Revision

$$x^{(n)} = f(x^{(n-1)}, x^{(n-2)}, \dots, x, t) + g(\quad) \cdot u \qquad (1)$$

$x_d = \dots$ Given, f, g are known

$u = ?$: $x \to x_d$

$u = ?$: $\tilde{x} \to 0$, $\tilde{x} = x - x_d$

↓

stable H. ODE of $\tilde{x}$, $\tilde{x} \to 0$

↓ Target

$$t(\tilde{x}^{(n)}, \tilde{x}^{(n-1)}, \dots) = 0 \qquad (2)$$

$u = ?$: (1) $\to$ (2)

$$u = \frac{1}{g}\Big(-f + \underbrace{x_d^{(n)} - h(\tilde{x}^{(n-1)}, \tilde{x}^{(n-2)}, \dots)}_{x^{(n)}}\Big)$$

$$t(\qquad)$$

e.g. $n = 2$

$$\ddot{x} = \underbrace{\cos t \cdot \dot{x} + 3x^2}_{f} + \overbrace{\sqrt{t} \cdot \cos x}^{g} \cdot u$$

$$u = \frac{1}{\sqrt{t}\cos x}\left(-\cos t\,\dot{x} - 3x^2 + \ddot{x}_d - 30\dot{\tilde{x}} - 200\tilde{x}\right)$$

$$\Rightarrow \ddot{\tilde{x}} + 30\dot{\tilde{x}} + 200\tilde{x} = 0 \quad \to \lambda_1 = -10, \quad \lambda_2 = -20$$

$\ddot{x} = f(\quad) + g(\quad)\cdot u$

$x_d = \dots \qquad \tilde{x} = x - x_d$

$u = ?$: $\left.\begin{array}{l} x \to x_d \ (\text{or } \tilde{x} \to 0) \\ \quad \text{AND} \\ \dot{\tilde{x}} \to 0 \end{array}\right\}$

$u = ?$: $s = \dot{\tilde{x}} + \lambda \tilde{x} \to 0$

$\lambda > 0$

$u = ?$: HODE of s is stable

Target t

$u = ?$: $\exists V$ $\begin{cases} V(s) > 0 \\ \dot{V}(s) < 0 \end{cases}$ L.F.

choose $V(s) = \frac{1}{2} \cdot s^2 > 0$

$\dot{V} = s \cdot \dot{s}$

$u = ?$: $s \cdot \dot{s} < 0$, if $\dot{s} = -s$ ($\dot{s} = -ks$)

$\Rightarrow \dot{V} = -s^2 < 0$

$u = ?$: $\dot{s} = -s \Rightarrow$

$$\ddot{\tilde{x}} + \lambda \dot{\tilde{x}} = -(\dot{\tilde{x}} + \lambda \tilde{x}) \Rightarrow$$

$$\ddot{x} - \ddot{x}_d + \lambda \dot{\tilde{x}} = -\dot{\tilde{x}} - \lambda \tilde{x} \Rightarrow$$

$$f + g \cdot u - \ddot{x}_d + \lambda \dot{\tilde{x}} = -\dot{\tilde{x}} - \lambda \tilde{x} \Rightarrow$$

$$u = \frac{1}{g}\left(-f + \ddot{x}_d - (\lambda + 1)\dot{\tilde{x}} - \lambda \tilde{x}\right) \Rightarrow$$

$$\ddot{\tilde{x}} + (\lambda + 1)\dot{\tilde{x}} + \lambda \tilde{x} = 0 \quad \begin{cases} r_1 = -1 \\ r_2 = -\lambda < 0 \end{cases}$$

Important

$$\ddot{x} = f + g\cdot u$$

$$x_d = \dots,\quad \tilde{x} = x - x_d,\quad s = \dot{\tilde{x}} + \lambda\tilde{x} \Rightarrow \dot{s} = \ddot{\tilde{x}} + \lambda\dot{\tilde{x}}$$

$$\Rightarrow \dot{s} = \ddot{x} - \ddot{x}_d + \lambda\dot{\tilde{x}}$$

$$\boxed{\dot{s} = f + g\cdot u - \ddot{x}_d + \lambda\dot{\tilde{x}}}$$

Nothing to do with L.F.

