

## **Chapter #3**

**EEE 8072**

### **Subsea Control and Communication Systems**

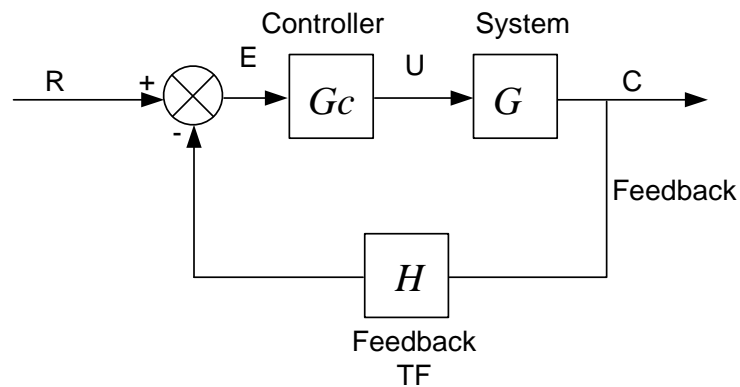
- **Closed loop systems**
- **Steady state error**
- **PID control**
- **Other controllers**

## Introduction

The general form for a CL system:

$$\frac{C}{R} = \frac{G'}{1 + G'H}, \text{ where } G' = G_c \times G, G_c = \text{TF of controller, } G \text{ TF of plant}$$

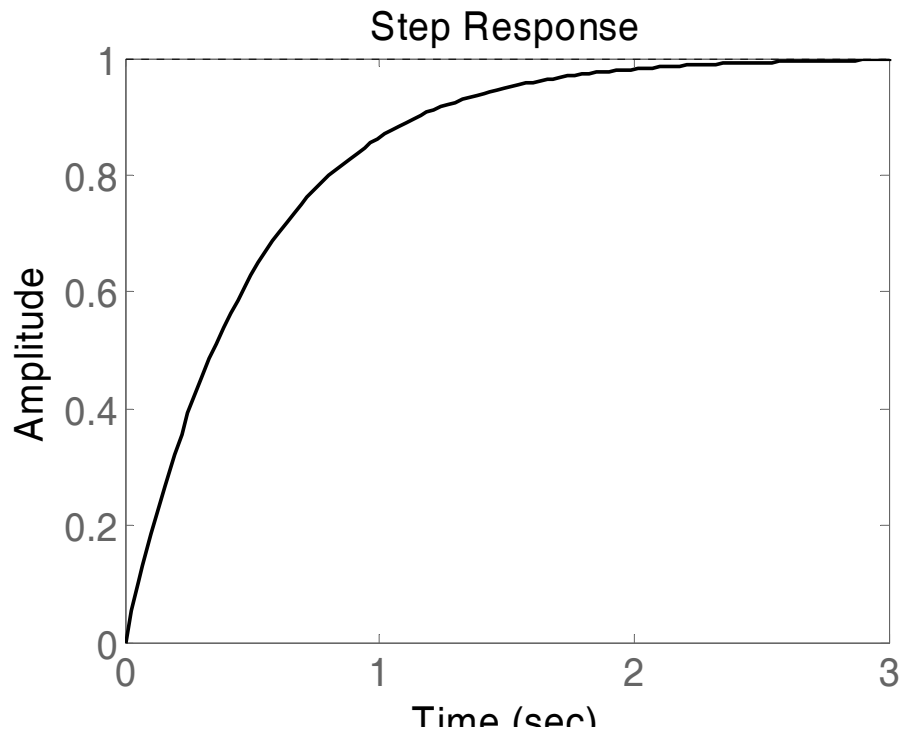
or Open Loop (OL) TF:



Our Task: DESIGN  $G_c$  and  $H$  (if applicable).

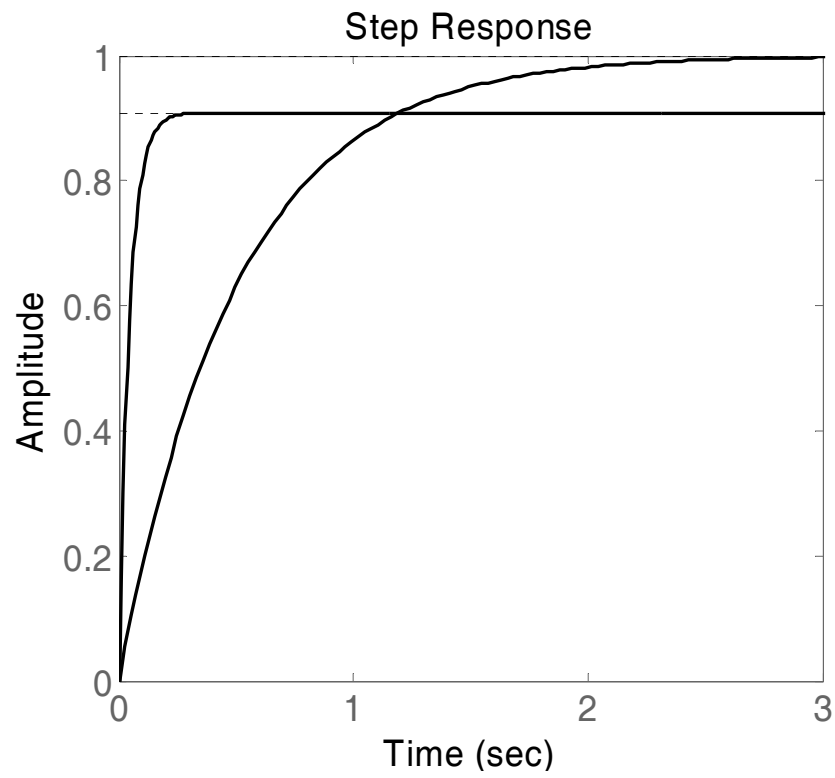
$$\text{OLFT: } G = \frac{2}{s + 2}$$

Open loop response (desired output=1):

