Linear Controller Design and State Space Analysis
EEE8013
Tutorial Exercise II

1. (a) Derive a state space representation of the mass spring system assuming that the system has 2 outputs: the displacement and the velocity.

   (b) If m=B=k=1 simulate the system response if the input force is 1 (unit step) assuming zero initial conditions.

   (c) Crosscheck you answer by simulating the numerical and the analytical solutions of the system based on ODE representation.

   (d) Justify the type of response you get.

2. Repeat Question 1 assuming initial conditions of x(0)=1, x'(0)=1.
3. Find the state space model of the following system:

$$\ddot{x} + 3\dot{x} + 2x = u(t)$$
$$y = x$$

   (a) Use the state space model to simulate the step response of the system assuming that x(0)=1, x'(0)=1.

   (b) Crosscheck you answer by simulating the numerical and the analytical solutions of the system based on ODE representation.

4. Find the state space model of the system:

$$\dddot{x} = -3\ddot{x} - 2\dot{x} - 5x + u_1 - 6u_2 + 7u_3$$
$$y_1 = \ddot{x} + u_2$$
$$y_2 = \ddot{x} + 3x + 5u_1$$
$$y_3 = x - 3\ddot{x} + 2u_2$$

Then simulate both numerical and state space representation asumming zero initial conditions and u_1=u_3=1 and u_2=2.

5. Simulate the homogeneous system:

$A=\begin{bmatrix}-2 & 2\\ 2 & -5\end{bmatrix}$ $B=\begin{bmatrix}0\\ 1\end{bmatrix}$ $C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ assuming that x(0)=1, x'(0)=0.

6. Use Matlab to find the TF of the following state space model:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & -0.5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u \text{ and } y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

7. Find the transfer function of the following system analytically and using Matlab:

$\mathbf{A}=\begin{bmatrix}2 & 0\\ 4 & 1\end{bmatrix}$, $\mathbf{B}=\begin{bmatrix}1\\ 1\end{bmatrix}$, $\mathbf{C}=\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$, **D** is the zero matrix.

8. Use Matlab to find the transfer function matrix of the system:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -25 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}u$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}u$$

9. Use Matlab to Find a state space representation of the following TF:

$$\frac{Y(s)}{U(s)}=G(s)=\frac{10s+10}{s^3+6s^2+5s+10}$$