Revision

$$x^{(n)} = f(x^{(n-1)}, x^{(n-2)}, \dots x, t) + g(\quad)u.$$

e.g. $\ddot{x} = f(\dot{x}, x, t) + g(\dot{x}, x, t)\cdot u.$

$$\ddot{x} + A\dot{x} + Bx = u.$$

$$\ddot{x} = \underbrace{-A\dot{x} - Bx}_{f} + \underbrace{1}_{g}\cdot u.$$

$$\ddot{x} = f(\quad) + g(\quad)\cdot u$$

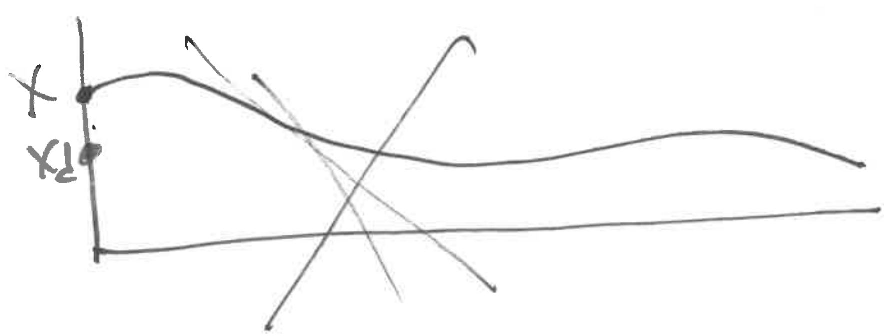
$$\Big\downarrow u = ?$$

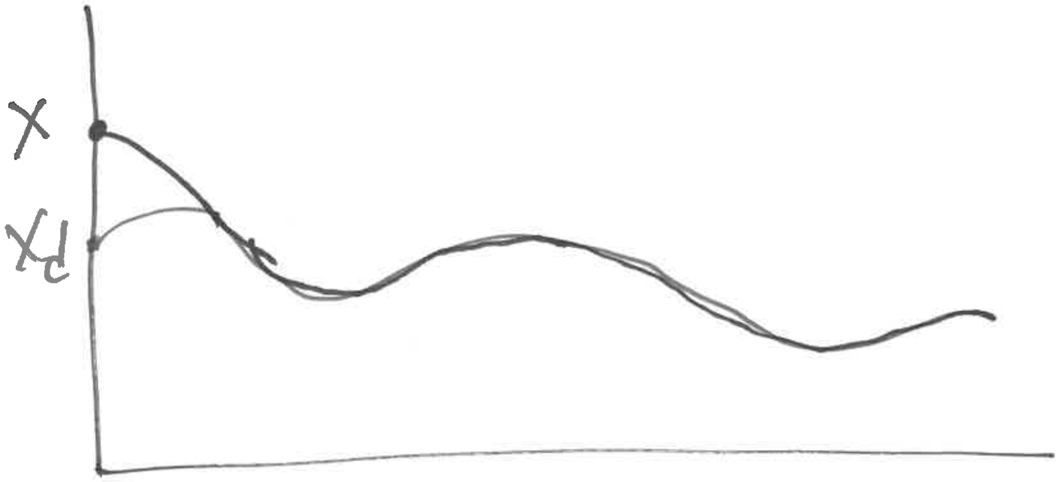
$$\ddot{\tilde{x}} + \underline{\quad}\cdot\dot{\tilde{x}} + \underline{\quad}\cdot\tilde{x} = 0, \quad \tilde{x} = x_d - x.$$