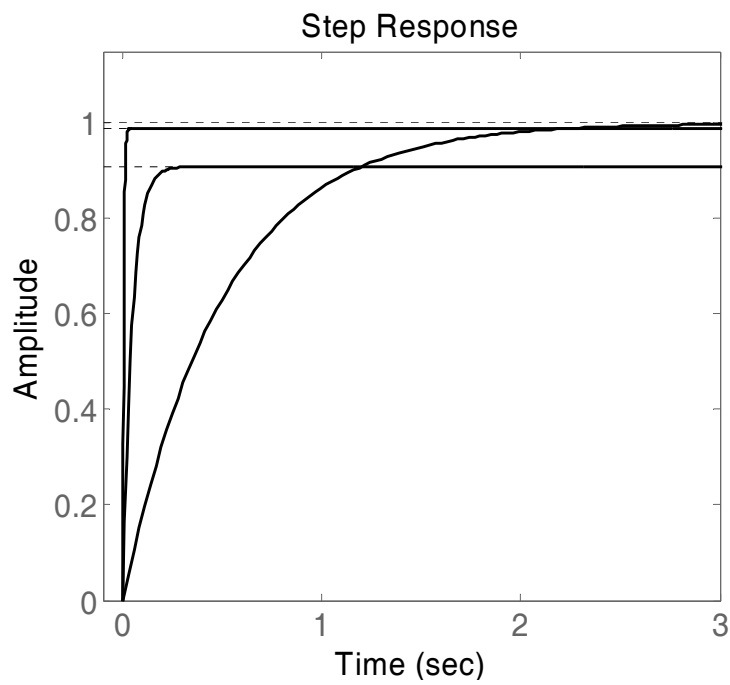


$$\text{CLTF: } G_{CL} = \frac{C}{R} = \frac{K}{s+2+K} \Rightarrow C_{ss} = \frac{K}{2+K}$$

Closed loop response (k=10):



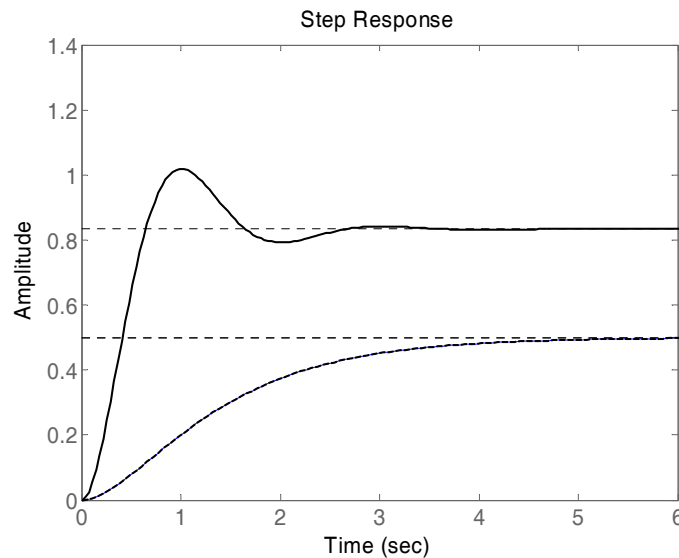
K=100:



## Higher order systems

Until now we have seen only 1<sup>st</sup> order systems with feedback. In these systems, the feedback and the controller influence the steady state and the time constant. The decrease of the time constant means that we moved the pole further into “minus infinity” area. Hence we changed the s-plane of the system. What is it going to happen if we use a 2<sup>nd</sup> order system in a feedback system?

$$G = \frac{1}{(s+1)(s+2)}$$



So apart from faster system and smaller steady state error we have oscillations!

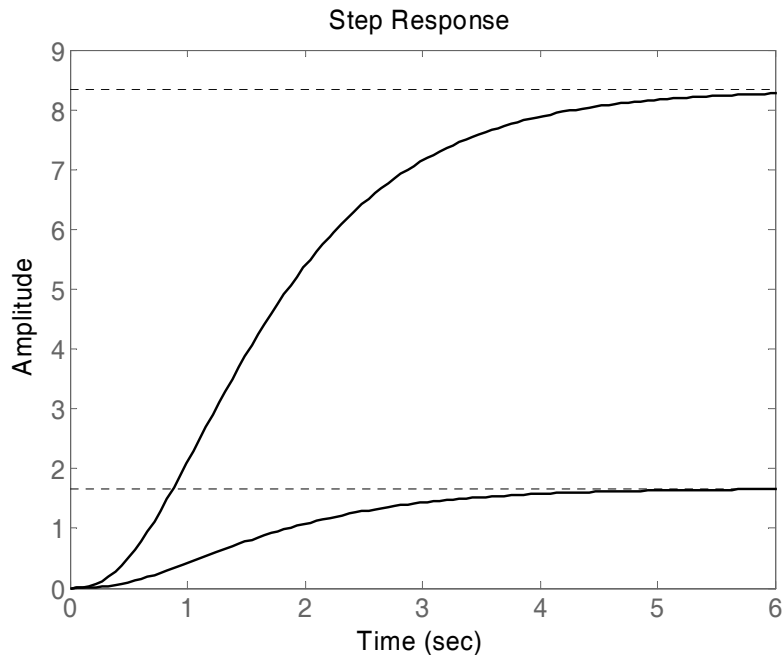
$$G_{CL} = \frac{k}{(s+1)(s+2)+k} = \frac{k}{s^2 + 3s + 2 + k}$$

CE:  $s^2 + 3s + 2 + k = 0$  but the general form:  $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$  so  
 $2\zeta\omega_n = 3$  and  $\omega_n^2 = 2 + k$  and for  $k=10$   $\omega_n^2 = 12 \Leftrightarrow \omega_n = \sqrt{12}$  and  
hence  $2\zeta\sqrt{12} = 3 \Rightarrow \zeta = 0.433$ :

So the feedback and the controller can completely change the location of the poles in the s-plane.

