T.F.

$$G(s) = \frac{Out(s)}{In(s)}$$

$$Out(s) = G(s) \cdot In(s)$$

$$G(s) = \frac{N(s)}{D(s)} \begin{array}{l} \rightarrow \text{zeros} \\ \rightarrow \text{poles} \end{array}$$

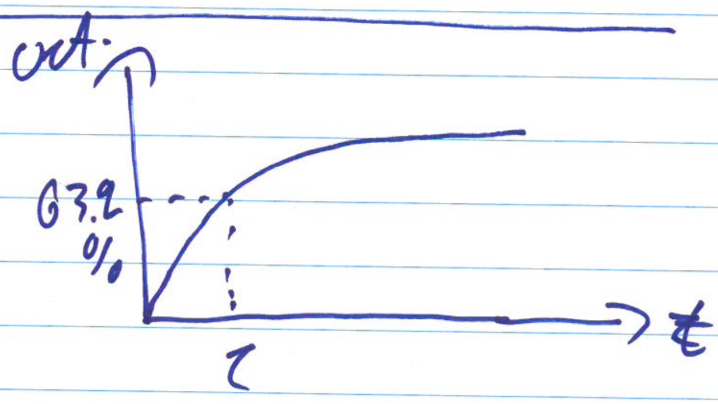
C.E. $\quad D(s) = 0$

if stable

$$Out_{ss} = \lim_{s \to 0} s \, Out(s)$$

$$= \lim_{s \to 0} s \, G(s) \cdot In(s)$$

$$G(s) = \frac{K}{\tau s + 1}$$



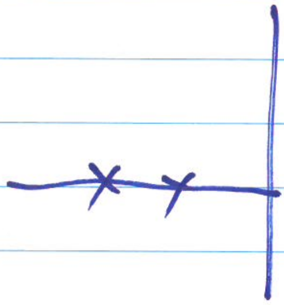
Supplementary Answer Paper

Student Number 45

$$G(s) = \frac{\omega_n^2}{s^2 + 2 \cdot Z \cdot \omega_n \cdot s + \omega_n^2}$$

