Revision

$$\dot{x} = A \cdot x \qquad e^{At} = I + At + \frac{(At)^2}{2!} + \dots$$

$x = T \cdot z$

$$\dot{z} = \hat{A} \cdot z$$

$T =$ eigenmatrix

$$e^{At} = T e^{\hat{A}t} \cdot T^{-1}$$

$n = 2$

- $\lambda_1 \neq \lambda_2 \Rightarrow e_1, e_2$

$T = [e_1 \;\; e_2]$

$$e^{\hat{A}t} = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

- $\lambda_1 = \lambda_2 = \lambda \to e$, b

$T = [e \;\; b]$

$$\hat{A} = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$

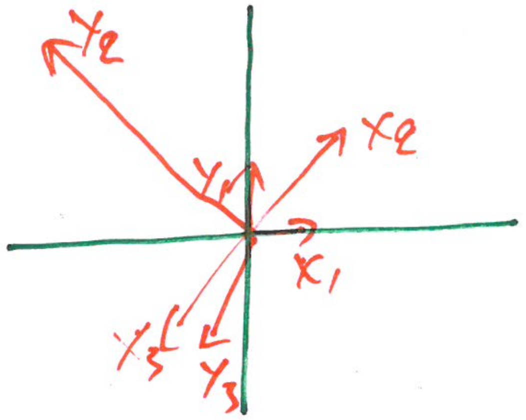
$$e^{\hat{A}t} = \begin{bmatrix} e^{\lambda t} & t e^{\lambda t} \\ 0 & e^{\lambda t} \end{bmatrix}$$

- $\lambda = a + bi \to e = \dots i = v + wi$

$T = [Re(e) \;\; Im(e)]$

$$\hat{A} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

$$e^{\hat{A}t} = e^{at} \cdot \begin{bmatrix} \cos bt & \dots \\ \dots & \dots \end{bmatrix}$$



$V = a\cdot x_i + b\cdot y_i$

$x_1 = [1 \quad 0]$

$y_1 = [0 \quad 1]$

$|x_1| = |y_1| = 1$

$x_1 \cdot y_1 = 0$

$x_2 = [\square \quad \square]$ $|x_2| \neq 1$

$y_2 = [\square \quad \square]$ $|y_2| \neq 1$

$x_2 \cdot y_2 = 0$

$x_3 = [\square \quad \square]$, $|x_3| \neq 1$

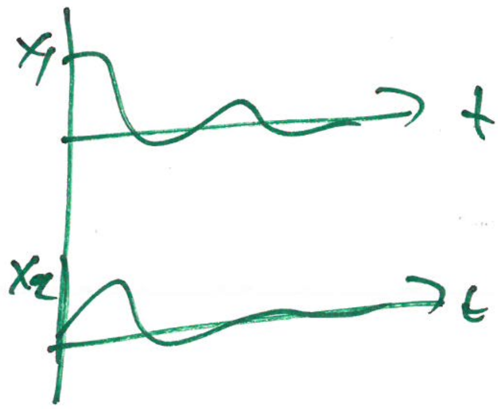
$y_3 = [\square \quad \square]$ $|y_3| \neq 1$

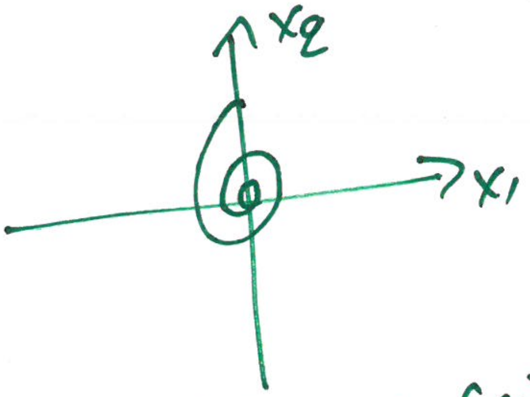
$x_3 \cdot y_3 \neq 0$

$\dot{x} = A\cdot x$ $\quad n = 2$

$x_1 = \dots$

$x_2 = \dots$





$x = x_1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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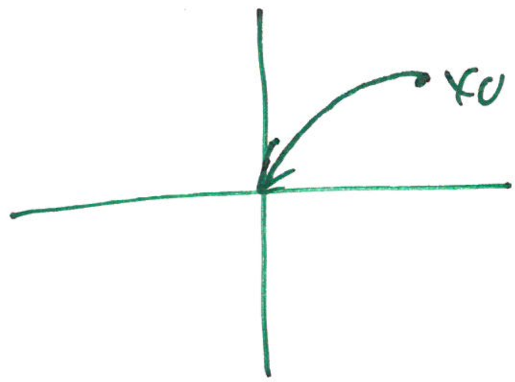
$$\dot{x} = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix} \cdot x \quad \begin{cases} \lambda_1 = -1 \rightarrow e_1 = [2 \;\; 1]^T \\ \lambda_2 = -6 \rightarrow e_2 = [1 \;\; -2]^T \end{cases}$$