stable ODE of $\tilde{x}$

if $t = 0$ $\quad x_d(0) \neq x(0) \Rightarrow \tilde{x}(0) \neq 0$

$$\Rightarrow \tilde{x} \to 0 \Rightarrow x \to x_d$$





Case 1: $\ddot{x} + A\dot{x} + Bx = u.$, $x_d = \dots$

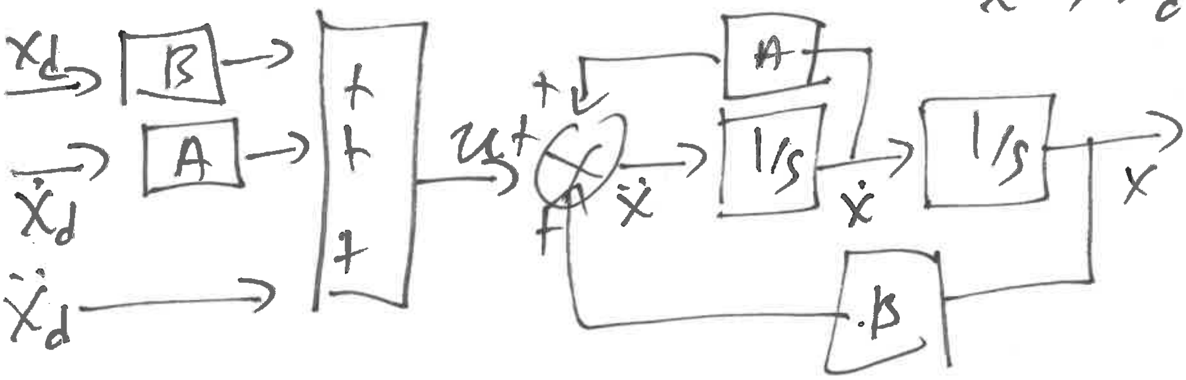
stable ODE

$$\ddot{\tilde{x}} + A\dot{\tilde{x}} + B\tilde{x} = 0$$

$$u = \ddot{x}_d + A\dot{x}_d + Bx_d$$

$\tilde{x} \to 0$

$x \to x_d$



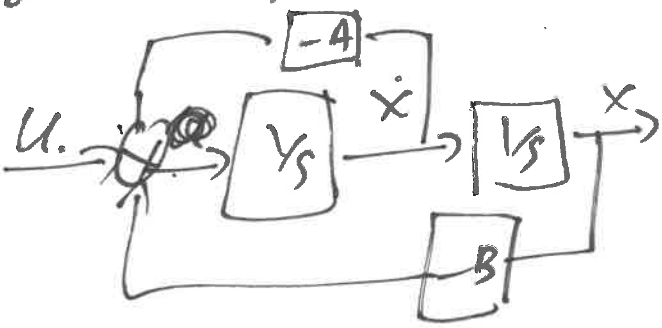
$$\ddot{x} + A\dot{x} + B\cdot x = u.$$

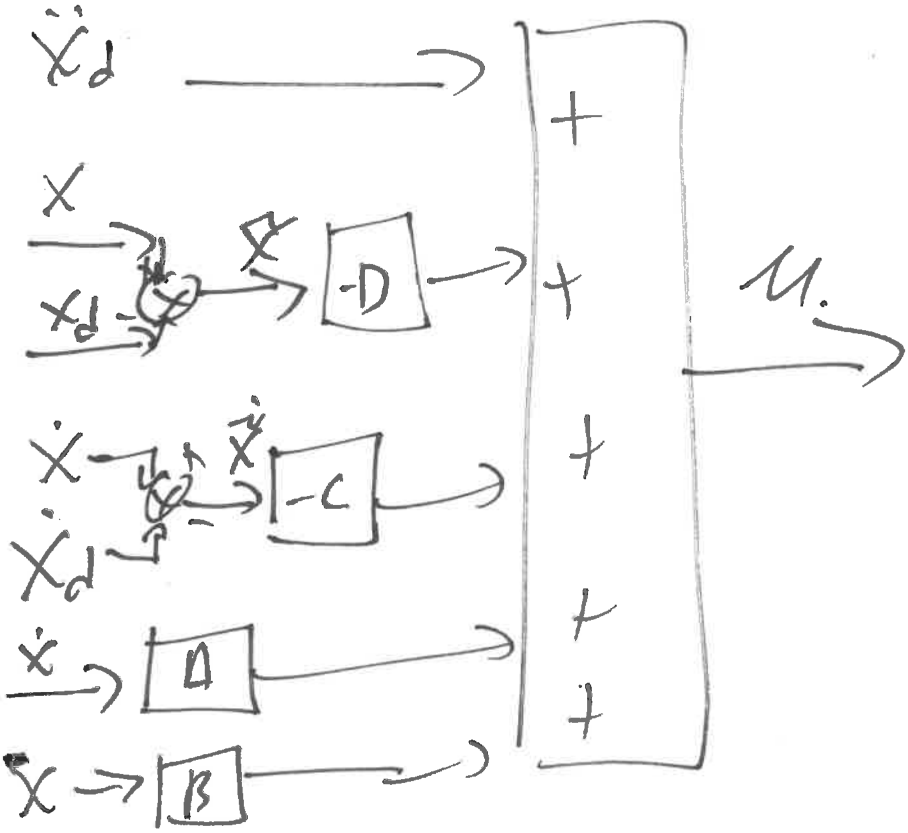
Unstable ODE

Target eigs for Error ODE → stable

$$\ddot{\tilde{x}} + C\cdot\dot{\tilde{x}} + D\tilde{x} = 0$$

$$\Rightarrow u = \ddot{x}_d + A\dot{x} + Bx - C\cdot\dot{\tilde{x}} - D\tilde{x}$$