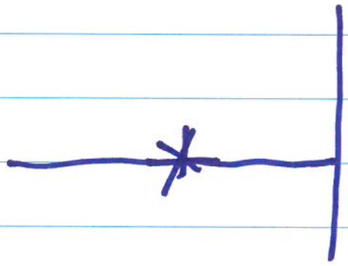
$Z > 1.$

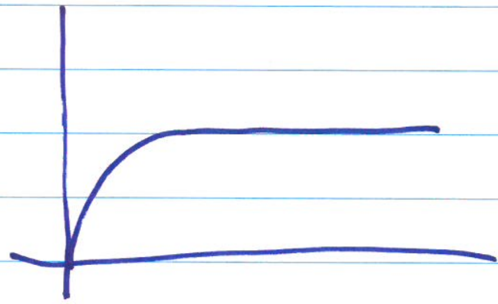
over



$Z = 1$

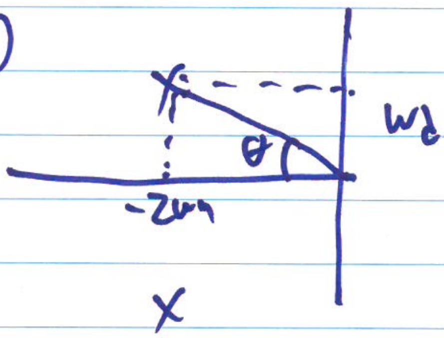
critically

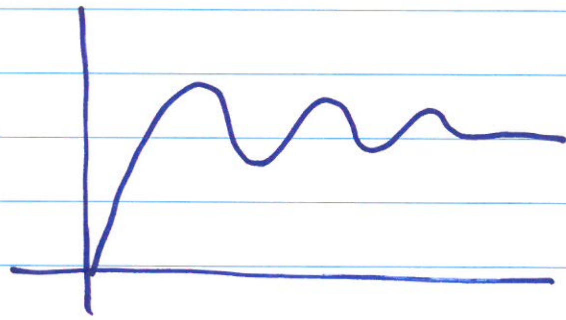




$Z \in (0, 1)$

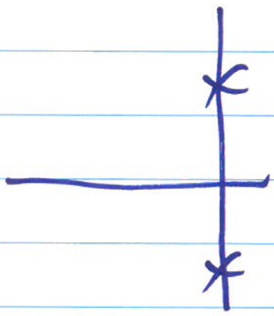
under

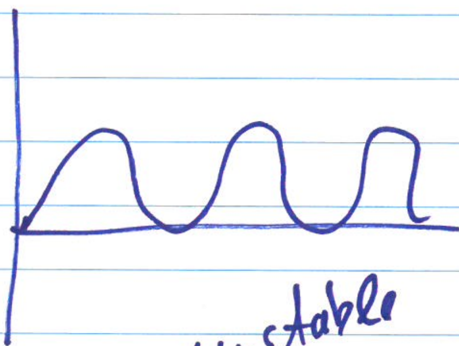




$Z = 0$

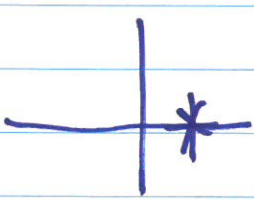
Marginal

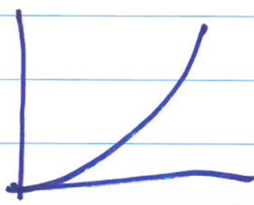




$Z = -1$

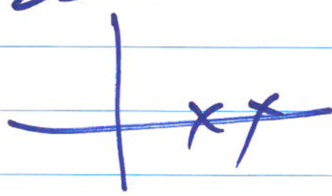
unstable





$Z < -1$

unstable



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Supplementary Answer Paper

Student Number: 46

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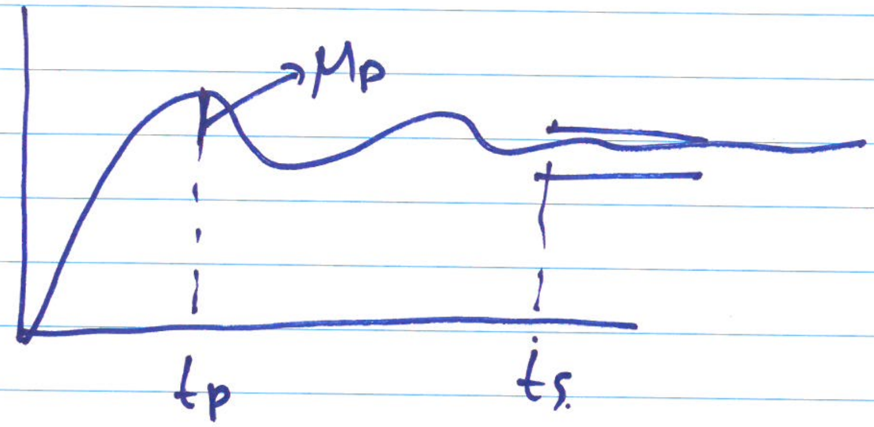

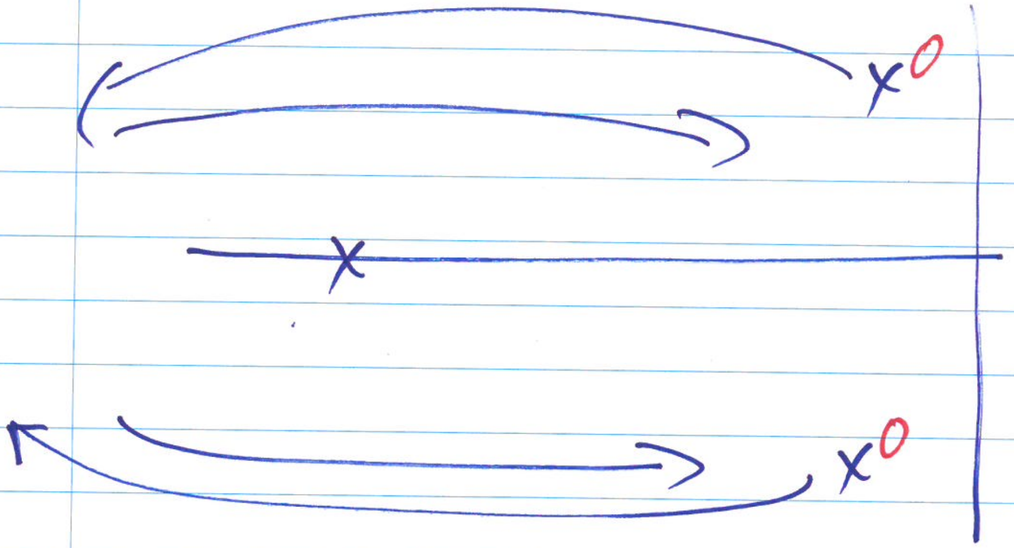

Supplementary Answer Paper

Student Number 47.