<u>if</u> $u = \frac{1}{g}\left(-f + \ddot{x}_d - (\lambda+1)\dot{\tilde{x}} - \lambda\tilde{x}\right)$

$$\dot{s} = \cancel{f} + \cancel{g}\frac{1}{\cancel{g}}\left(-\cancel{f} + \cancel{\ddot{x}_d} - (\lambda+1)\dot{\tilde{x}} - \lambda\tilde{x}\right) - \cancel{\ddot{x}_d} + \lambda\dot{\tilde{x}}$$

$$\dot{s} = -(\lambda+1)\dot{\tilde{x}} - \lambda\tilde{x} + \lambda\dot{\tilde{x}}$$

$$= -\dot{\tilde{x}} - \lambda\tilde{x} = -s$$

$$\boxed{\begin{aligned} V(s) &= \tfrac{1}{2}s^2 > 0 \\ \dot{V}(s) &= s\cdot\dot{s} = -s^2 < 0 \;\dots \end{aligned}}$$

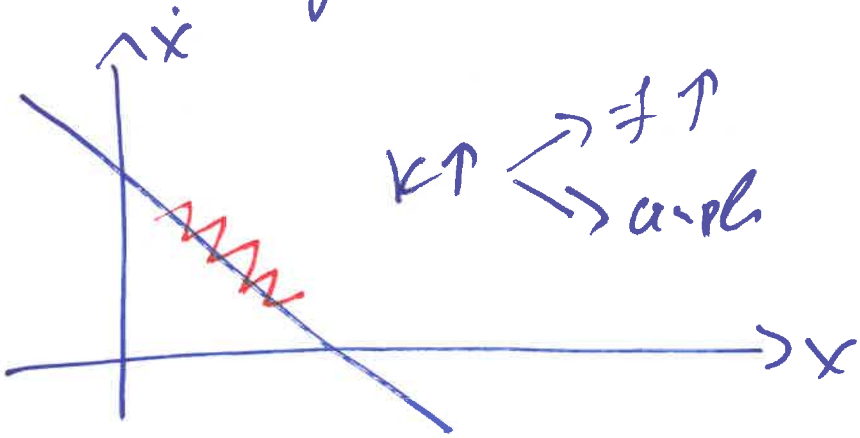
Only L.F.

$\dot{s} = \cancel{-k \cdot s} \quad \rightarrow \quad \dot{s} = -k \cdot sign(s)$

$V(s) = \frac{1}{2} s^2 , \quad \dot{V} = s \cdot \dot{s} \Rightarrow \dot{V} = -k|s| < 0$

$u = ?$: $\dot{s} = -k \cdot sign(s)$

$$\Rightarrow u = \frac{1}{g}\left(-f + \ddot{x}_d - \lambda \dot{\tilde{x}} - k\, sign(s)\right)$$



$k \uparrow$ → $f \uparrow$
→ ampl.

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uncertainty

$\ddot{x} = f + g \cdot u$ → real sys

$\ddot{x} = \hat{f} + \hat{g} u$ → Model

$|f - \hat{f}| < F$
$|g - \hat{g}| < G$
bounded

structure is same

$\ddot{x} = -3\dot{x} - 2x + u$

$\ddot{x} = -3.1\dot{x} - 2.5x + u$

Assume $g = \hat{g} = 1$

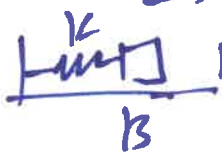
$|f - \hat{f}| = |0.1\dot{x} + 0.5x| < F$

$\dot{V} = \ldots\ldots \qquad k > F$

MRAS

$$\ddot{x} = f(\dot{x}, x, t) + u$$

(e.g.



