Revision

$$\left.\begin{aligned}\dot{x} &= Ax + B\cdot u \\ y &= C\cdot x + D\cdot u\end{aligned}\right\} \xrightarrow[T=\text{inv}]{x = T\cdot z} \quad \dot{z} = \underbrace{T^{-1}\cdot A\cdot T}_{\hat{A}}\cdot z + \underbrace{T^{-1}\cdot B}_{\hat{B}}\cdot u$$

$$z = T^{-1}x \qquad y = \underbrace{C\cdot T}_{\hat{C}}\cdot z + \underbrace{D}_{\hat{D}}\cdot u$$

Both models $\to$ same T.F.

$\to$ same CTRB/OBSV

$$e^{At} = T e^{\hat{A}t} T^{-1} \quad \text{or} \quad e^{\hat{A}t} = T^{-1} e^{At}\cdot T$$

If $T =$ eigenmatrix

Assume $n=2$

- $T = [e_1 \quad e_2]$ • $T = [e \quad b]$ • $T = [Re(e) \quad Im(e)]$

$b \Downarrow$ gen eig

☒ $T = [e_1 \quad e_2] \to \hat{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ $\hat{e}_1 = [1 \quad 0]^T$, $\hat{e}_2 = [0 \quad 1]^T$

$$\Rightarrow e^{\hat{A}t} = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$$

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$\lambda_1 = a + bi$, $\lambda_2 = \bar{\lambda}_1$, $a, b \in \mathbb{R}$

$\lambda_1, \lambda_2 \in \mathbb{C}$

$\Downarrow$

$e_1 = [v + wi]$, $e_2 = \bar{e}_1$, $v, w \in \mathbb{R}^{n \times 1}$

$e_1, e_2 \in \mathbb{C}^{n \times 1}$

assume $n = 2$

$A \cdot e_1 = \lambda_1 e_1 \Leftrightarrow$

$$A \cdot [v + wi] = (a + bi) \cdot [v + wi].$$

$$= av + awi + b \cdot vi - bw$$

$$Av + A \cdot wi = (av - bw) + (aw + bv) \cdot i \Rightarrow$$

$$\left.\begin{aligned} A \cdot v &= av - bw \\ Aw &= aw + bv \end{aligned}\right\} \Rightarrow A \cdot [v \quad w] = [v \quad w] \cdot \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

$$A \cdot T = T \cdot \hat{A}$$

$$A = T \cdot \hat{A} \cdot T^{-1}$$

$e_1 = [1 \quad i]^T$

$e_2 = [1 \quad -i]^T$

$$\hat{A} = \begin{bmatrix} Re(\lambda) & Im(\lambda) \\ -Im(\lambda) & Re(\lambda) \end{bmatrix}$$

$$e^{\hat{A}t} = e^{at} \cdot \begin{bmatrix} \cos bt & \sin bt \\ -\sin bt & \cos bt \end{bmatrix}$$

$$e^{At} = T e^{\hat{A}t} T^{-1}$$

$\bullet\ \lambda = \lambda_1 = \lambda_2 \rightarrow e\ ,\ b \rightarrow$ gen eig.

$(A - \lambda I)\cdot e = 0$ $\qquad e = (A - \lambda I)\cdot b.$

$Ae = \lambda e$ $\qquad e = A\cdot b - \lambda b$

$Ab = e + \lambda b.$

$$A\cdot T. = A\cdot[e \quad b] =$$

$$= [Ae \quad A\cdot b].$$

$$= [Ae \quad e + \lambda b]$$

$$= [\lambda e \quad e + \lambda b]$$

$$= \underbrace{[e \quad b]} \cdot \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$

$$= T\cdot \hat{A}$$

$$\hat{A} = T^{-1} A\cdot T$$

$$e^{\hat{A}t} = \begin{bmatrix} e^{\lambda t} & te^{\lambda t} \\ 0 & e^{\lambda t} \end{bmatrix}$$

V.S.

A set of objects (vectors)

$V = \{V_1, V_2, \ldots\ldots\}$

1) if $V_1, V_2 \in V \Rightarrow V_1 + V_2 \in V$

2) if $V_1, V_2 \in V \Rightarrow V_1 + V_2 = V_2 + V_1 \in V$

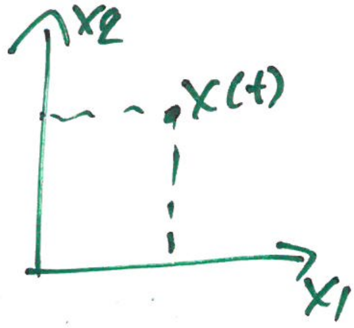
3) $\exists \; 0 : \; 0 + V_1 = V_1 + 0$

e.g. $V = \{x^2, 3x^2, 3.1x^2, -x^2 \ldots\ldots\}$

$= \{a \cdot x^2 \mid \quad a \in \mathbb{R}$

$$\dot{x} = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix} \cdot x$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



$$\lambda_1 = -1 \rightarrow e_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\lambda_2 = -6 \rightarrow e_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x = c_1 e_1 e^{\lambda_1 t} + c_2 e_2 e^{\lambda_2 t}$$

$$= \underbrace{c_1 e^{-t}}_{\downarrow} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \underbrace{c_2 e^{-6t}}_{\downarrow} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$= a(t) \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + b(t) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