$$x(t) = \underbrace{c_1 e^{-t}}_{\downarrow} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \underbrace{c_2 e^{-6t}}_{\downarrow} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

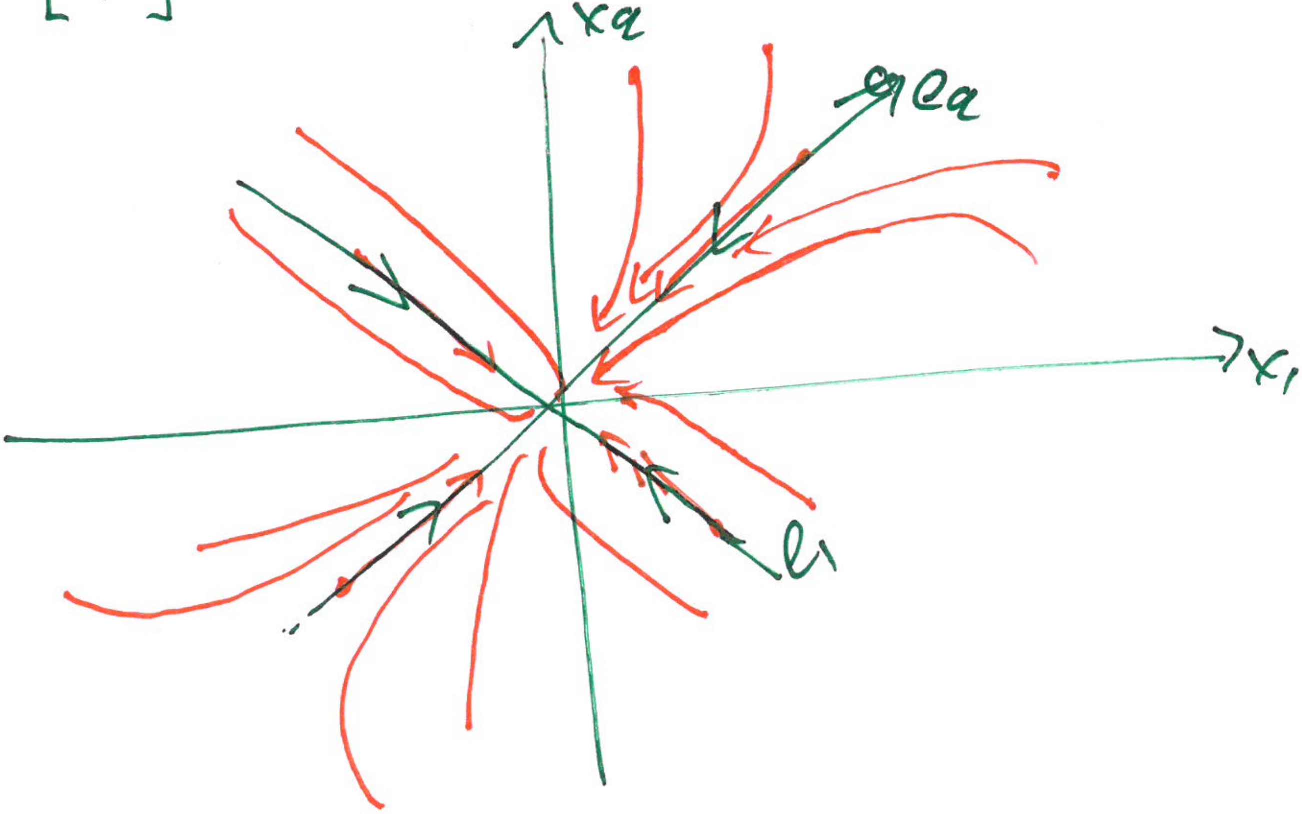
$$x(t) = a(t) \cdot e_1 + b(t) \cdot e_2$$