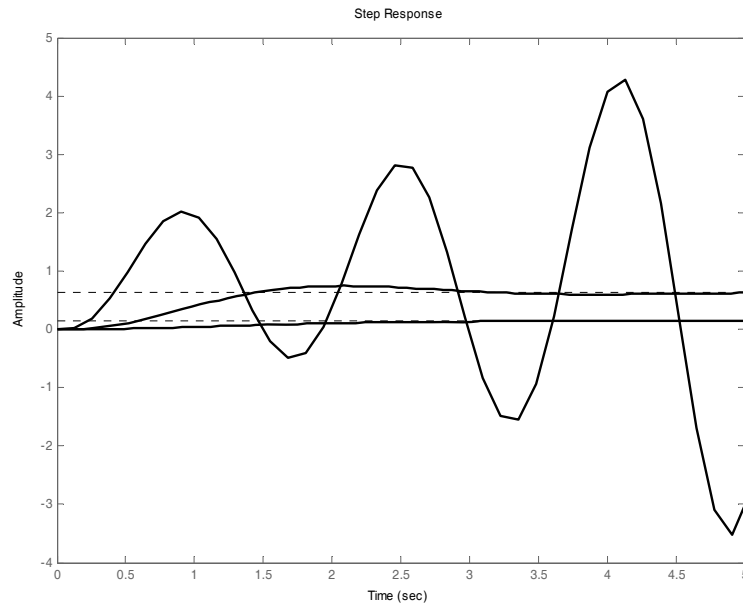
Example: 
$$G = \frac{K}{(s+1)(s+2)(s+3)}$$

The open loop response for various gains is:



The open loop system will be stable for all values of K since they do not influence the poles of the system.

The response of closed loop system for  $K=1, 10, 100$  is:



Hence the feedback may introduce instability.

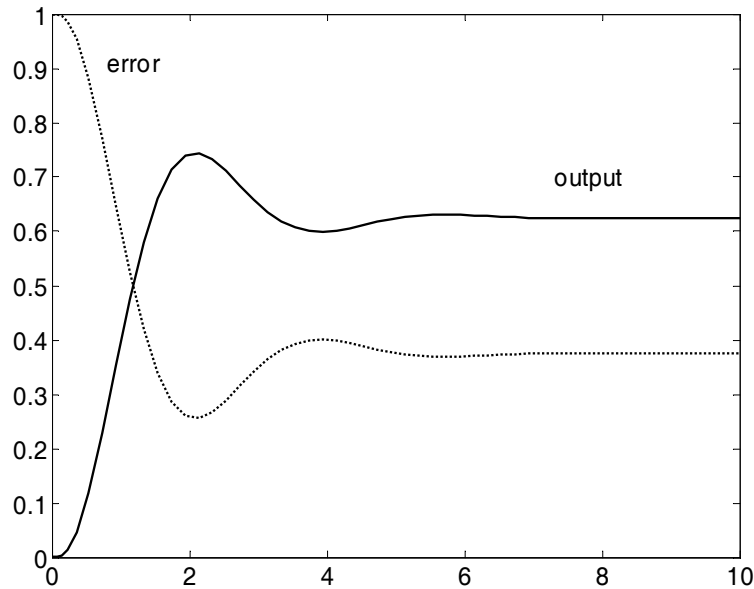
To understand why we have these changes solve the CE.

Properties of feedback systems:

1. Minimise steady state error.
2. Faster system.
3. Less sensitive to system uncertainties.
4. Introduce instability (even for negative feedback).
5. Expensive (we need to feedback the signal, i.e. use a sensor).

## PID control

Consider the system:  $G(s) = \frac{K}{(s+1)(s+2)(s+3)}$  for  $K=10$ :

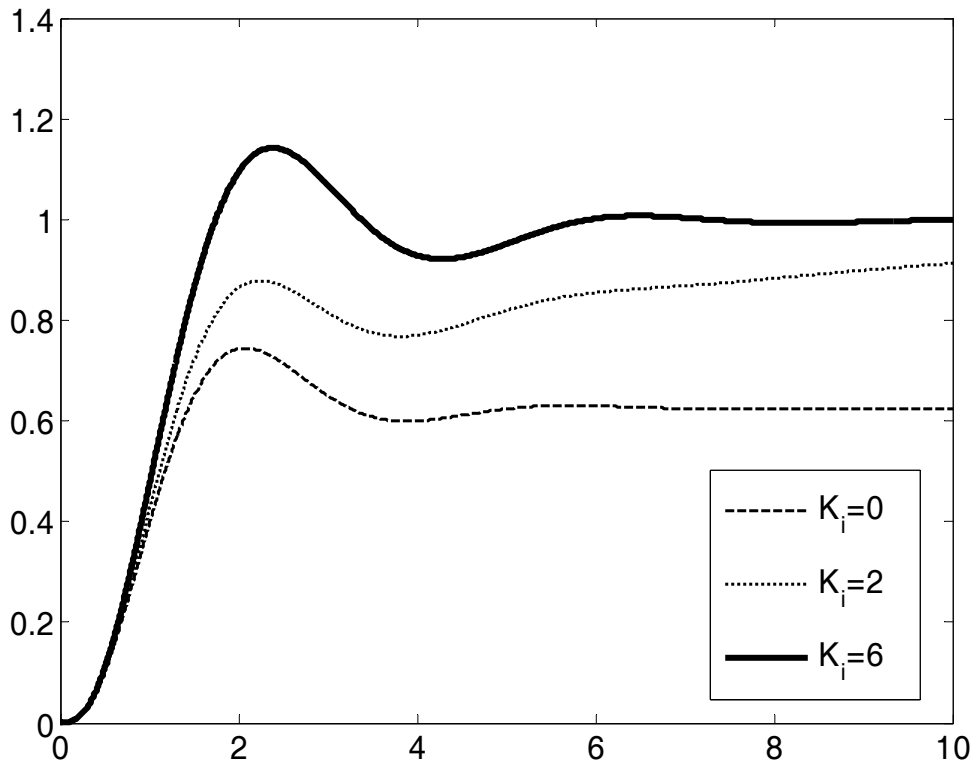
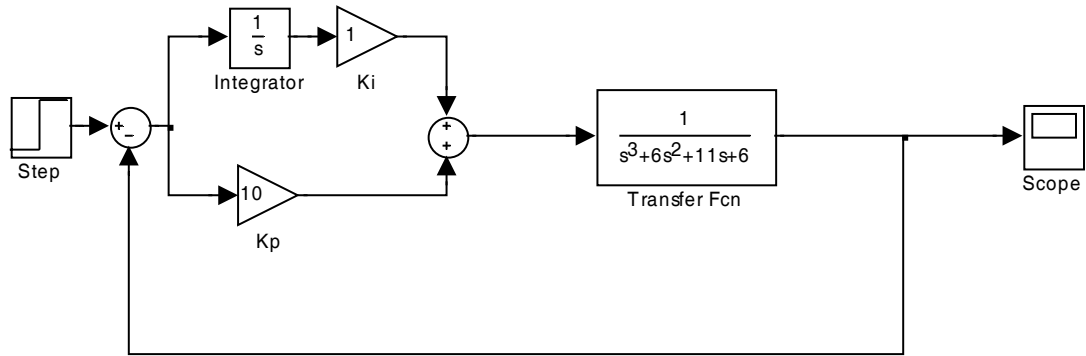