new basis

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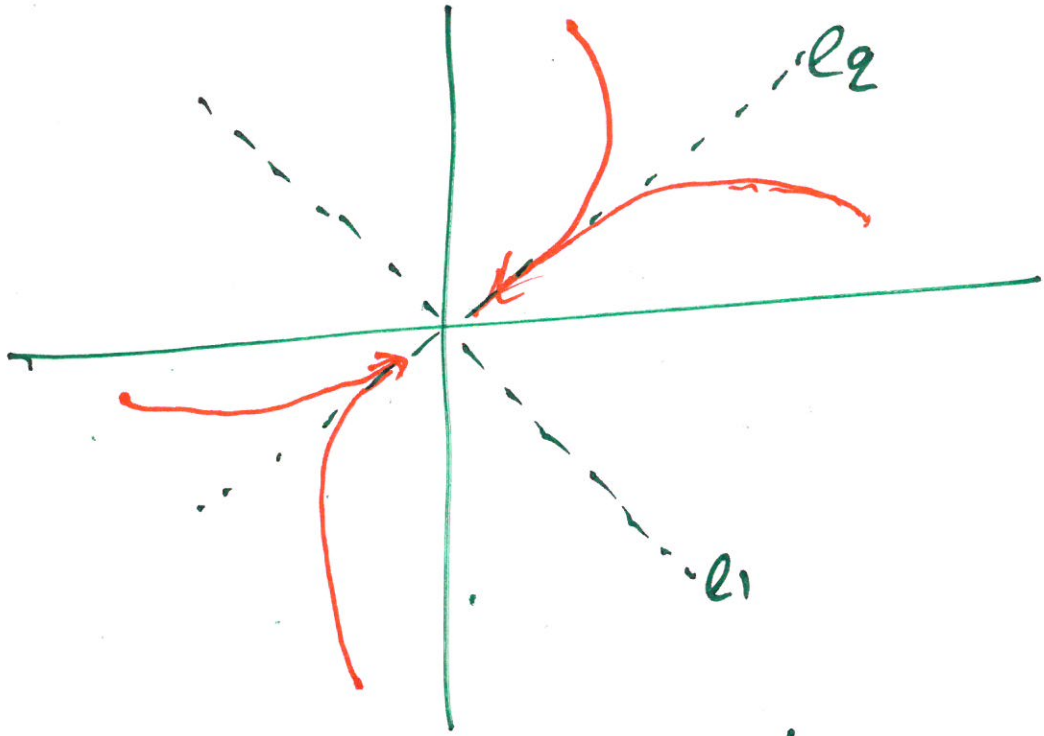
$$\dot{x} = \begin{bmatrix} -5.5 & 4.5 \\ 4.5 & -5.5 \end{bmatrix} x$$

$\lambda_1 = -10 \quad e_1 = [-1 \quad 1]$

$\lambda_2 = -1 \quad e_2 = [1 \quad 1]$



$$x = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-10t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t}$$

very fast $\quad x \simeq c_2 e^{-t} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

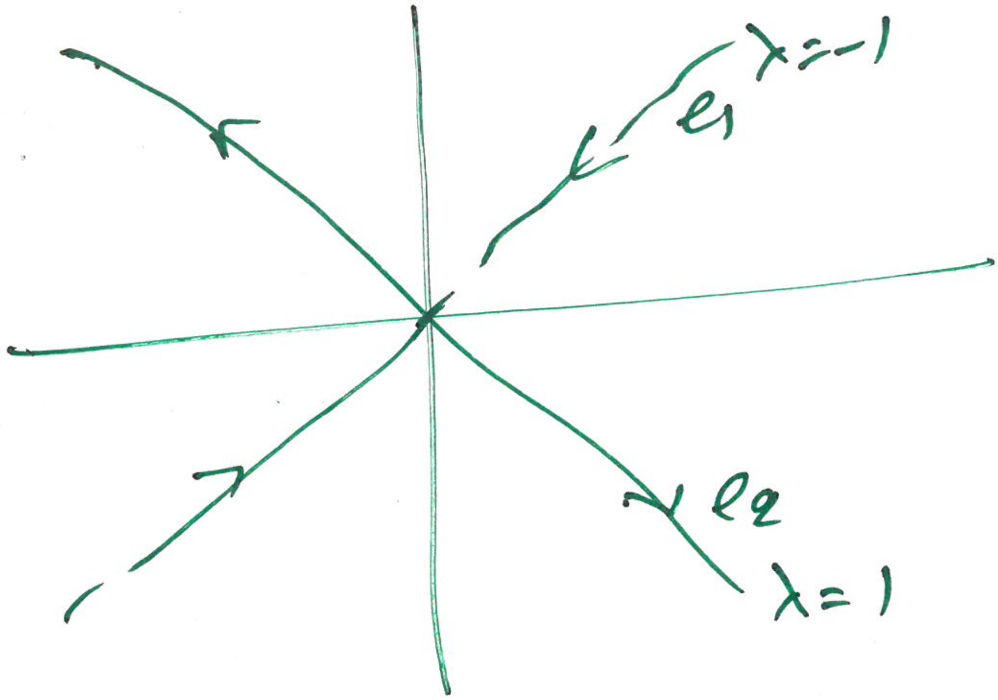
$$= a(t) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

If $\quad x = a(t) \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} + b(t) \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Assume that at $t = t_1 \qquad x = b(t_1) \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

i.e. $a(t_1) = 0$

$c_1 e^{-10t_1} = 0 \Rightarrow c_1 = 0$



$x = c_1 e^{-t} \cdot e_1 + c_2 e^{t} \cdot e_2$

If. x_0 : $c_1, c_2 \neq 0$

If x_0 : $c_1 \neq 0$, $c_2 = 0$