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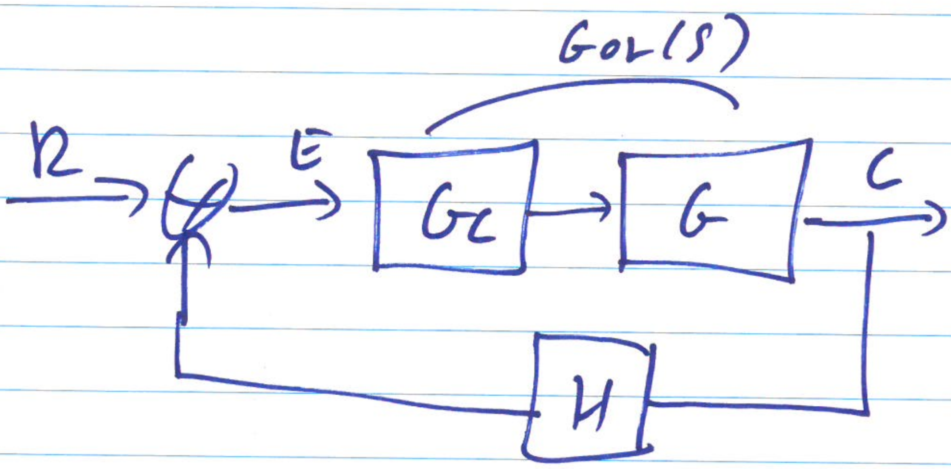


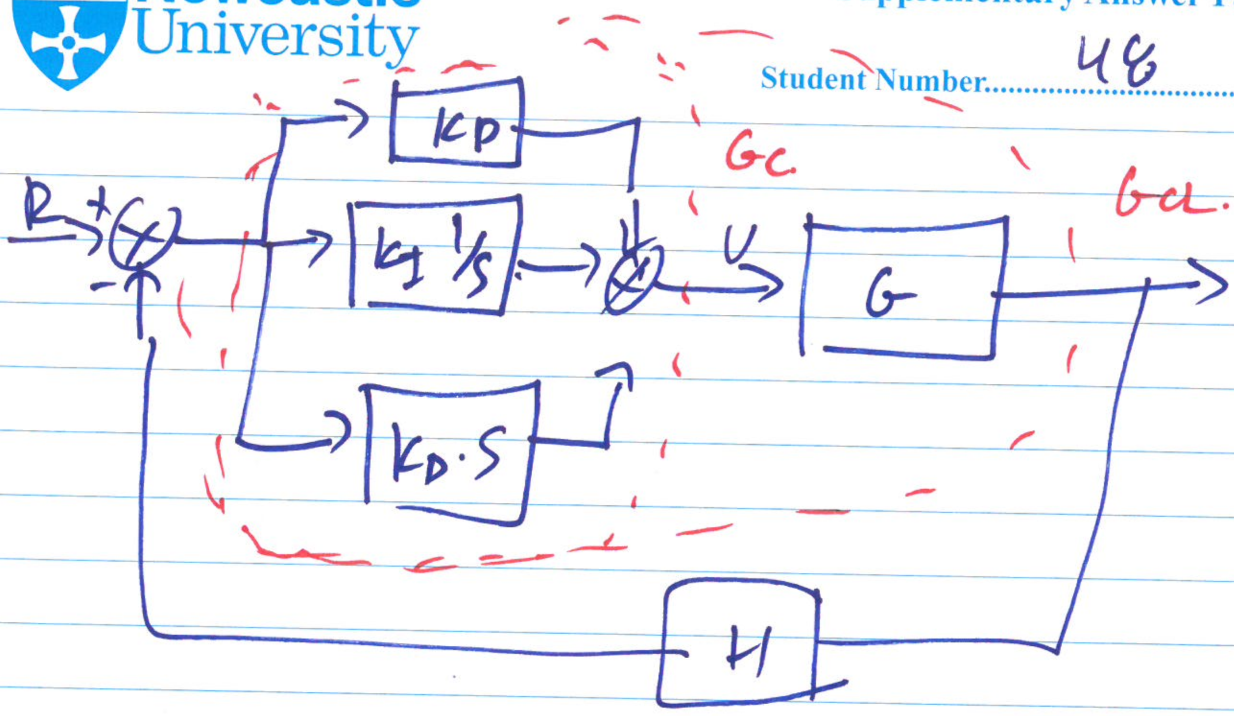
$$\frac{C}{R} = \frac{G_c \cdot G}{1 + G_c \cdot G \cdot H} = \frac{G_{OL}}{1 + G_{OL} \cdot H}$$

$G_c = K$

1st: Faster $t_s \downarrow$
$E_{ss} \downarrow$

2nd $t_s \downarrow$ $E_{ss} \downarrow$ Oscilations

3rd $t_s \downarrow$ $E_{ss} \downarrow$ Osc. + instability



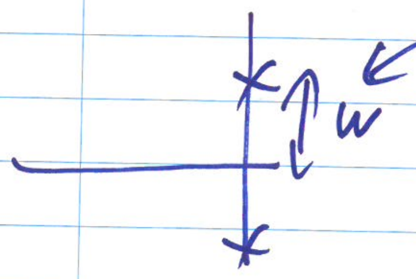
$G_{OL} = G\left(K_P + K_I \frac{1}{s} + K_D \cdot s\right)$