$m = 1$

$$\ddot{x} = -B\dot{x} - Kx + F$$

$$f(\dot{x}, x, t) = -B\dot{x} - Kx$$

$$\ddot{x} = f(\dot{x}, x, t, P_1, P_2 \dots\dots)$$

$$\ddot{x} = f(\dot{x}, x, t, P) \quad , \quad P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \end{bmatrix}$$

Assume

$$f = f_1(\dot{x}, x, t)\cdot P_1 + f_2(\dot{x}, x, t)\cdot P_2 + \dots\dots$$

$f_1 = \dot{x}$, $P_1 = -B$, $f_2 = x$, $P_2 = -K$

e.g.

$$f = \underbrace{(\dot{x} + \cos x)}_{f_1}\cdot \underbrace{7}_{P_1} + \underbrace{x\cdot|\dot{x}|}_{f_2}\,\underbrace{(-m)}_{P_2} + \underbrace{\cos t}_{P_3}\cdot \underbrace{\cos x}_{f_3}\cdot 3$$

$$f = \sum_i f_i P_i$$

Assume $f_i \to$ known

$P_i \to$ unknown + constant

$$\ddot{x} = \sum_i f_i p_i + u.$$

$$x_d = \dots, \quad \tilde{x} = x - x_d.$$

$$s = \dot{\tilde{x}} + \lambda \tilde{x}$$

$$\dot{s} = \ddot{x} - \ddot{x}_d + \lambda \dot{\tilde{x}}$$

$$= f + g \cdot u - \ddot{x}_d + \lambda \dot{\tilde{x}}$$

$$\boxed{\dot{s} = \sum_i f_i p_i - \ddot{x}_d + \lambda \dot{\tilde{x}} + u}$$

$$\boxed{\dot{s} = u + F \cdot P}$$

$$F = [f_1 \quad f_2 \quad \dots \quad \ddot{x}_d \quad \dot{\tilde{x}}]$$

$$P = [p_1 \quad p_2 \quad \dots \quad -1 \quad \lambda]^T$$

if $P \to$ known

$$\Rightarrow u = -F \cdot P - ks \Rightarrow \dot{s} = -ks$$

So if $V = \frac{1}{2} \underset{>0}{s^2} \Rightarrow \dot{V} = -s^2 < 0$

same analysis as before

$$+ \quad f = \sum_i f_i p_i$$

If $p \to$ known $\dot{s} = u + F \cdot p$

then $u = -F \cdot p - ks$

$\Rightarrow$ problem solved $(\dot{s} = -ks)$

$p \to$ unknown, $\hat{p} \to$ estimate

$\Rightarrow u = -F \cdot \hat{p} - ks$

$$\Rightarrow \dot{s} = u + F \cdot p = -F \cdot \hat{p} - ks + F \cdot p$$
$$= F \cdot (p - \hat{p}) - ks$$
$$= F \tilde{p} - ks \quad , \tilde{p} = p - \hat{p}$$

If $V = \frac{1}{2} s^2 \Rightarrow \dot{V} = s \cdot \dot{s}$
$$= -ks^2 + F \tilde{p} \cdot s$$

$u = ?$: $s \to 0$ AND $\tilde{p} \to 0$

assume $\tilde{p} \in \mathbb{R}^k \Rightarrow k+1$ MODE $\to$ stable

$\Rightarrow k+1$ L.F.

e.g.