To increase the type of the OLTF (which is  $G_C \times G$ ) we add an integrator:

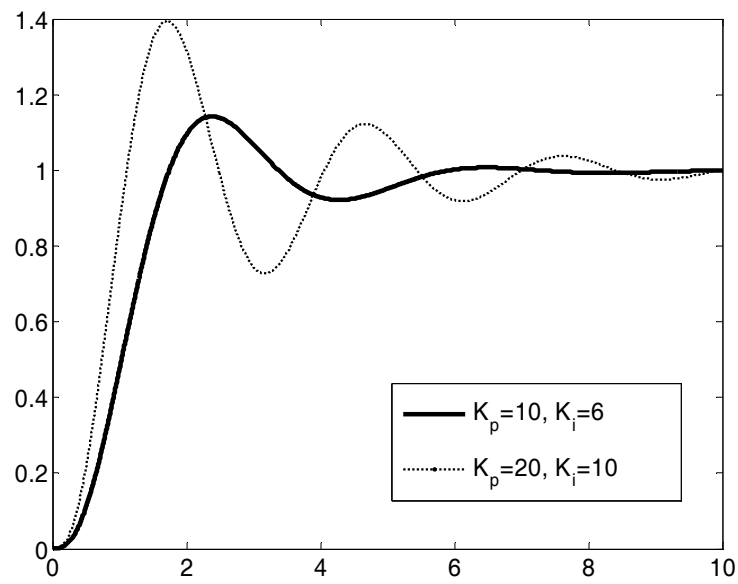
$$G_c(s) = K_p + K_i \frac{1}{s} \Rightarrow G_{OL}(s) = \frac{sK_p + K_i}{s} \frac{1}{(s+1)(s+2)(s+3)}$$

(This is the so-called PI controller).

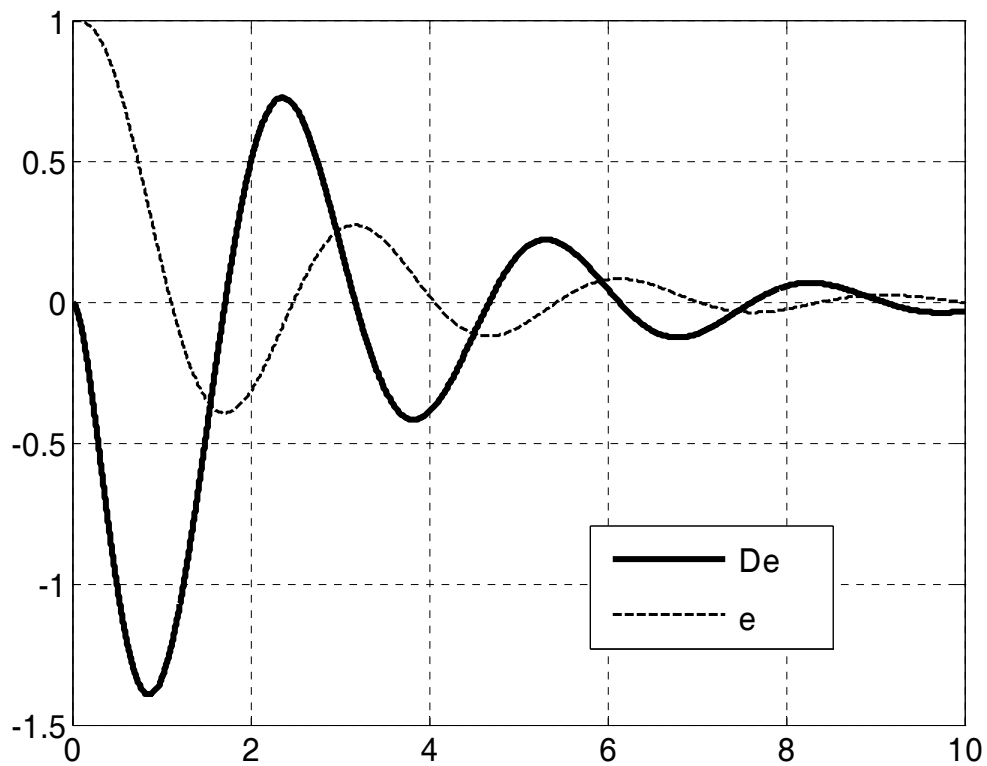




Further increase of Kp, Ki:



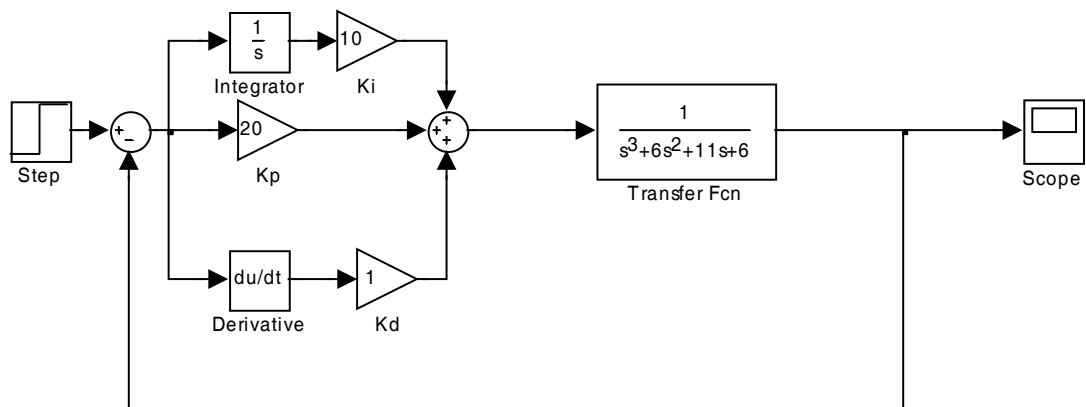
Check the derivative of error, i.e. the rate of change of e:

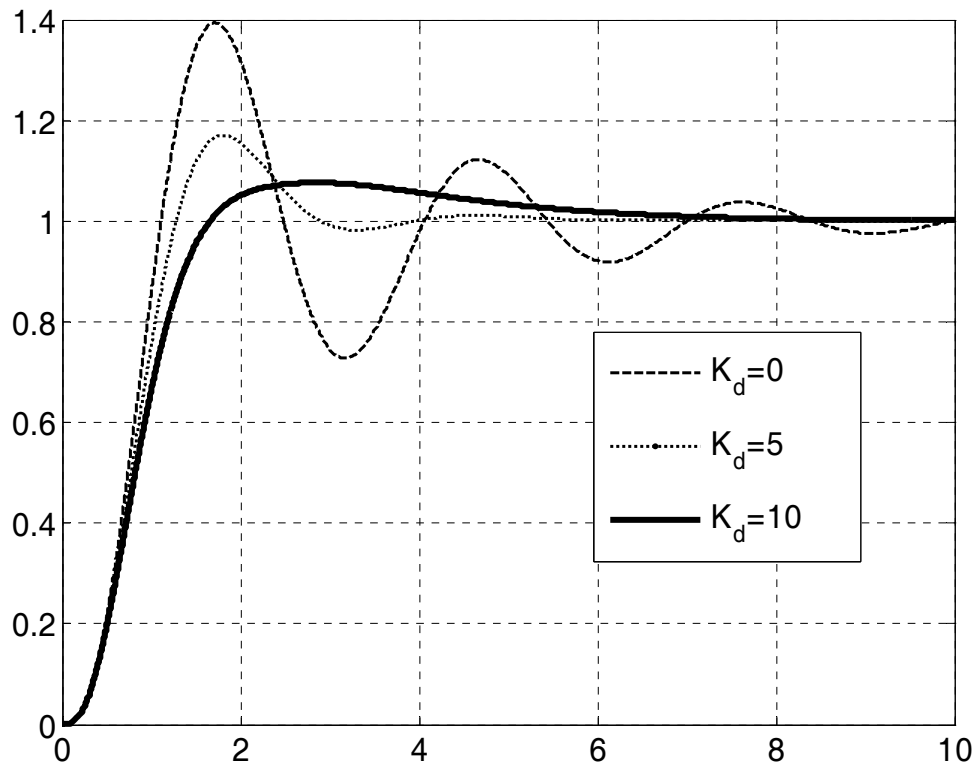


Maximum value of De just before e=0. So De can control the oscillations:

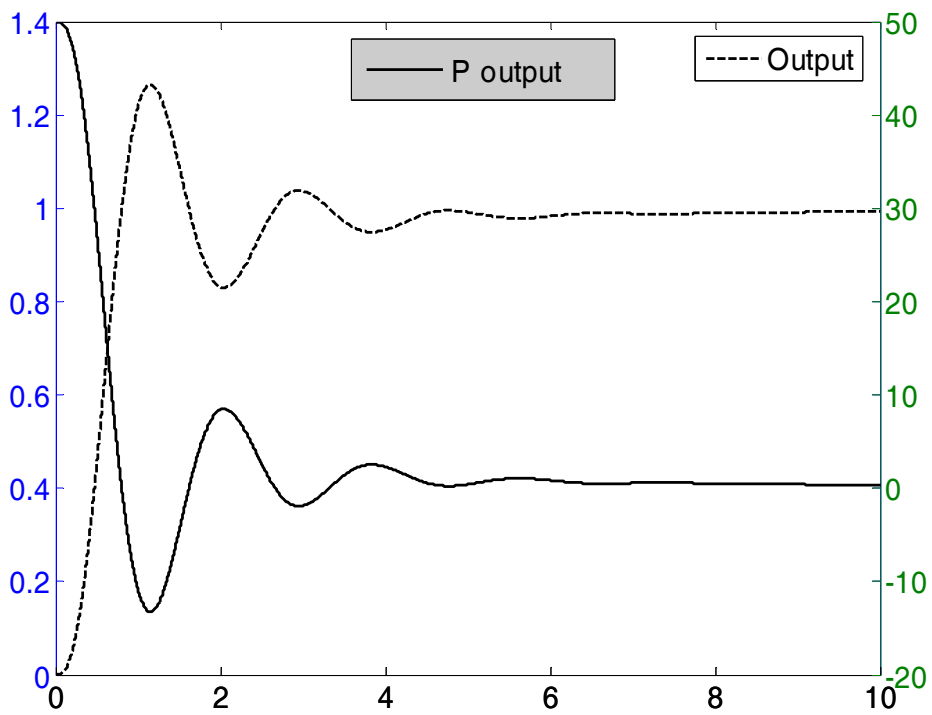
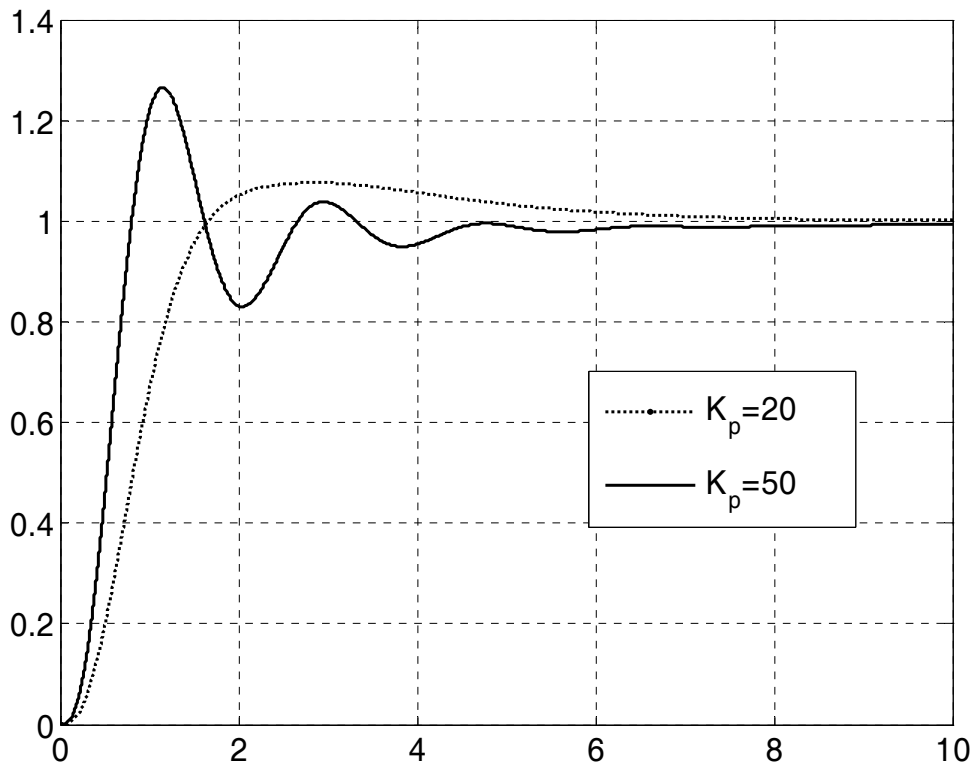
$$G_c(s) = K_p + K_i \frac{1}{s} + K_d s \Rightarrow$$