$G_C = K_P\left(1 + \frac{1}{T_i s} + T_D s\right)$

sometimes $U(t) = K_P\left(e(t) + k_I \int e\, dt + k_d \frac{de}{dt}\right)$

ZNII $K_I = K_D = 0$

$K_P \uparrow$ Marginally stable K_{cr} P_{cr}



use Table

$w = \frac{2\pi}{P_{cr}}$

C.L.T.F. $G_{CL}(s) \rightarrow$ C.L.C.E. $D_{CL}(s) = 0$

$s = jw$ $D_{CL}(jw) = 0$

Number of questions only to be written in this margin

$$G(s) = \frac{1}{s^2 + 11s - 34}$$

$$G_c(s) = \frac{K_p s + K_I}{s}$$

$\omega_n = 6$ rad/s

$Z = 0.5$

$$G_{OL} = G \cdot G_c$$

$$\frac{K_p s + K_I}{s(s^2 + 11s - 34)}$$

$$G_{CL} = \frac{G_{OL}}{1 + G_{OL}} = \frac{K_p s + K_I}{s(s^2 + 11s - 34) + K_p s + K_I}$$

C.L.C.E. $s(s^2 + 11s - 34) + K_p s + K_I = 0$

$$s^3 + 11s^2 - 34s + K_p s + K_I = 0$$

$\rightarrow$ $s^3 + 11s^2 + (-34 + K_p)s + K_I = 0$

G.C.E. $(s+a) \cdot (s^2 + 2 \cdot Z \cdot \omega_n \cdot s + \omega_n^2) = 0$

$\rightarrow$ $s^3 + (2 \cdot Z \cdot \omega_n + a) \cdot s^2 +$

$+ (2 \cdot Z \cdot \omega_n \cdot a + \omega_n^2) s + a\,\omega_n^2 = 0$

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Supplementary Answer Paper

Student Number: 50

Number of questions only to be written in this margin

$$\left.\begin{aligned} 11 &= 2 \cdot 2 \cdot \omega_n + a \\ -34 + K_P &= 2 \cdot 2 \cdot \omega_n \cdot a + \omega_n^2 \\ K_I &= a \cdot \omega_n^2 \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} 11 &= 2 \cdot 0.5 \cdot 6 + a \\ -34 + K_P &= 2 \cdot 0.5 \cdot 6 \cdot a + 36 \\ K_I &= a \cdot 36 \end{aligned}\right\} \Rightarrow$$

$a = 5$

$K_P = 100$

$K_I = 180$

$M_P = \dots$ $\qquad t_s$

Number of questions only to be written in this margin

Assume $M_p = 0.1 \rightarrow Z = ?$

$$0.1 = \exp\left(-\frac{Z\cdot\pi}{\sqrt{1-Z^2}}\right)$$

$$\ln(0.1) = \ln(\exp(\quad)).$$

$$-2.3 = -\frac{Z\pi}{\sqrt{1-Z^2}}$$

$$(-2.3)^2 = \left(\frac{-Z\pi}{\sqrt{1-Z^2}}\right)$$

$$5.29 = \frac{Z^2\cdot\pi^2}{1-Z^2}$$

$$(1-Z^2)\,5.29 = (1-Z^2)\,\frac{Z^2\cdot\pi^2}{1-Z^2}$$

$$5.3 - 5.3\cdot Z^2 = \pi^2\cdot Z^2.$$

$$5.3 = (\pi^2 + 5.3)\cdot Z^2$$

$$Z^2 = \frac{5.3}{\pi^2+5.3} = 0.35$$

$$\Rightarrow Z = \sqrt{0.35}\ldots\ldots$$

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Supplementary Answer Paper

Student Number 52

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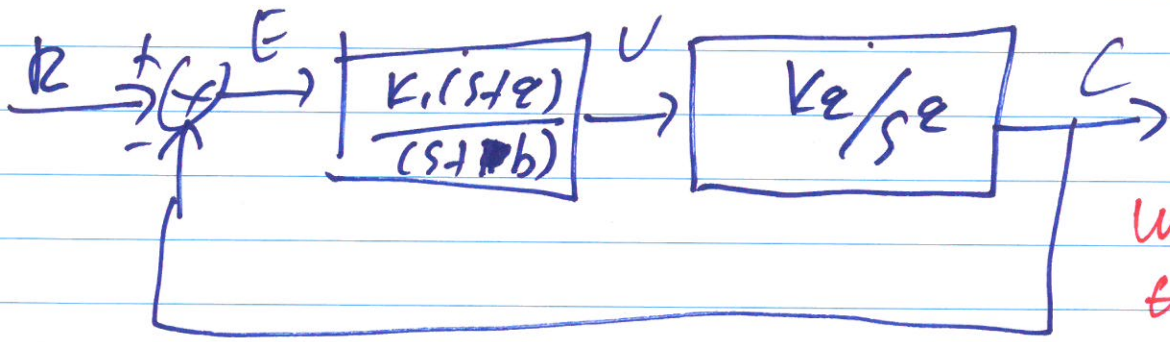


