$$\frac{d}{dt}\left(\frac{1}{2}\vec{p}^T H \cdot \vec{p}\right) =$$

$$\frac{1}{2}\underbrace{\dot{\vec{p}}^T H \vec{p}}_{\text{scalar}} + \frac{1}{2}\underbrace{\vec{p}^T H \dot{\vec{p}}}_{\text{scalar}}$$

$(scalar)^T = scalar$

$$(\dot{\vec{p}}^T \cdot H \cdot \vec{p})^T = \vec{p}^T \cdot H \cdot \dot{\vec{p}}$$

$$(\vec{p}^T H \dot{\vec{p}})^T = \dot{\vec{p}}^T H \vec{p} \longrightarrow \text{No needed}$$

$$\vec{p}^T H \cdot \dot{\vec{p}} = \dot{\vec{p}}^T H \vec{p}$$

$$\frac{d}{dt}(\quad) = \dot{\vec{p}}^T H \vec{p}$$

$$\dot{\vec{p}} = \cancel{\dot{p}}^{\,0} - \dot{\hat{p}}$$

$$\frac{d}{dt}(\quad) = -\dot{\hat{p}}^T \cdot H \vec{p}$$

$$\dot{V} = s.s - \dot{\hat{p}}^T H \vec{p}$$

Gen. Revision

HOS $x^{(5)} + \dots$

- $x = \dots$ prove that is a soln
- Find the C.E.
- Prove $r = \dots$ is an eig.

- $x_1 = \dots$ $x_2 = \dots$
- prove $x_1, x_2 \dots$ are L.I. solns
- $r_1 = \dots$ $r_2 = \dots$ $\Rightarrow$ $x_{gen}(t) = \dots$

  - $r_1 \neq r_2 \neq r_3$
  - $r_1 = r_2 \in \mathbb{R}$
  - $r_1 = \overline{r_2}$
  - $r_1 = \overline{r_2}$, $r_3 = \overline{r_4}$, $r_1 = r_3$

- $\dot{x} = A \cdot x$, $\lambda_1 = \dots$ $\lambda_2 = \dots$

  $x_{gen}(t) = \dots$

  $\lambda_1 \neq \lambda_2$

  $\lambda_1 = \overline{\lambda_2}$

- $\dot{x} = Ax$, 2×2, $\lambda_1 = -1 + i$, $\lambda_2 = 3 - i$

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\left.\begin{aligned} \dot{x} &= A x + B \cdot u \\ y &= C \cdot x + D \cdot u. \end{aligned}\right\} \Rightarrow \begin{aligned} \dot{z} &= \hat{A} \cdot z + \hat{B} \cdot u \\ y &= \hat{C} \cdot z + \hat{D} \cdot u. \end{aligned}$$

$$\hat{A} = T^{-1} \cdot A \cdot T$$

$$\boxed{e^{At} = T \cdot e^{\hat{A}t} \cdot T^{-1}} \qquad \boxed{\underline{T = \text{Inv}}}$$

$$\text{eig}(A) = \text{eig}(\hat{A})$$

$$\text{CTRB}(A) \neq \text{CTRB}(\hat{A})$$

$$\text{rank}(\text{CTRB}(\hat{A})) = \text{rank}(\text{CTRB}(\hat{A}))$$

- $T = \text{eigm.}$ $\quad \hat{A} \quad r_1 = \ldots \quad r_2 = .$

$A: -1, -2, -3, -4, -5$

$$\hat{A} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ \vdots & & \vdots & \vdots & \end{bmatrix}$$

$A: -1, -1, -2$

$$\hat{A} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad \underline{\underline{\text{or}}} \quad \hat{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$-1 + i \qquad -1 - i \qquad -3$

