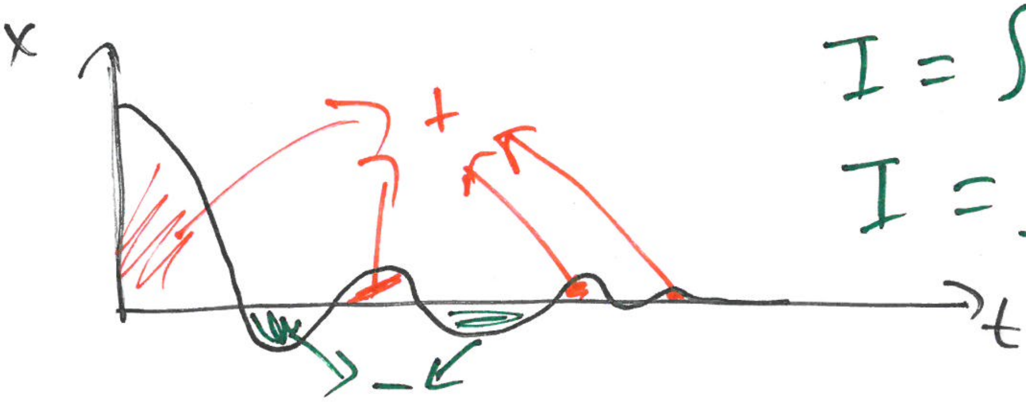
$$I_x = \int_0^\infty e^{-\frac{R}{L}\cdot t}\cdot i(0)\,dt = i(0)\cdot\int_0^\infty \left(\frac{e^{-R/L\cdot t}}{-R/L}\right)' dt$$

$$= \boxed{i(0)\cdot\frac{L}{R}}$$

• $x(t) = e^{-6\cdot\frac{R}{L}\cdot t} \Rightarrow$

$$I_x = \boxed{i(0)\cdot\frac{L}{R}\,\frac{1}{6}}$$

6 time faster



$$I = \int |x(t)|\,dt$$

$$I = \int x^2(t)\,dt$$

if $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$ $\quad I_x = \int (x_1^2 + x_2^2 + \dots\dots +) \, dt$

$$I_x = \int (2 \cdot x_1^2 + 1.5 \cdot x_2^2 + x_3^2 \dots) \, dt$$

$$= \int X^T \cdot Q \cdot X \, dt$$

Imp of speed

$$X^T = [x_1 \quad x_2 \quad x_3 \dots]$$

$$X^T \cdot Q \cdot X = [x_1 \quad x_2 \quad x_3 \dots] \cdot Q \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{bmatrix}$$

$$Q = \begin{bmatrix} \boxed{2} & 0 & 0 & 0 \\ & \boxed{1.5} & & \\ 0 & & \boxed{1} & \\ 0 & 0 & & \end{bmatrix}$$

$K = ?$: $\quad I_x = \int X^T \cdot Q \cdot X \, dt \rightarrow$ small

$u = -k \cdot x$ $\quad u = -5 \cdot R \cdot i$ $\quad \left(eig = -6\frac{R}{L}\right)$ 90

$i(0) = 1\,A$ $\quad R = 1\,\Omega$

$u(0) = -5\ V$

$eig = -10\,001\,\frac{R}{L}$

$u = -10000\ R \cdot i$

$u(0) = -10\ kV$

$I_u = \int u^2 dt$

I want $I_x + I_u \rightarrow$ small

$$I = \int (x^T \cdot Q \cdot x + u^2)\, dt$$

In general $I = \int (\underbrace{x^T \cdot Q x}_{\text{speed}} + \underbrace{U^T \cdot R \cdot U}_{\text{energy}})\, dt$

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