$\omega_n = 6$ rad/s

$\theta = 60$

$K_1 K_2 = ?$

$b = ?$

O.L.T.F

$$G_{OL}(s) = \frac{K_1(s+2)\cdot K_2}{s^2(s+b)}$$

$$= \frac{K(s+2)}{s^2(s+b)} \qquad K = K_1 \cdot K_2.$$

C.L. T.F.

$$G_{CL}(s) = \frac{G_{OL}}{1+G_{OL}} = \frac{K(s+2)}{s^2(s+b)+K(s+2)}$$

C.L.C.E $\quad s^2(s+b) + K(s+2) = 0$

$$s^3 + b\,s^2 + Ks + 2K = 0$$

G.C.E $\quad s^3 + (2\zeta\omega_n + a)\cdot s^2 + (2\cdot\zeta\cdot\omega_n\cdot a + \omega_n^2)s + a\cdot\omega_n^2.$

Compare

$2\cdot\zeta\cdot\omega_n + a = b.$

$2\cdot\zeta\cdot\omega_n\cdot a + \omega_n^2 = K.$

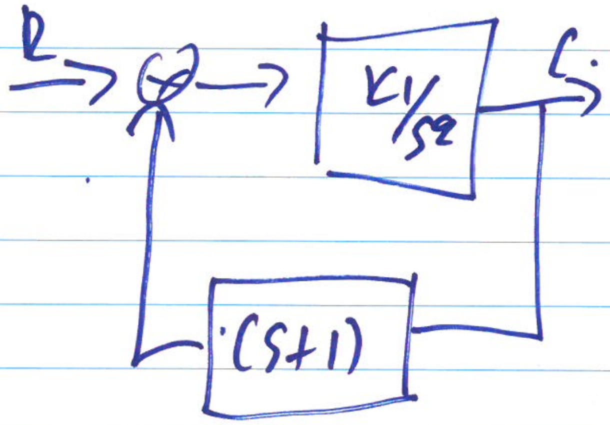
$a\cdot\omega_n^2 = 2K.$

Supplementary Answer Paper

Student Number: 59.

$z = ?$ $\quad z = \cos\theta = \cos 60 = 1/2.$

$\Rightarrow b = 9$	$6 = a + b.$
$a = 3$	$6 \cdot a + 36 = k.$
$k = 54.$	$a \cdot 36 = 2k$



$K_1 = ?$

$M_p = 0.05$

OLTF $\quad G_{OL} = K_1/s^2$

C.L.T.F $\quad G_{CL} = \dfrac{G_{OL}}{1 + G_{OL} \cdot H.} = \dfrac{K_1/s^2}{1 + K_1/s^2 \cdot (s+1)}$

$$= \frac{K_1}{s^2 + K_1(s+1)}$$

C.L. C.E. / G. CE $\quad s^2 + K_1 s + K_1 = 0$

$s^2 + 2 \cdot z \cdot \omega_n s + \omega_n^2.$

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Supplementary Answer Paper

Student Number 53

Number of questions only to be written in this margin

$K_1 = 2 \cdot \zeta \cdot \omega_n$

$K_1 = \omega_n^2$

$M_p = \exp \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = 0.05$

$\frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = -3$

$\frac{\zeta^2 \cdot \pi^2}{1-\zeta^2} = 9$

$\zeta^2 \cdot \pi^2 = 9 - 9\zeta^2$

$\zeta^2(\pi^2+9) = 9$

$\zeta = \sqrt{\frac{9}{\pi^2+9}} = 0.7$

$K_1 = 1.4$

$$G(s) = \frac{1}{s(0.02s+1)(0.01s+1)}$$

Design a PID controller.

O.L.T.F. $G_c(s) = K_P + K_I \frac{1}{s} + K_D \cdot s$

$$K_I = K_D = 0$$

$$G_c(s) = K_P$$

$$G_{OL} = \frac{K_P}{s(0.02s+1)(0.01s+1)}$$

C.L.T.F. $G_{CL} = \frac{G_{OL}}{1+G_{OL}} = \frac{K_P}{s(0.02s+1)(0.01s+1) + K_P}$

C.L.C.E $s(0.02s+1)(0.01s+1) + K_P = 0$

set $s = j\omega$

$$j\omega(0.02j\omega+1)(0.01j\omega+1) + K_P = 0$$

$$(0.02j^2\omega^2 + j\omega)(0.01j\omega+1) + K_P = 0$$

$$(-0.02\omega^2 + j\omega)(0.01j\omega+1) + K_P = 0$$

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Supplementary Answer Paper

Student Number: 55

$$-0.02\cdot 0.01 \, w^2 \cdot jw - 0.02\cdot w^2 + jw\cdot 0.01 jw + jw + Kp = 0$$