$$A = \begin{bmatrix} -1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

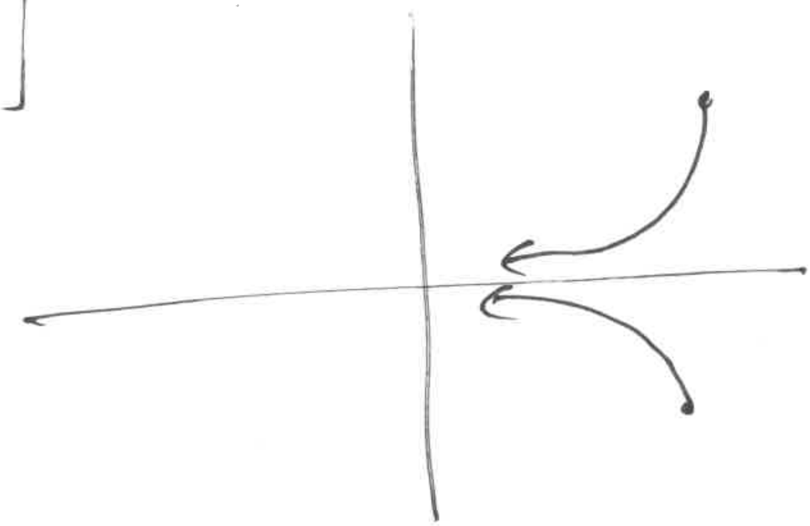
$A = -1, -2 \quad -3$

$$e^{\hat{A}t} = \begin{bmatrix} e^{-t} & 0 & 0 \\ 0 & e^{-2t} & \\ \ldots & \ldots & \ldots \end{bmatrix}$$

$A = \ldots \quad \lambda_1 = \quad \to \quad e_1 = [\quad]$

$\qquad\quad \lambda_2 = \quad \to \quad e_2 = [\quad]$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -10 \end{bmatrix}$$

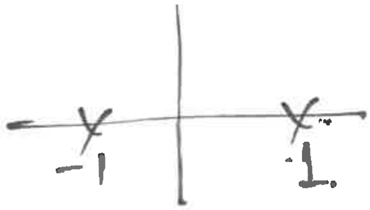


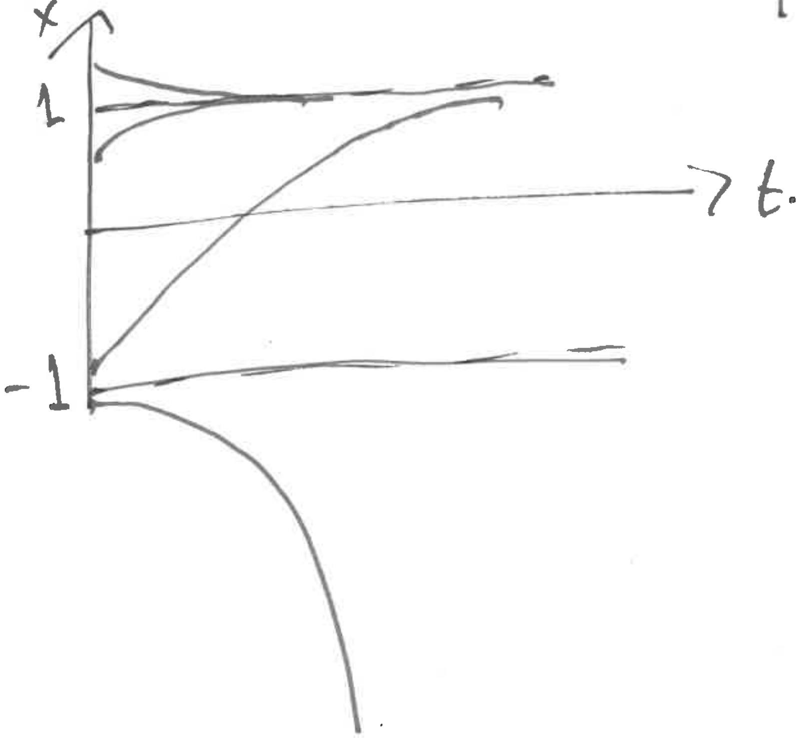
$A = \ldots \quad \lambda = -3$

Ch. 4.

- F.P. $\dot{x} = 0$
- stability of F.P.s.