$$A = \begin{bmatrix} -5.5 & 4.5 \\ 4.5 & -5.5 \end{bmatrix}$$



$\lambda_1 = -10$ $\qquad$ $\lambda_2 = -1$

$e_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\qquad$ $e_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



$$x = c_1 e^{-t} \cdot e_1 + c_2 e^{-10t} \cdot e_2$$

$$x_0 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

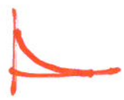
$$x_0 = c_1 \cdot 1 \cdot e_1 + c_2 \cdot 1 \cdot e_2 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$c_1 \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \Rightarrow$$

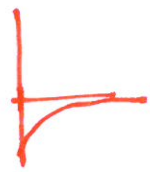
$$c_1 = -0.25, \quad c_2 = 0.75$$

$$\Rightarrow x(t) = -0.25 e^{-t} \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} + 0.75 \cdot e^{-10t} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \Rightarrow \left.\begin{matrix} c_1 = 0 \\ c_2 = 2 \end{matrix}\right\} \Rightarrow x(t) = 0 + 2 \cdot e^{-10t} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



$$x_0 = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \Rightarrow \left.\begin{matrix} c_1 = 0 \\ c_2 = -2 \end{matrix}\right\} \Rightarrow x(t) = -2 e^{-10t} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



$$x_0 = \begin{bmatrix} -2 \\ 2 \end{bmatrix} \Rightarrow \begin{matrix} c_2 = 0 \\ c_1 = 2 \end{matrix}$$

$x_0 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$ $\quad e^{-t} \quad e^{-10t}$

	e^{-t}	e^{-10t}
$t=0 \to$	1	1
$t=0.1 \to$	0.9048	0.3679
$t=0.2 \to$	0.8187	0.1353
$\vdots$		
$t=1 \to$	0.3679	$4.54 \cdot 10^{-5}$

$$x = c_1 e^{-10t} \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 \cdot e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$x(0) = \dots$

$x(0.1) = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}$

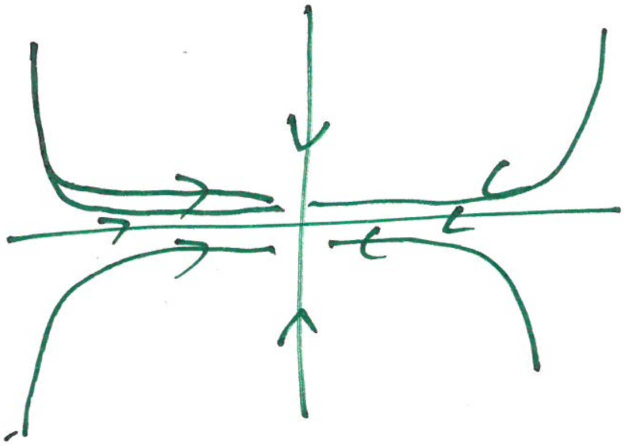
$$\hookrightarrow c_1 \cdot e^{-10 \cdot 0.1} \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 \, e^{-0.1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} c_1 &= -0.1359 \\ c_2 &= 0.2763 \end{aligned}$$

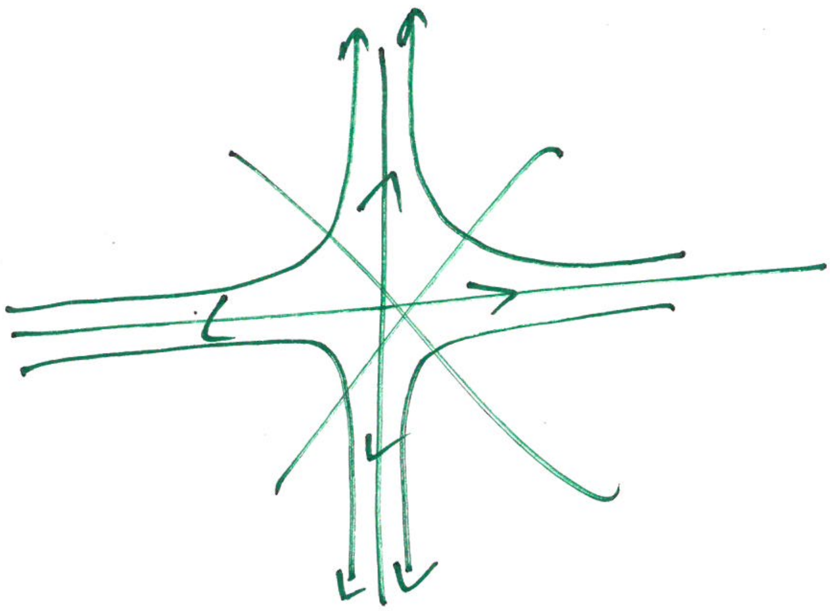
$x_{0.1} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix} = x(0.1) \Rightarrow$ $\quad c_1 = 0$, $\quad c_2 = 0.221$