$$-2\cdot 10^{-4} w^3 j - 0.02\cdot w^2 - 0.01 w^2 + jw + Kp = 0$$

$$-0.03\cdot w^2 + Kp = 0$$

$$w - 2\cdot 10^{-4}\cdot w^3 = 0 \Rightarrow$$

$$1 = 2\cdot 10^{-4}\cdot w^2 \Rightarrow \dots \quad w = 70.71 \text{ rad/s}$$

$$Kp = +0.03\cdot w^2 = 150$$

$$Pcr = \frac{2\pi}{70.71} = 0.08 \text{ s}$$

$$Kp = 0.6\cdot Kcr = 0.6\cdot 150 = 90$$

$$Ti = 0.5\cdot Pcr = 0.5\cdot 0.08 = 0.04 \text{ s}$$

$$Td = 0.125\cdot Pcr = 0.01 \text{ s}$$

$$Gc(s) = Kp\left(1 + \frac{1}{Ti}\frac{1}{s} + Td\, s\right)$$

$$= 90\left(1 + \frac{1}{0.04}\frac{1}{s} + 0.01\cdot s\right)$$

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Supplementary Answer Paper

Student Number: 56.

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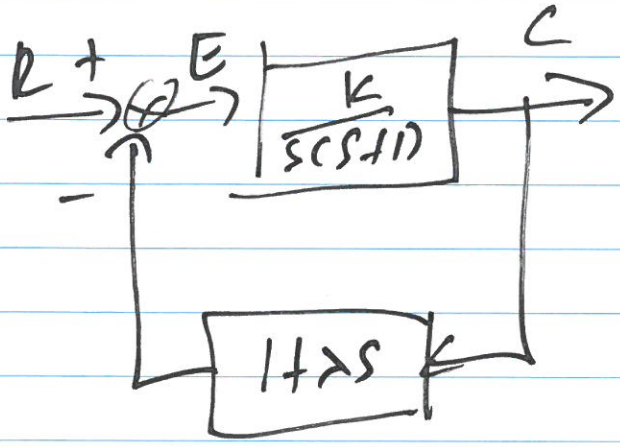

$K = ?$

$Z = ?$

$M_p = 40\% = 0.4$

$T_p = 1s$

O.L.T.F. $G_{OL} = \dfrac{K}{S(S+1)}$

C.L.T.F. $G_{CL} = \dfrac{G_{OL}}{1 + G_{OL} \cdot H}$

$$= \frac{\frac{K}{S(S+1)}}{1 + \frac{K}{S(S+1)} \cdot (1+\lambda S)}$$

$$= \frac{K}{S(S+1) + K(1+\lambda S)}$$

C.E $S^2 + S + K + K\lambda S = 0$

$S^2 + (1 + K\lambda) \cdot S + K = 0$

$S^2 + 2 \cdot Z \cdot \omega_n \cdot S + \omega_n^2 = 0$

$1 + K\lambda = 2 \cdot Z \cdot \omega_n$

$K = \omega_n^2$

$$M_p = 0.4 = \exp\left(\frac{-z\pi}{\sqrt{1-z^2}}\right)$$

$$\ln(0.4) = \frac{-z\pi}{\sqrt{1-z^2}}$$

$$-0.92 = \frac{-z\pi}{\sqrt{1-z^2}}$$

$$0.8464 = \frac{z^2 \cdot \pi^2}{1-z^2}$$

$$0.8464 - 0.8464 \cdot z^2 = z^2 \pi^2$$

$$0.8464 = z^2 (\pi^2 + 0.8464)$$

$$\Rightarrow \quad z = 0.279$$

$$T_p = 1 = \frac{\pi}{\omega_d} \qquad \omega_d = \omega_n \sqrt{1-z^2}$$

$$\omega_d = \frac{\pi}{1} \Rightarrow \omega_d = \pi \text{ rad/sec.}$$

$$\pi = \omega_n \sqrt{1-z^2} \quad \Rightarrow \quad \omega_n = 3.27 \text{ rad/s.}$$

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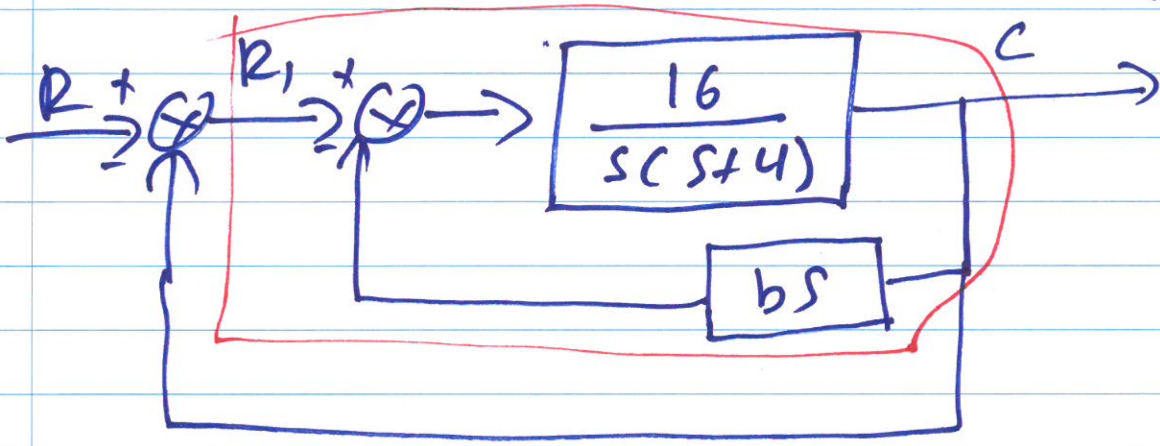
Supplementary Answer Paper

