Revision

37

$$A=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \quad A=\begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix} \quad A=\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$$

2 L.I. 1 L.I. 2 L.I.

$$A=\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$ 2. L.I.

$$e^{At}=\cdots$$

$$A=\begin{bmatrix} a & 0 & 0 & \cdots \\ & a & & \\ & & a & \\ & & & \ddots \\ & & & & a \end{bmatrix}$$

0s or 1s

# of zeros + 1 = LI

$$A=\begin{bmatrix} a & b & 0 & 0 \\ -b & a & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -b & a \end{bmatrix}$$ → 0

4. L.I.

$$A=\begin{bmatrix} a & b & 1 & 0 \\ -b & a & 0 & 1 \\ 0 & 0 & a & b \\ 0 & 0 & -b & a \end{bmatrix}$$ → I

2 L.I.

$$\left.\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}\right\} \xrightarrow{x = T\cdot z} \begin{aligned}\dot{z} &= \hat{A}\cdot z + \hat{B}\cdot u \\ y &= \hat{C}\cdot z.\end{aligned}$$

Better

T = constant

T = inv

$\hat{A} = T^{-1} A \cdot T$

$\hat{B} = T^{-1} \cdot B$

$\hat{C} = C \cdot T$

eigenvalues = same

NOT eigenvectors

$G_x = G_z$

$e^{\hat{A}t}$ = easy $\rightarrow$ $e^{AT} = T e^{\hat{A}t} \cdot T^{-1}$

A

- $\lambda_1 = a, \lambda_2 = b, \lambda_1 \neq \lambda_2$, Target $\hat{A} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

- $\lambda_1 = a = \lambda_2$, Target $\hat{A} = \begin{bmatrix} a & \phi \\ 0 & a \end{bmatrix}$

  e_1 L.I.

- $\lambda_1 = \lambda_2 = a$ Target $\hat{A} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

  e_1, e_2 L.I.

- $\lambda = a \pm bi$ Target $\hat{A} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$

I know, A, $\hat{A}$, T = ? $\hat{A} = T^{-1} A T$

I want to find a T:

$$T \cdot \underbrace{\hat{A}}_{\text{known}} = \underbrace{A}_{\text{known}} T$$

$\lambda_1 \neq \lambda_2$ $\quad \hat{A} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$a \neq b$

$e_1 \quad e_2$ L.I.

$$(A - \lambda I) e = 0$$

$$A \cdot e = \lambda \cdot e$$

$$\left.\begin{aligned} A \cdot e_1 &= \lambda_1 e_1 \\ A \cdot e_2 &= \lambda_2 e_2 \end{aligned}\right\} \Rightarrow A \underbrace{[\overset{2\times 1}{e_1} \quad \overset{2\times 1}{e_2}]}_{\substack{T \\ 2\times 2}} = [e_1 \quad e_2] \cdot \underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}}_{\hat{A}}$$

$A = \dots$ ~~ab~~

$\lambda_1 = \lambda_2 \rightarrow e$

Target $\hat{A} = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$

$\boxed{A \cdot e = \lambda \cdot e}$ $+ 0 \cdot b$

$e = (A - \lambda I) b$

$= Ab - \lambda b$

$\boxed{Ab = e + \lambda b}$

$\rightarrow A \cdot [e \quad b]^T$

$= [e \quad b] \cdot \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$

$A = \dots$ $\lambda = a + bi$ $\quad \lambda, a, b \in \mathbb{R}$

$e = [v + wi]$ $\quad e, v, w \in \mathbb{R}^{n \times 1}$

$a + bi = c + di$
$a = c$
$b = d$

$\hat{A} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$

$A \cdot e = \lambda \cdot e$

$A \cdot [v + wi] = (a + bi) \cdot (v + wi)$

$Av + Awi = a \cdot v + awi + biv - bw$

$= a \cdot v - bw + (aw + bv) \cdot i$

$Av = av - bw$

$Aw = aw + bv$ $\longrightarrow$

$$A \cdot \underbrace{[v \quad w]}_{T} = [v \quad w] \cdot \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

Vector space

A set of objects that satisfy some rules.