$x(0.1) = \begin{bmatrix} -0.2 \\ -0.2 \end{bmatrix} \Rightarrow$ $\quad c_1 = 0$, $\quad c_2 = -0.221$

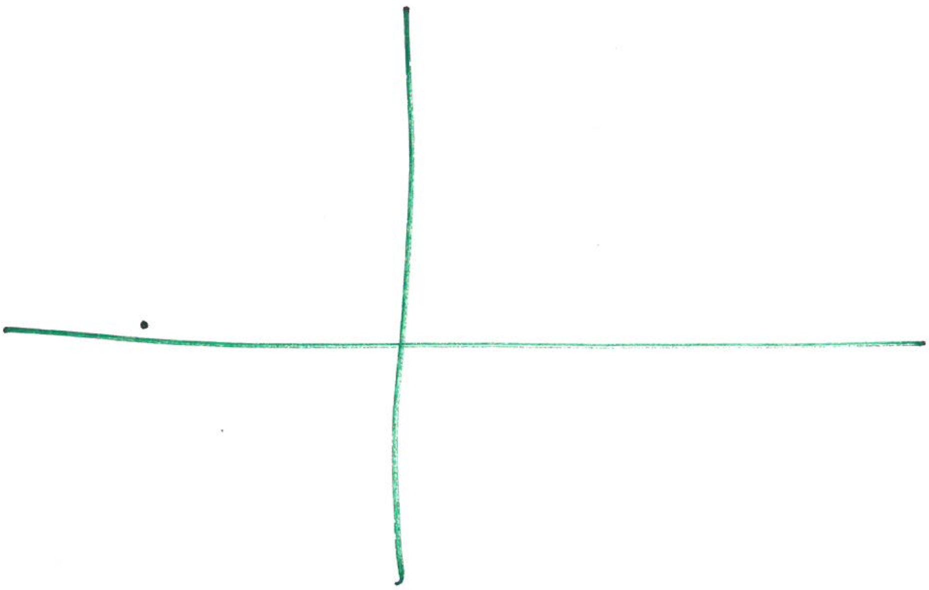
$$\hat{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{matrix} \nearrow e_1 = [1 \;\; 0]^T \\ \searrow e_2 = [0 \;\; 1]^T \end{matrix}$$



$\lambda_1, \lambda_2 < 0$



$\lambda_1, \lambda_2 > 0$



$A = \begin{bmatrix} & \\ & \end{bmatrix} \to \lambda_1 = \lambda_2 = \lambda \to e$

$b \to$ gen.