$\dot{x} = -1 + x^2$



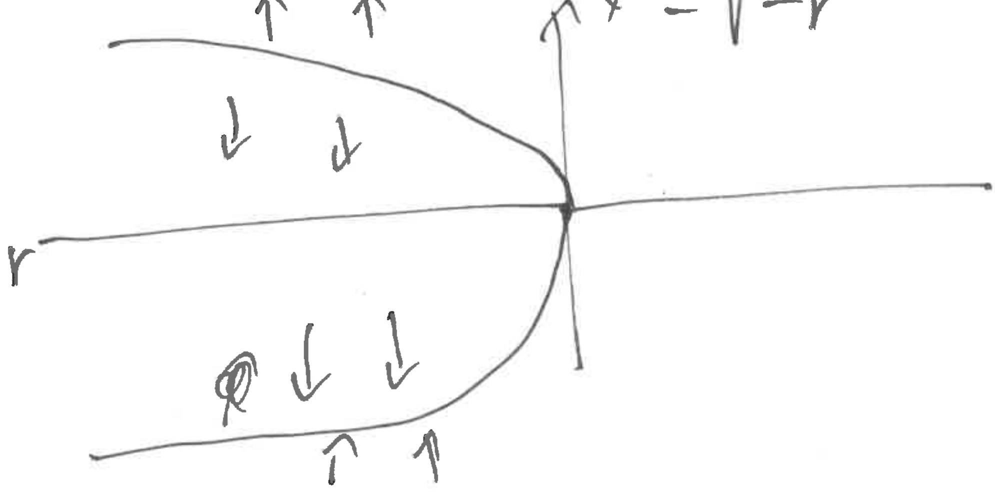


$\dot{x} = r + x^2$ $\quad \dot{x} = 0 \Rightarrow x = \pm\sqrt{-r}$

No F.P. for $r > 0$

2 F.P. for $r < 0$ $\quad +\sqrt{-r}$

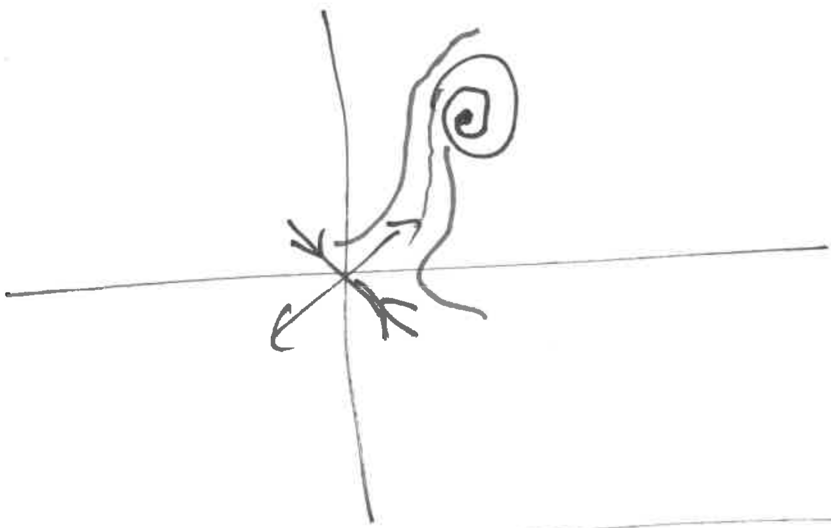
$-\sqrt{-r}$



Lyapunov Theorem

$\dot{x} = f(\quad)$ **AND** $V(x)$

prove that $\dot{x} = f(\quad)$
is stable



Chapter 5

$\ddot{x} - 3\dot{x} + 2x = u$

$u = ?$: $\dddot{x} + 5\ddot{x} + 4\dot{x} = 0$

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if $K > F$, $|f - \hat{f}| < F$
then sliding M.C. is stable

Rev of Ch5

$$x^{(n)} = f(x^{(n-1)}, x^{(n-2)}, \dots, \dot{x}, t) + g(\quad) \cdot u.$$

$x_d = \dots$ $\quad u = ? : x \to x_d$

$\tilde{x} = x - x_d$ $\quad u = ? : \tilde{x} \to 0$

CASE 1: I want the error:

$$\tilde{x}^{(n)} + h(\dots) = 0$$

$$x^{(n)} = \cancel{f} + \cancel{g} \cdot u$$

$$u = \frac{1}{\cancel{g}}(\cancel{-f} + x_d^{(n)} - h(\quad))$$

$$\tilde{x}^{(n)} + h(\quad) = 0$$

$\ddot{x} = -3\dot{x} - 2x + u$ $\quad g = 1, \; f = -3\dot{x} - 2x$

Target $\quad \ddot{\tilde{x}} + 3\dot{\tilde{x}} + 2\tilde{x} = 0$

so $\quad h(\dot{\tilde{x}}, \tilde{x}, t) = 3\dot{\tilde{x}} + 2\cdot\tilde{x}$

$$u = \underbrace{\frac{1}{1}}_{g}\left(\underbrace{3\dot{x} + 2x}_{-f} + \ddot{x}_d - h\right)$$