$$\square + \square = \square$$

a subset of this set → Basis
L.C. can describe all other elements

$V = \{$ all 2nd order polynomials $\}$

$v_1 = x^2 + x + 1 \qquad v_2 = 3x^2 + 5 \qquad v_3 = 3x - 2$

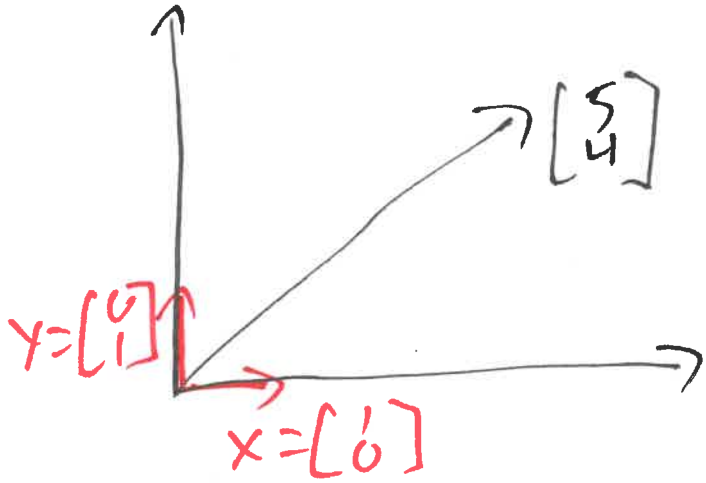
$v_1 + v_2 = 4 \cdot x^2 + x + 5 \quad \in V$

$B_1 = \{x^2, x, 1\}$ Basis

$v_1 = 1 \cdot x^2 + 1 \cdot x + 1 \cdot 1$

$v_2 = 3 \cdot x^2 + 0\, x + 5 \cdot 1$

$B_2 = \{3x^2, 5x, \sqrt{\pi}\cdot 1\}$



$$B_1 = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$$

$$\begin{bmatrix}5\\4\end{bmatrix} = 5\cdot\begin{bmatrix}1\\0\end{bmatrix} + 4\cdot\begin{bmatrix}0\\1\end{bmatrix}$$

$$B_2 = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\2\end{bmatrix}\right\}$$

$$\begin{bmatrix}5\\4\end{bmatrix} = 5\cdot\begin{bmatrix}1\\0\end{bmatrix} + 2\cdot\begin{bmatrix}0\\2\end{bmatrix}$$

$$B_3 = \left\{\begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}5\\2\end{bmatrix}\right\}$$

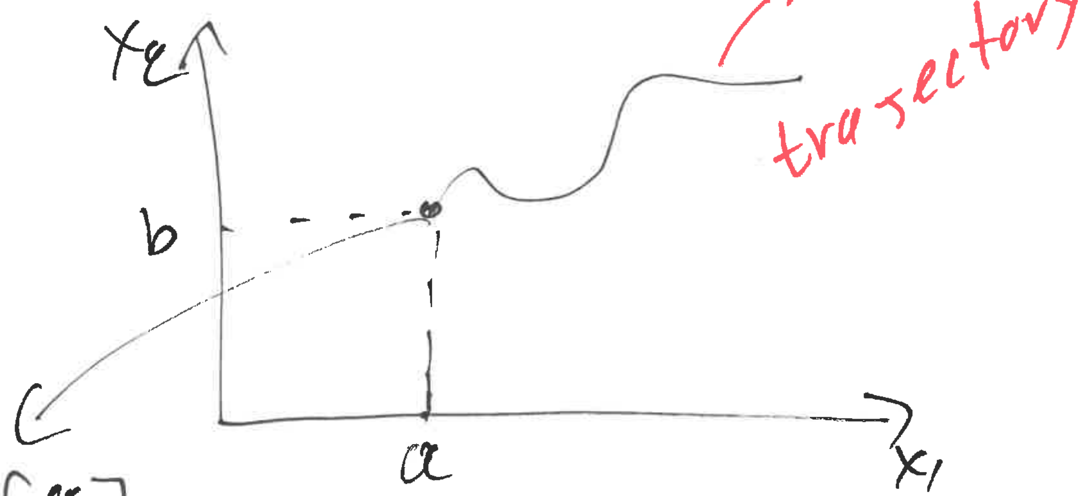
$$\begin{bmatrix}5\\4\end{bmatrix} = a\cdot\begin{bmatrix}1\\3\end{bmatrix} + b\begin{bmatrix}5\\2\end{bmatrix} \Rightarrow$$

$$\left.\begin{aligned} 5 &= a\cdot 1 + b\cdot 5 \\ 4 &= 3\cdot a + 2b \end{aligned}\right\} \Rightarrow \ldots$$

State Space

?