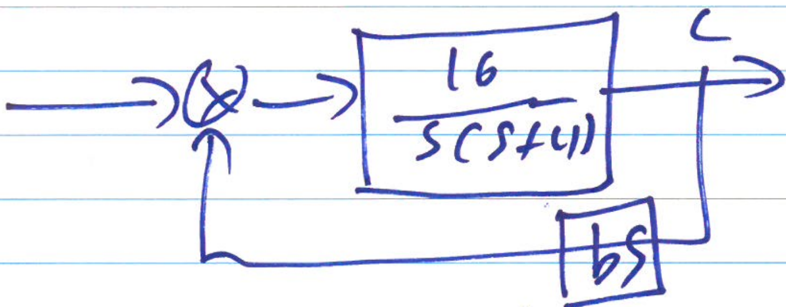
Student Number 58

$1+K\cdot\lambda = 2\cdot 0.279\cdot 3.27$

$K = 3.27^2 = 10.69$

$\lambda = 0.08.$





O.L.T.F. $G = \frac{16}{s(s+4)}$

C.L.TF $G_c = \frac{G}{1+G\cdot bs} = $ ~~$\frac{16}{s(s+4)+16}$~~.

$$\frac{\frac{16}{s(s+4)}}{1+\frac{16}{s(s+4)}\cdot bs} = \frac{16}{s(s+4)+16\cdot bs}$$

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Supplementary Answer Paper

Student Number 59

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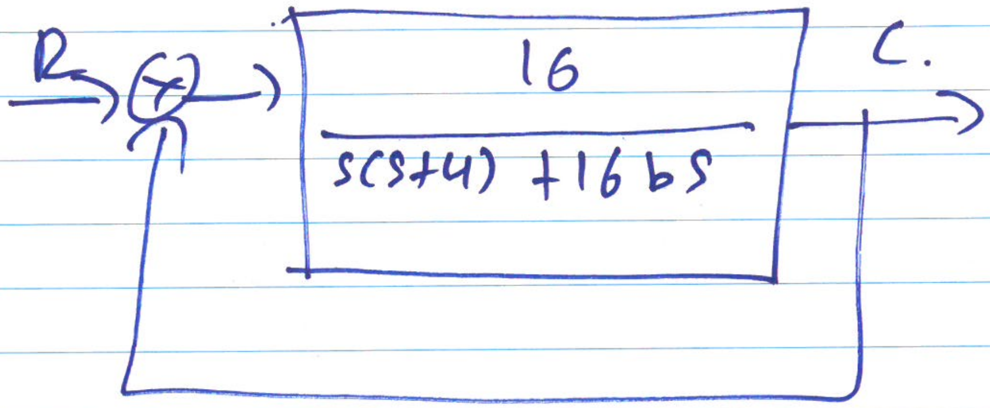