$$= \left(3\dot{x} + 2x + \ddot{x}_d - 3\dot{\tilde{x}} - 2\tilde{x}\right)$$

$$\ddot{x} = \cancel{-3\dot{x}} \cancel{-2x} + \cancel{3\dot{x}} + \cancel{2x} + \ddot{x}_d - 3\dot{\tilde{x}} - 2\tilde{x}$$

$$\ddot{\tilde{x}} + 3\dot{\tilde{x}} + 2\tilde{x} = 0$$

CASE 2 $\quad \tilde{x} \to 0$

$\dot{\tilde{x}} \to 0$

$s = \dot{\tilde{x}} + \lambda \tilde{x}, \quad \lambda > 0$

$s \to 0$

$s = 0 \quad \dot{\tilde{x}} \neq 0 \quad \text{or} \quad \tilde{x} \neq 0$

$\dot{\tilde{x}} + \lambda \tilde{x} = 0$

$\dot{\tilde{x}} = -\lambda \tilde{x} \Rightarrow \tilde{x}(t) = e^{-\lambda t} \cdot \tilde{x}(0).$

$\tilde{x} \to 0$

$\dot{\tilde{x}} \to 0$

$\dot{x} = f(\quad) + g(\quad) \cdot u$

$u = ? \quad : \quad s \to 0$

$u = ? \quad : \quad$ H. ODE of s = stable

If $V(s) > 0, \quad \dot{V} < 0$ then $\uparrow$

$V(s) = \frac{1}{2} s^2$

$u = ? \quad : \quad \dot{V} < 0, \; \dot{V} = s \cdot \dot{s}$

If $\dot{s} = -s \quad \dot{V} = -s^2 < 0$

$u = ? \quad : \quad \dot{s} = -s$

$$s = \dot{\tilde{x}} + \lambda \tilde{x}$$

$$\dot{s} = -s$$

$$\underbrace{\ddot{\tilde{x}}}_{\underbrace{\ddot{x}}_{f+gu} - \ddot{x}_d} + \lambda \dot{\tilde{x}} = -\dot{\tilde{x}} - \lambda \tilde{x}$$

$$f + gu - \ddot{x}_d + \lambda \dot{\tilde{x}} = -\dot{\tilde{x}} - \lambda \tilde{x}$$

$$u = \frac{1}{g}\left(-f + \ddot{x}_d - (\lambda+1)\dot{\tilde{x}} - \lambda \tilde{x}\right) \Longrightarrow$$

$$\ddot{\tilde{x}} + (\lambda+1)\dot{\tilde{x}} + \tilde{x} = 0$$

CASE 3: $V(s) = \frac{1}{2} s^2$ $\qquad \dot{V} = s \cdot \dot{s}$

$\cancel{\dot{s} = -s}$ $\qquad \dot{s} = -k \cdot \operatorname{sign}(s)$

$\dot{V} = -k \cdot |s| < 0$

$$u = \frac{1}{g}\left(-k\,\operatorname{sign}(s) - f + \ddot{x}_d - \lambda \dot{\tilde{x}}\right)$$

$\ddot{x} = f + g \cdot u$ sys

$\ddot{x} = \hat{f} + \hat{g} \cdot u$ Model

$|f - \hat{f}| < F \Longrightarrow k > F$

MRAS

$$\ddot{x} = f(\quad) + g(\quad)\cdot u.$$

$$g = 1.$$

$$f(\dot{x}, x, t) = \underbrace{f_1(\dot{x}, x, t)}_{\text{known}} \cdot p_1 + \underbrace{f_2}_{\text{known}} \cdot p_2 + \dots$$

unknown const.