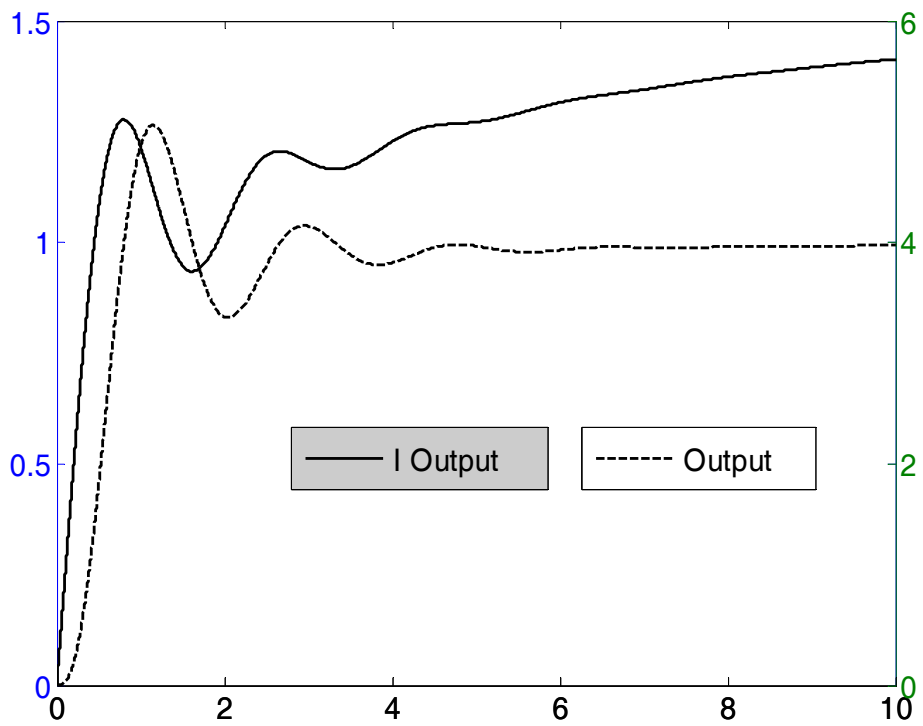
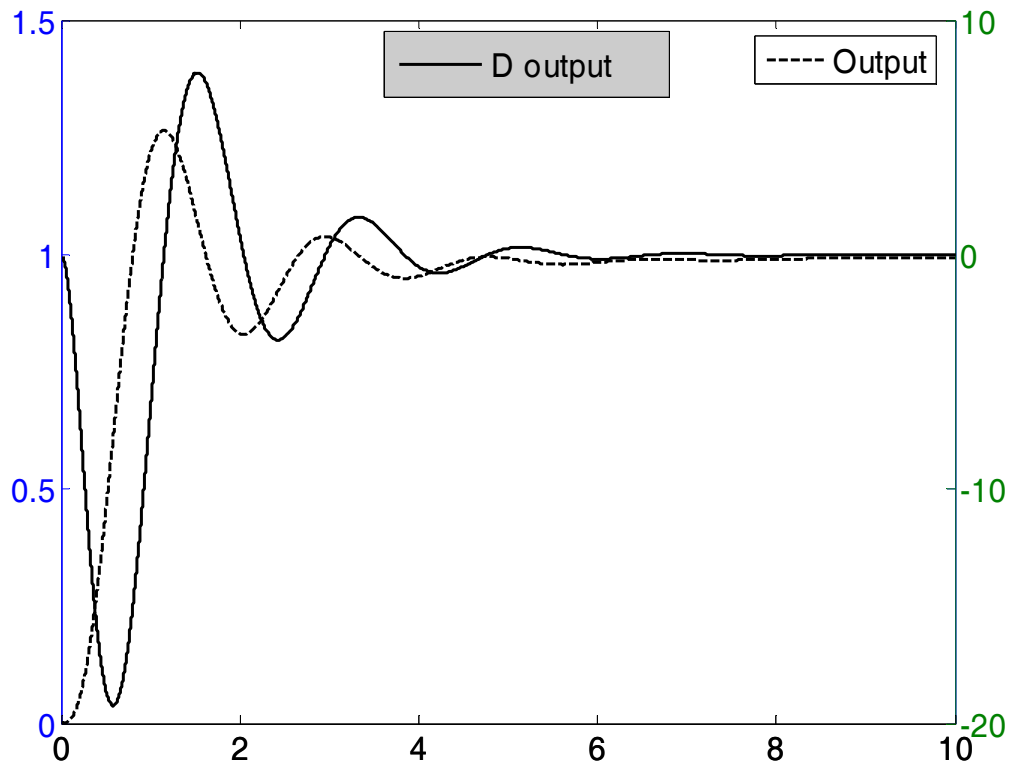
$$G_{OL}(s) = \frac{s^2 K_d + s K_p + K_i}{s} \frac{1}{(s+1)(s+2)(s+3)}$$





Since I have no oscillations I can increase  $K_p$  a little bit more to make the system faster:





Another way to write the PID controller:

$$G_{CL}(s) = K_p + \frac{K_i}{s} + K_d s = K_p \left( 1 + \frac{K_i}{K_p} \frac{1}{s} + \frac{K_d}{K_p} s \right)$$

$$G_{CL}(s) K_p \left( 1 + \frac{1}{T_i s} + T_d s \right)$$

### Tuning of PID controllers

1. Trial and error.
2. Ziegler Nichols I
3. Ziegler Nichols II
4. Root locus
5. Frequency response
6. Other advanced control methods

#### **Trial and error:**

P: Faster system, in some cases reduces the error (can cause instability).