$V_s = \frac{1}{2} s^2, \quad V_{p_1} = \frac{1}{2} p_1^2 \quad V_{p_2} = \frac{1}{2} p_2^2$

Group as one ODE

$$V = \frac{1}{2}s^2 + \frac{1}{2}\tilde{p}_1^2 + \frac{1}{2}\tilde{p}_2^2 + \ldots$$

$$V = \frac{1}{2}s^2 + \frac{1}{2}\tilde{p}_1^2 h_1 + \frac{1}{2}\tilde{p}_2^2 h_2 + \ldots$$

$$V = \frac{1}{2}s^2 + \frac{1}{2}\underbrace{\tilde{p}^T}_{1\times k}\underbrace{H}_{k\times k}\cdot\underbrace{\tilde{p}}_{k\times 1}, \quad H = \begin{bmatrix} h_1 & 0 & 0 & \ldots \\ 0 & h_2 & \ldots & \\ \ldots & \ldots & \ldots & \end{bmatrix}$$

$$3^{-1} = \frac{1}{3} \qquad 3^T = 3$$

$$-5^T = -5$$

$$(\text{scalar})^T = \text{scalar}.$$

$A \in \mathbb{R}^{1\times 3}$ $\qquad A\cdot B \in \mathbb{R}^{1\times 1}$

$B \in \mathbb{R}^{3\times 1}$

In general $(A\cdot B)^T = B^T\cdot A^T$ $\qquad (x^T)^T = x$

$$(A\cdot B)^T = B^T\cdot A^T$$

$$= A\cdot B$$

$$(f\cdot g)' = f'g + f\cdot g'$$

$$V = \frac{1}{2} s^2 + \frac{1}{2} \tilde{p}^T H \tilde{p}$$

$\dot{V} = ?$

$$\dot{V} = s \cdot \dot{s} + \frac{1}{2} \underbrace{\underbrace{\dot{\tilde{p}}^T}_{1\times k} \underbrace{H}_{k\times k} \cdot \underbrace{\tilde{p}}_{k\times 1}}_{1\times 1} + \frac{1}{2} \underbrace{\tilde{p}^T H \cdot \dot{\tilde{p}}}_{1\times 1}$$

$$\left(\dot{\tilde{p}}^T H \tilde{p}\right)^T = \dot{\tilde{p}}^T H \cdot \tilde{p}$$

L.A. $= \tilde{p}^T \underbrace{H}_{\text{diag.}} \dot{\tilde{p}}$

Hence

$$\dot{V} = s \cdot \dot{s} + \dot{\tilde{p}}^T H \tilde{p}$$

$$\tilde{p} = p - \hat{p}$$

$$\dot{\tilde{p}} = \dot{p} - \dot{\hat{p}} = -\dot{\hat{p}}$$

$$\left.\begin{aligned} &\dot{V} = s \cdot \dot{s} - \dot{\hat{p}}^T H \tilde{p} \\ &\text{But } \dot{s} = -ks + F \cdot \tilde{p} \end{aligned}\right\} \Rightarrow \dot{V} = s(-ks + F \cdot \tilde{p}) - \dot{\hat{p}}^T H \tilde{p}$$

$$\dot{V} = -ks^2 + s F \tilde{p} - \dot{\hat{p}}^T H \tilde{p}$$

$$\dot{V} = -ks^{\varepsilon} + sF\hat{p} - \dot{\hat{p}}^T H \hat{p}$$

If $\quad s \cdot F \cdot \hat{p} - \dot{\hat{p}}^T H \hat{p} = 0$ ✗

if $\quad sF - \dot{\hat{p}}^T H = 0$

then $\quad sF\hat{p} - \dot{\hat{p}}^T H \hat{p} = 0$

problem is solved.

$$sF - \dot{\hat{p}}^T H = 0 \quad \text{(I want)}$$

$$\dot{\hat{p}}^T H = sF$$

$$\dot{\hat{p}}^T = sFH^{-1}$$

$$\hat{p}^T = \int sFH^{-1}dt$$

$$\hat{p} = \left(\int sFH^{-1}dt\right)^T \rightarrow \text{"estimator"}$$

$$u = -F\hat{p} - sk \rightarrow \text{controller}$$