$$s = \dot{\tilde{x}} + \lambda \tilde{x}$$

$$V = \frac{1}{2} s^2 \qquad \dot{V} = s \cdot \dot{s}$$

$$\dot{s} = \ddot{\tilde{x}} + \lambda \dot{\tilde{x}}$$

$$= f + u - \ddot{x}_d + \lambda \dot{\tilde{x}}$$

$$= f_1 p_1 + f_2 p_2 + \dots - \ddot{x}_d \cdot 1 + \dot{\tilde{x}} \cdot \lambda + u.$$

$$= F \cdot P + u$$

$$F = [f_1 \quad f_2 \quad \dots \quad \ddot{x}_d \quad \dot{\tilde{x}}]$$

$$P = [p_1 \quad p_2 \quad \dots \quad -1 \quad \lambda]^T$$

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$$\dot{s} = F \cdot P + u.$$

$$\dot{V} = s \cdot \dot{s}$$

$\Rightarrow$

if $u = -F \cdot \cancel{P} - k \cdot s$

$$\dot{s} = F \cdot P - F \cdot P - ks = -ks \Rightarrow$$

$$\boxed{\dot{V} = -ks^2}$$

$$u = -F \cdot \hat{P} - ks$$

$$\dot{s} = F \cdot P + \underbrace{- F \cdot \hat{P} - ks}$$

$$= -k \cdot s + F \cdot \tilde{P}, \quad \tilde{P} = P - \hat{P}$$

Before $s \to 0$

I must $\tilde{P} \to 0$

$$V = \frac{1}{2} s^2 + h_1 \frac{1}{2} \tilde{P}_1^2 + \overset{h_2}{\frac{1}{2}} \tilde{P}_2^2 + \ldots\ldots$$

$$= \frac{1}{2} s^2 + \frac{1}{2} \tilde{P}^T \cdot H \cdot \tilde{P}$$

$$\dot{V} = s \cdot \dot{s} + \frac{d}{dt}\left(\frac{1}{2} \tilde{P}^T \cdot H \cdot \tilde{P}\right)$$

$$\left.\begin{aligned} \dot{s} &= u + F\cdot p \\ u &= \underbrace{-F\cdot\hat{p} - ks} \end{aligned}\right\} \Rightarrow$$

$$\dot{s} = -F\cdot\hat{p} - ks + F\cdot p$$

$$= F\cdot\tilde{p} - ks$$

$$\dot{V} = s\cdot\dot{s} - \dot{\hat{p}}^T H\tilde{p}$$

$$= s\cdot(F\tilde{p} - ks) - \dot{\hat{p}}^T H\tilde{p}$$

$$= -ks^2 - \underbrace{sF\tilde{p} - \dot{\hat{p}}^T H\tilde{p}}$$

$$-sF\cancel{\tilde{p}} = \dot{\hat{p}}^T H\cancel{\tilde{p}}$$

$$-sF = \dot{\hat{p}}^T H$$

$$\boxed{\dot{\hat{p}}^T = -sF\cdot H^{-1}} \quad \text{est of } \hat{p}$$

$$u = -ks - F\hat{p}$$

$$\ddot{x} = 3\dot{x} + 2x + u.$$

$$f_1 = \dot{x} \qquad p_1 = 3$$

$$f_2 = x \qquad p_2 = 2.$$

$$F = [\dot{x} \quad x \quad \ddot{x}_d \quad \dot{\tilde{x}}]$$

$$P = [3 \quad 2 \quad -1 \quad \lambda]^T.$$

$$\cancel{\dot{x} = \begin{bmatrix}-2 & 0\\ 0 & -3\end{bmatrix}\cdot x \quad 0 \quad -A^{-1}\cdot B\cdot u}$$

$$\dot{x} = \underbrace{A}_{2}x + B\cdot u \quad , \quad \dot{x} = 0 \Rightarrow A\cdot x_{EP} + B\cdot u = 0$$

$$x_{EP} = -\underbrace{A^{-1}\cdot B\cdot u}_{2\times 1}$$

$$|\lambda I - A| = 0$$

$A = 2\times 2$

$B = 2\times 2$ $\quad 2\times 1$

$u = 2\times 1$

$$A = \begin{bmatrix}-2 & 0\\ 0 & -3\end{bmatrix} \qquad B = \begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix}$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix}-3 & 0\\ 0 & -2\end{bmatrix} \qquad u = \begin{bmatrix}1\\ 1\end{bmatrix}$$

$$x_{EP} = \frac{1}{6}\begin{bmatrix}-3 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix}\cdot\begin{bmatrix}1\\ 1\end{bmatrix}$$