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



$$X = \begin{bmatrix} a \\ b \end{bmatrix}$$

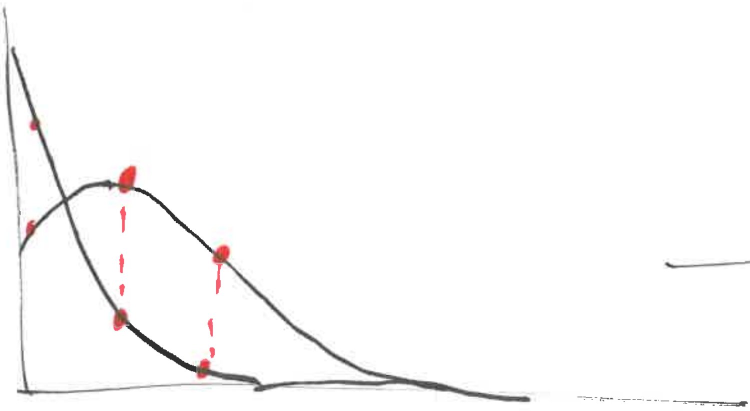
$$X(t)$$

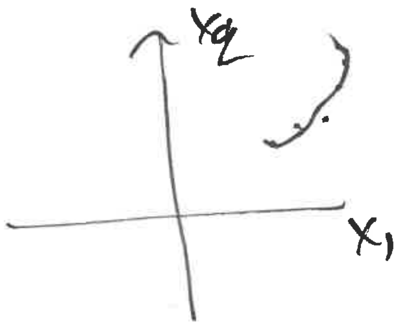
$$\boxed{\begin{aligned} \dot{X} &= (A - BK)\cdot X \\ X &\to 0 \end{aligned}}$$

$$\dot{X} = AX, \quad A = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix}$$

$$\to \lambda_1 = -1, \quad e_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\to \lambda_2 = -6 \quad e_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$





+ Matlab

$$A = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix} \qquad \dot{x} = A \cdot x$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$-1 \qquad -6$$

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x = \underbrace{c_1 \cdot e^{-t}}_{a(t)} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \underbrace{c_2 \, e^{-6t}}_{b(t)} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x = a \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + b \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

x given as a L.C. of eigenvectors

Form eigenbasis

$$X = C_1 \cdot e^{-t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + C_2 \cdot e^{-6t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \dots \quad C_1 = \dots \quad C_2 = \dots$$

$$X_0 = \cancel{3}\,k \cdot e_1 = \cancel{\begin{bmatrix} 6 \\ 3 \end{bmatrix}}$$

$$C_1 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + C_2 \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = k \cdot e_1$$

$$C_1 = k$$
$$C_2 = 0$$

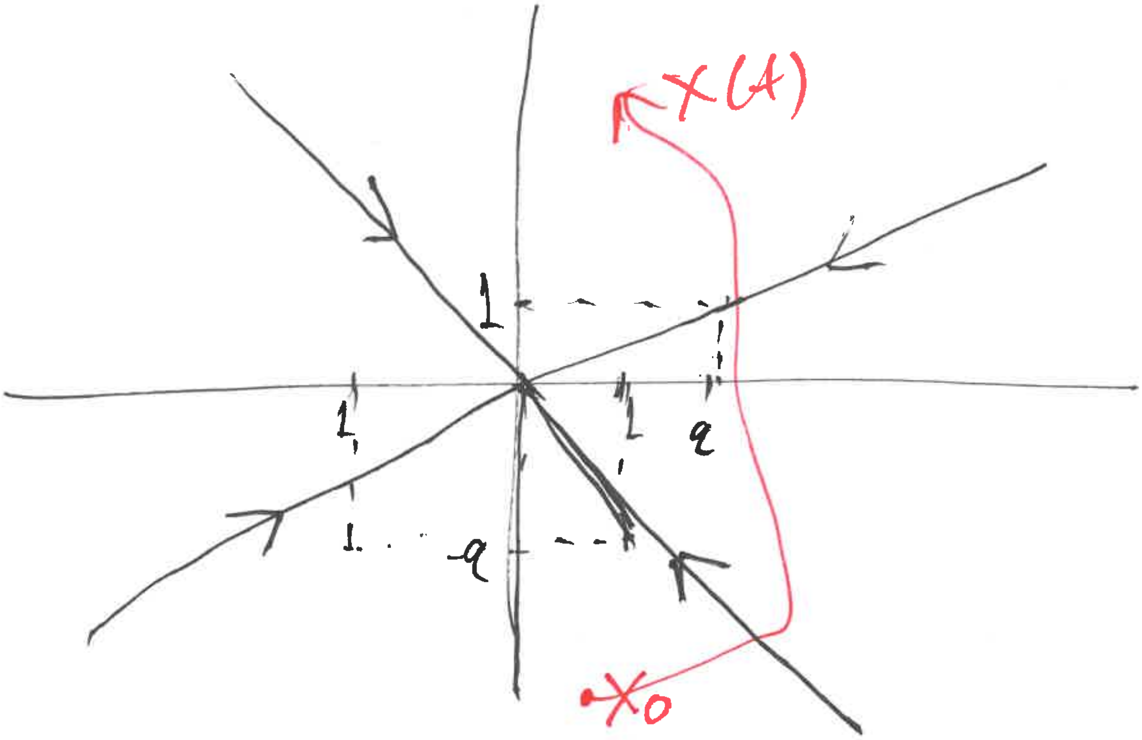
$$X = k \cdot e^{-t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