$b = ?$ $\quad z = 0.8$

. - - - -

$\dot{x} + 29x = 0$ $\quad e^{-kt}(x_0 + \int_0^t e^{kt_1} u dt$

$\dot{x} - \sqrt{3}\, x = 1$

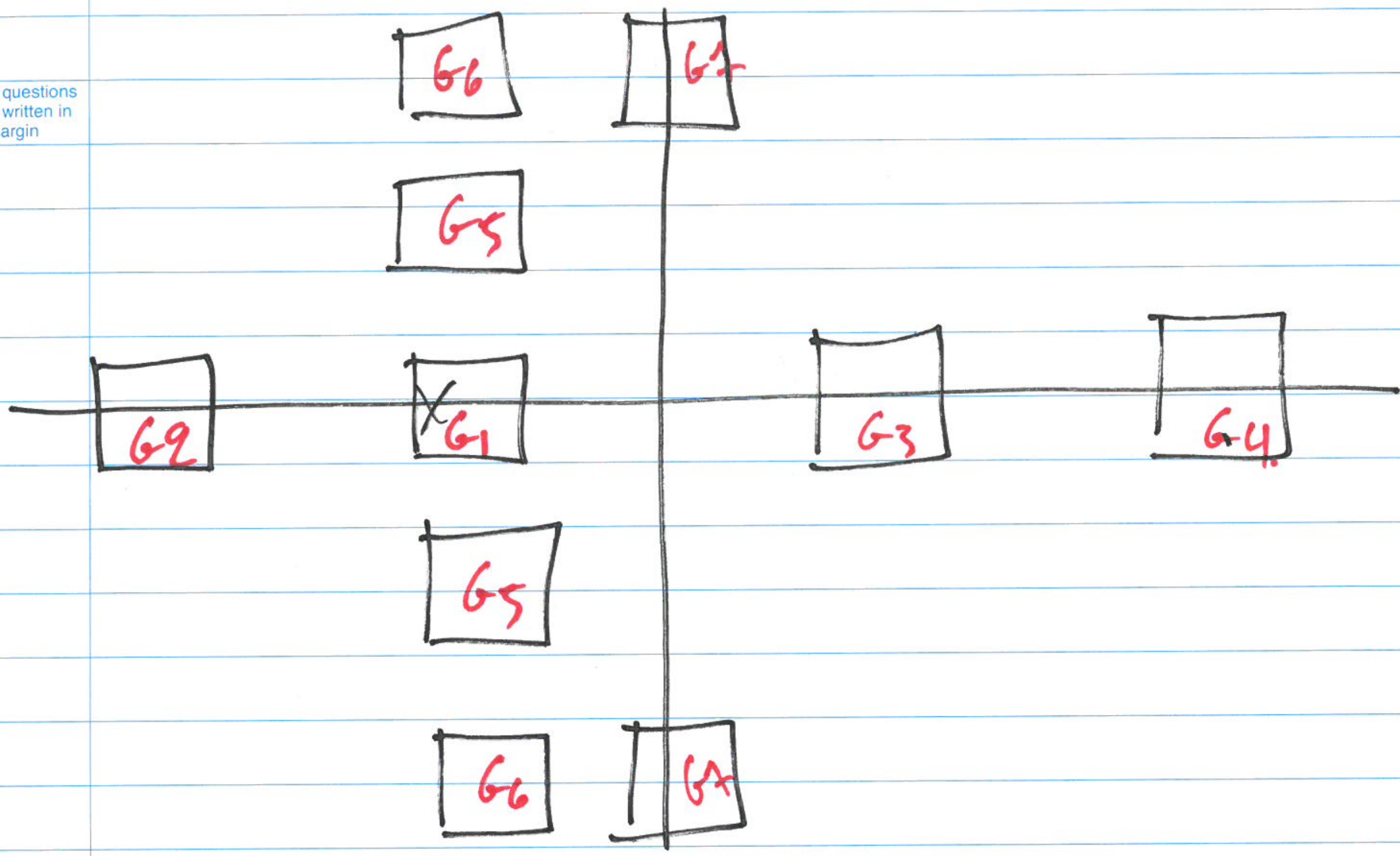
$\dot{x} = \pi.$

stable conv. to zero

unstable as $-\sqrt{3} < 0$

$\int \dot{x} = \int \pi \Rightarrow x = \pi \cdot t + x_0$

unstable



$$G_1 = \frac{1}{s+1} \qquad G_2 = \frac{1}{s+2} \qquad G_3 = \frac{1}{s-1}$$

$$G_4 = \frac{1}{s-5}$$

$$G_5 = \frac{1}{s^2+2s+101} \qquad G_8 = \frac{1}{s^2+2s+26}$$

$$G_7 = \frac{1}{s^2+100}$$

Number of questions only to be written in this margin

$G_1: s=-1$ $G_2: s=-2$ $G_3: s=1$

$G_4: s=+5$ $G_5: s=-1\pm j$ $G_6=-1\pm 10j$

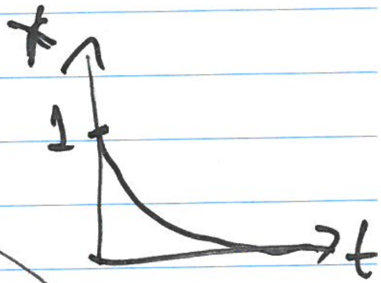
$G_7=0\pm 10j$

$\ddot{x}+15\cdot\dot{x}+50x=0$ $x_0=1$, $\dot{x}_0=0$

$x=e^{rt}$

$r^2+15\cdot r+50=0$

$r_{1,2}=\dfrac{-15\pm\sqrt{15^2-4\cdot 50}}{2}$ → $r_1=-5$, $r_2=-10$



$x=C_1\cdot e^{-5t}+C_2\cdot e^{-10t}$

$x(0)=C_1+C_2=1$

$\dot{x}=-5\cdot C_1 e^{-5t}-10C_2\cdot e^{-10t}$

$\dot{x}(0)=-5C_1-10C_2=0$

$-5C_1=10C_2$, $C_1=-2C_2$

$-2C_2+C_2=1 \Rightarrow C_2=-1$

$C_1=+2$