I: Reduces the steady state error, increases the number of oscillations.

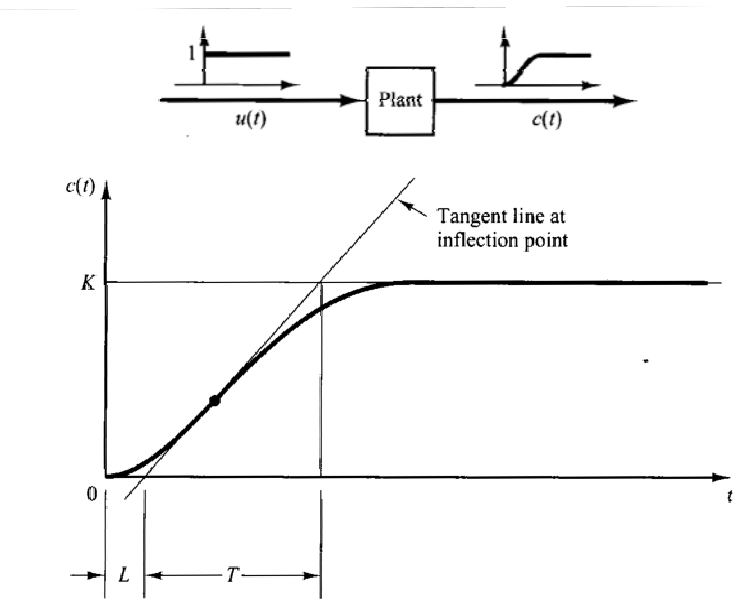
D: Reduces the oscillations.

## Ziegler Nichols I

Assume a system with no delays (we do not study these systems) and with no-complex conjugate poles.



Its open loop step response may look like (obtained experimentally or from simulations):



This can be modelled as:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Based on that we have the following table:

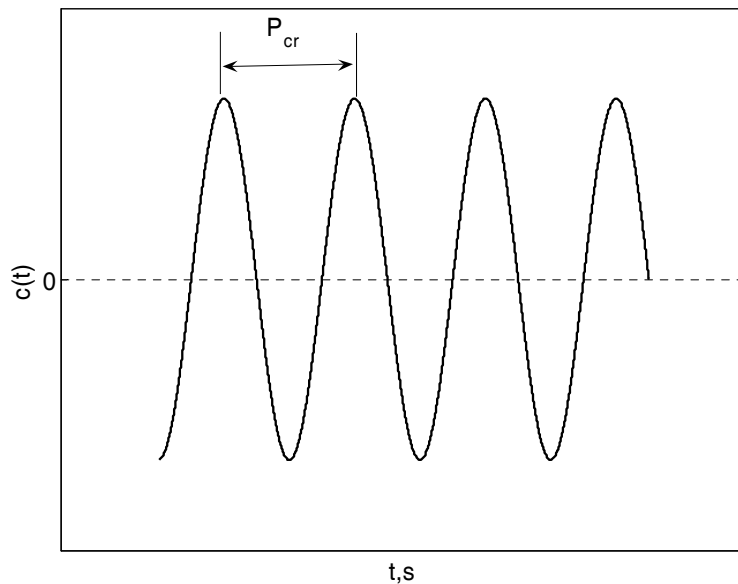


| Type of controller | $K_p$    | $T_i$    | $T_d$  |
|--------------------|----------|----------|--------|
| P                  | $T/L$    | $\infty$ | 0      |
| PI                 | $0.9T/L$ | $L/0.3$  | 0      |
| PID                | $1.2T/L$ | $2L$     | $0.5L$ |

### Ziegler Nichols II

Initially assume  $K_i=K_d=0$ .

Increase  $K_p$  until the system is marginally stable. Record the  $K_p=K_{cr}$  and the frequency of oscillations:



| Type of controller | Kp           | Ti                    | Td            |
|--------------------|--------------|-----------------------|---------------|
| P                  | $0.5K_{cr}$  | $\infty$              | 0             |
| PI                 | $0.45K_{cr}$ | $\frac{1}{1.2}P_{cr}$ | 0             |
| PID                | $0.6K_{cr}$  | $0.5P_{cr}$           | $0.125P_{cr}$ |

These methods aim at achieving an overshoot of 25%.

### Root locus method

With the RL we specifically target a pole location at the s-plane, i.e. we target damping factors, natural and damped frequencies.

Example:

The OLTF is  $G(s) = \frac{1}{s^2 + 11s - 34}$  use a PI controller.

$$G_{OL}(s) = \left( K_p + \frac{K_i}{s} \right) \frac{1}{s^2 + 11s - 34} = \frac{sK_p + K_i}{s(s^2 + 11s - 34)}$$

$$G_{CL}(s) = \frac{sK_p + K_i}{s(s^2 + 11s - 34) + sK_p + K_i}$$

This is a 3<sup>rd</sup> order system = 2<sup>nd</sup> order x 1<sup>st</sup> order:

$$\text{CE: } s(s^2 + 11s - 34) + sK_p + K_i = (s + a)(s^2 + 2\zeta\omega_n s + \omega_n^2)$$

$$s^3 + 11s^2 + s(-34 + K_p) + K_i = s^3 + (2\zeta\omega_n + a)s^2 + (2\zeta\omega_n a + \omega_n^2)s + a\omega_n^2$$

$$\left\{ \begin{array}{l} 11 = 2\zeta\omega_n + a \\ -34 + K_p = 2\zeta\omega_n a + \omega_n^2 \\ K_i = a\omega_n^2 \end{array} \right\}$$

$$\text{Assume that the design specs are: } \left\{ \begin{array}{l} \omega_n = 6 \\ \zeta = 0.5 \end{array} \right\}$$

$$\text{So: } \left\{ \begin{array}{l} 11 = 6 + a \\ -34 + K_p = 6a + 36 \\ K_i = 36a \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} a = 5 \\ K_p = 100 \\ K_i = 180 \end{array} \right\}$$

Homework:

$$\text{Find the PID gains : the CLTF of } G(s) = \frac{1}{s^2 + 6s + 16} \text{ has } \left\{ \begin{array}{l} \omega_n = 6 \\ \zeta = 0.5 \end{array} \right\}$$

and a real pole at  $a = -5$ .