$x = c_1 \cdot (et + b) e^{\lambda t} + c_2 \cdot e^{\lambda t} \cdot e$

$x_0 = c_1 \, b \cdot 1 + c_2 \cdot e$

if at $t = 0$ $\quad x_0 = k \cdot e \Rightarrow c_1 = 0 \to e \to$ inv

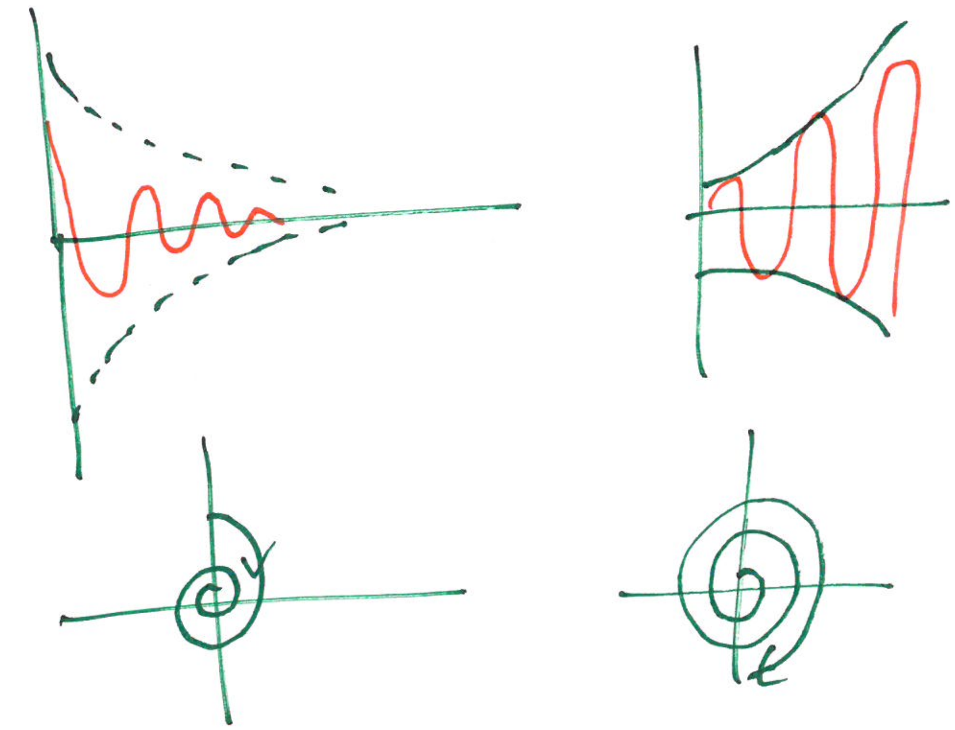
$x_0 = k \cdot b \Rightarrow c_2 = 0 \to b \not\to$ inv.

$A = \begin{bmatrix} & \\ & \end{bmatrix} \to \lambda = a + bi \to e = [v + wi]$

$e \cdot e^{\lambda t}, \quad \bar{e} e^{\bar{\lambda} t}$

$x = c_1 \, e \cdot e^{\lambda t} + c_2 \cdot \bar{e} \, e^{\bar{\lambda} t}$

$e^{\lambda t} = e^{at} \cdot e^{bit} = e^{at} (\cos bt + i \sin bt)$



$\dot{x} = f(x,u)$, $f, x \in \mathbb{R}^{n\times 1}$

$u \in \mathbb{R}^{p\times 1}$

Ch. 4

$$\dot{x} = -x^2$$

$$x(t) = \frac{1}{t+c}$$

$$\dot{x} = -\frac{(t+c)'}{(t+c)^2} = -\frac{1}{(t+c)^2}$$

$$\dot{x} = -x^2$$

$$-x^2 = -\left(\frac{1}{t+c}\right)^2 = -\frac{1}{(t+c)^2}$$

$$x = 3\,\frac{1}{t+c} \Rightarrow ?$$

$$\dot{x} = -3\,\frac{1}{(t+c)^2}$$

$$-x^2 = -\left(\frac{3}{t+c}\right)^2 = -9\,\frac{1}{(t+c)^2}$$

F.P.s

$\dot{x} = 0$

L.S. $\dot{x} = A x + B \cdot u = 0 \Rightarrow A \cdot x = -B \cdot u$

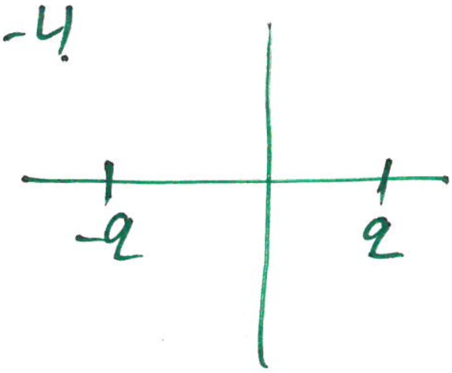
$$x_{EP} = -A^{-1} B \cdot u$$

N.L.

$$\left.\begin{array}{l} \dot{x} = x^2 - 4 \\ \dot{x} = 0 \end{array}\right\} \Rightarrow x^2 - 4 = 0 \Rightarrow x = \pm 2$$

$\dot{x} = x^2 + 4 \Rightarrow x^2 = -4$

$\nexists\, x : x^2 = -4$



$\dot{x} = x^2 - a$

- $a > 0 \Rightarrow x_{EP} = \pm\sqrt{a}$
- $a = 0 \Rightarrow x_{EP} = 0$
- $a < 0$ $\nexists$ F.P.