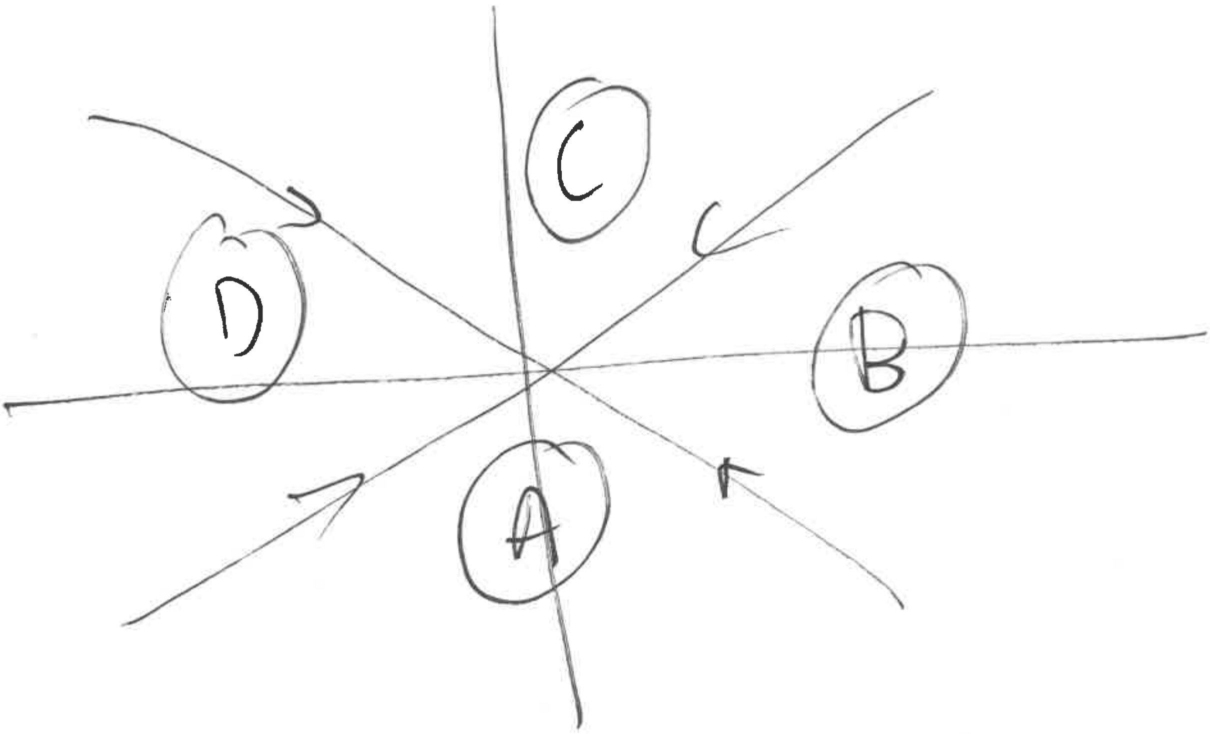
if $X_5 = 20 \cdot e_1 = C_1 \cdot e^{-5} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + C_2 \cdot e^{-30} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$$C_2 = 0$$

if at $t = t_1$ you are on e_1 (or e_2) then you will stay on e_1 forever (or e_2)

e_1 (or e_2) is invariant under time


x(t)
1
1
1
2
1
-2
x0


C
D
B
A

48

$A = \ldots \qquad e_1 = [2 \quad 1]^T \quad \lambda_1 = -1$

$e_2 = [1 \quad -2]^T \quad \lambda_2 = -6$

$$X = c_1 \, e^{-t} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \, e^{-6t} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$