$\dot{x} = a - x^2$

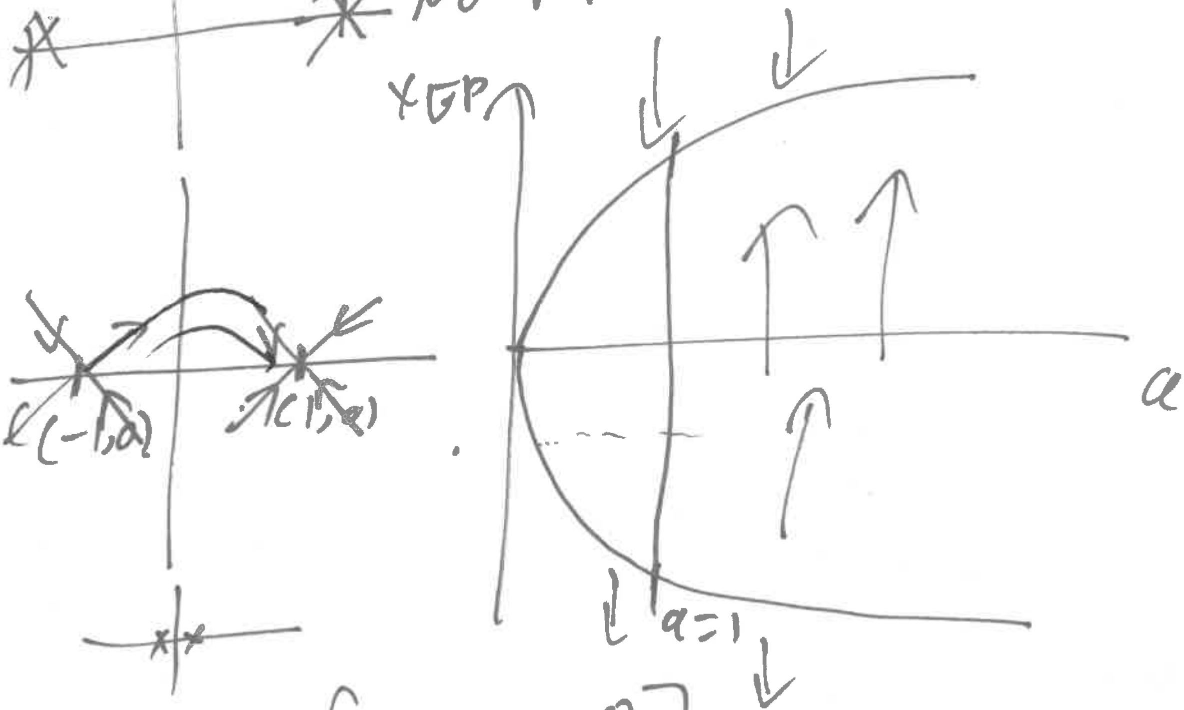
$\dot{y} = -y$

F.P.s

$$\left.\begin{aligned}\dot{x} &= 0\\ \dot{y} &= 0\end{aligned}\right\} \begin{aligned} a - x^2 &= 0\\ -y &= 0\end{aligned}$$

$x^* = \pm\sqrt{a}$. $a > 0$

No F.P. $a < 0$



$$J = \begin{bmatrix} -2x & 0 \\ 0 & -1 \end{bmatrix}$$

$$A(\sqrt{a},0) = \begin{bmatrix} -2\sqrt{a} & 0 \\ 0 & -1 \end{bmatrix} \begin{cases} \to -2\sqrt{a} < 0 \\ \to -1 < 0 \end{cases}$$ Stable Node.

$$A(-\sqrt{a},0) = \begin{bmatrix} 2\sqrt{a} & 0 \\ 0 & -1 \end{bmatrix} \begin{cases} \to 2\sqrt{a} > 0 \\ \to -1 < 0 \end{cases}$$ saddle

$\dot{x} = a(y-x)$, $\dot{y} = bx - y - xz$ ~~98~~ 96

$\dot{z} = x \cdot y - c \cdot z$ $\quad a, b, c > 0$

$\dot{x} = 0$
$\dot{y} = 0$
$\dot{z} = 0$
$(0,0,0) = (x,y,z).$

$$J = \begin{bmatrix} -a & a & 0 \\ b-z & -1 & -x \\ y & x & -c \end{bmatrix}$$

$$A(0,0,0) = \begin{bmatrix} -a & a & 0 \\ b & -1 & 0 \\ 0 & 0 & -c \end{bmatrix}$$

$|A - rI| = 0 \Rightarrow$

$$r = \frac{-(1+a) \pm \sqrt{(1+a)^2 - 4(-ab+a)}}{2}$$