*Solution:*  $K_p = 50$ ,  $K_i = 180$ ,  $K_d = 5$ .

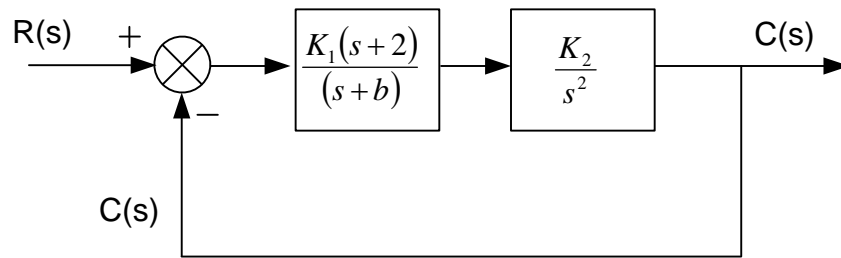
### Frequency response – Advanced methods

We will not study these.

## Other controllers

By using the previous (root locus) method we can design more general controllers.

Example:



$$K_1 K_2 = ? \quad \omega_n = 6$$

$$b = ? \quad \theta = 60^\circ$$

$$\left. \begin{array}{l} \omega_n = 6 \\ \zeta = \cos(60^\circ) \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \omega_n = 6 \\ \zeta = 0.5 \end{array} \right\}$$

$$G(s) = \frac{K_1(s+2)}{s+b} \frac{K_2}{s^2}$$

$$\frac{C(s)}{R(s)} \stackrel{K_1 K_2 = K}{=} \frac{K(s+2)}{(s+b)s^2 + K(s+2)}$$

$$CE : s^3 + s^2 b + Ks + 2K = 0$$

$$CE : s^3 + s^2 b + Ks + 2K = (s+a)(s^2 + 2s\zeta\omega_n + \omega_n^2)$$

$$CE : s^3 + s^2b + Ks + 2K = (s + a)(s^2 + 2s\zeta\omega_n + \omega_n^2)$$

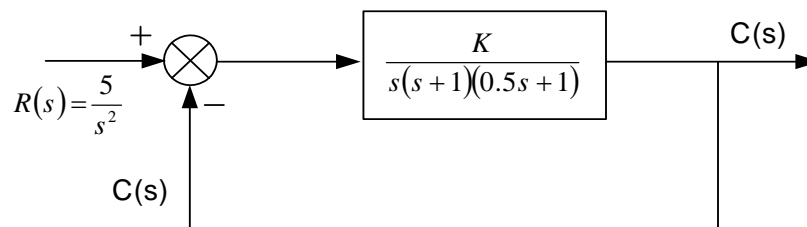
$$CE : s^3 + s^2b + Ks + 2K = s^3 + (2\zeta\omega_n + a)s^2 + (2\zeta a\omega_n + \omega_n^2)s + a\omega_n^2$$

$$\left\{ \begin{array}{l} b = 2\zeta\omega_n + a \\ K = 2\zeta a\omega_n + \omega_n^2 \\ 2K = a\omega_n^2 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} b = 9 \\ a = 3 \\ K = 54 \end{array} \right\}$$

Example:

$$G_{OL}(s) = \frac{K}{s(s+1)(0.5s+1)}, \text{ Unity feedback and input: } r(t)=5t$$

- a) If  $K=1.5$ , find the steady state error
- b) The system must have steady state error,  $E_{ss} < 0.1$  find the value of  $K$



$$\frac{C(s)}{R(s)} = \frac{K}{s(s+1)(0.5s+1) + K} \Rightarrow C(s) = \frac{5}{s^2} \frac{K}{s(s+1)(0.5s+1) + K}$$

$$E(s) = R(s) - C(s) \Rightarrow E(s) = \frac{5}{s^2} \left( 1 - \frac{K}{s(s+1)(0.5s+1) + K} \right)$$

$$E(s) = \frac{5}{s} \left( \frac{(s+1)(0.5s+1)}{s(s+1)(0.5s+1) + K} \right)$$

$$E_{ss} = \lim_{s \rightarrow 0} \left( s \frac{5}{s} \left( \frac{(s+1)(0.5s+1)}{s(s+1)(0.5s+1) + K} \right) \right)$$

$$E_{ss} = \frac{5}{K}$$

$$\text{a) } E_{ss} = \frac{5}{1.5} = 3.33\dots$$

$$\text{b) } E_{ss} < 0.1 \Rightarrow \frac{5}{K} < 0.1 \Rightarrow K > 50$$

Example:

$$G_{OL}(s) = \frac{K}{s(0.02s+1)(0.01s+1)}$$

a) Find the value of K such as the system is marginally stable

b) Find the frequency of oscillations at that point

$$\frac{C(s)}{R(s)} = \frac{K}{s(0.02s+1)(0.01s+1)+K}$$

$$CE: s(0.02s+1)(0.01s+1)+K=0$$

For marginally stable system:  $s = 0 + j\omega$

$$CE: j\omega(0.02j\omega+1)(0.01j\omega+1)+K=0$$

$$CE: j(\omega - 0.02 \times 0.01\omega^3) + (-0.01\omega^2 - 0.02\omega^2 + K) = 0 + 0j$$

$$\left. \begin{array}{l} \omega - 0.02 \times 0.01\omega^3 = 0 \\ -0.03\omega^2 + K = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \omega = 0 \\ \omega = 70.71 \text{ rad / s} \end{array} \right\} \text{ and } K = 150$$