$\ddot{x} = f(\quad) + g(\quad)\cdot u$

$u = ?$ $\quad \tilde{x} \to 0$ <u>AND</u> $\dot{\tilde{x}} \to 0$

$$s = \dot{\tilde{x}} + \lambda \tilde{x}, \quad \lambda \in \mathbb{R}^+$$

$u = ?$: ODE of $s \to$ stable
or $s \to 0$

if $\left.\begin{array}{l} V(s) > 0 \\ \dot{V}(s) < 0 \end{array}\right\} \Rightarrow$ ODE of $s \to$ stable

$u = ?$: $V(s) > 0$, $\dot{V}(s) < 0$

$$V(s) = \frac{1}{2} s^2 > 0$$

$$\dot{V}(s) = s \cdot \dot{s}$$

$u = ?$: $s \cdot \dot{s} < 0$

$s = -\dot{s} \Leftrightarrow \dot{s} = -s$

$u = ?$: $\dot{s} = -s$

$s = \dot{\tilde{x}} + \lambda \tilde{x} \to -s = -\dot{\tilde{x}} - \lambda \tilde{x}$

$\dot{s} = \ddot{\tilde{x}} + \lambda \dot{\tilde{x}}$

$$\dot{s} = \ddot{\tilde{x}} + \lambda\dot{\tilde{x}}$$

$$\ddot{x} - \ddot{x}_d$$

$$f + g\cdot u$$

$$\dot{s} = f + g\cdot u - \ddot{x}_d + \lambda\dot{\tilde{x}} = -s = -\dot{\tilde{x}} - \lambda\tilde{x}$$

$$f + g\cdot u - \ddot{x}_d + \lambda\dot{\tilde{x}} = -\dot{\tilde{x}} - \lambda\tilde{x}$$

$$\cancel{g}\cdot u = \left(-f + \ddot{x}_d - \lambda\dot{\tilde{x}} - \dot{\tilde{x}} - \lambda\tilde{x}\right)\frac{1}{g}$$

$$u = \frac{1}{g}\left(-f + \ddot{x}_d - (1+\lambda)\cdot\dot{\tilde{x}} - \lambda\tilde{x}\right)$$

$$\ddot{x} = f + g\cdot u$$

$$= \cancel{f} + \cancel{g}\,\frac{1}{\cancel{g}}\left(-\cancel{f} + \ddot{x}_d - (1+\lambda)\dot{\tilde{x}} - \lambda\tilde{x}\right)$$

$$\ddot{x} - \ddot{x}_d + (1+\lambda)\dot{\tilde{x}} + \lambda\tilde{x} = 0$$

$$\ddot{\tilde{x}} + (1+\lambda)\dot{\tilde{x}} + \lambda\tilde{x} = 0$$

$$r^2 + (1+\lambda)r + \lambda = 0$$

$$\Delta = (\lambda+1)^2 - 4\lambda = (\lambda-1)^2$$

$$r_{1,2} = \frac{-(1+\lambda) \pm (\lambda-1)}{2} = \frac{-1-\lambda \pm (\lambda-1)}{2} \begin{cases} \to -1 \\ \to -\lambda \end{cases}$$

$$u = \frac{1}{g}\left(-f + \ddot{x}_d - (1+\lambda)\dot{\tilde{x}} - \lambda\tilde{x}\right).$$

$$\ddot{x} = -A\dot{x} - Bx + u.$$

$$f = -A\dot{x} - Bx$$

$$g = 1$$

ODE of $\tilde{x}$ $\quad \ddot{\tilde{x}} + (1+\lambda)\dot{\tilde{x}} + \lambda\tilde{x} = 0$

$\rightarrow -1$

$\rightarrow -\lambda$

$$\ddot{x} = f + g \cdot u.$$

$u = ?$: $\frac{dV}{dt} < 0$, $\frac{dV}{dt} = s \cdot \dot{s}$, $\dot{s} = -s$

$k \in \mathbb{R}^+$

if. $\dot{s} = -k \cdot \mathrm{sign}(s) \Rightarrow$

$$\frac{dV}{dt} = s \cdot \dot{s} = s \cdot \left(-k \cdot \mathrm{sign}(s)\right).$$

$$= -k \cdot \underline{s \cdot \mathrm{sign}(s)}$$

$$= -k \cdot |s|$$

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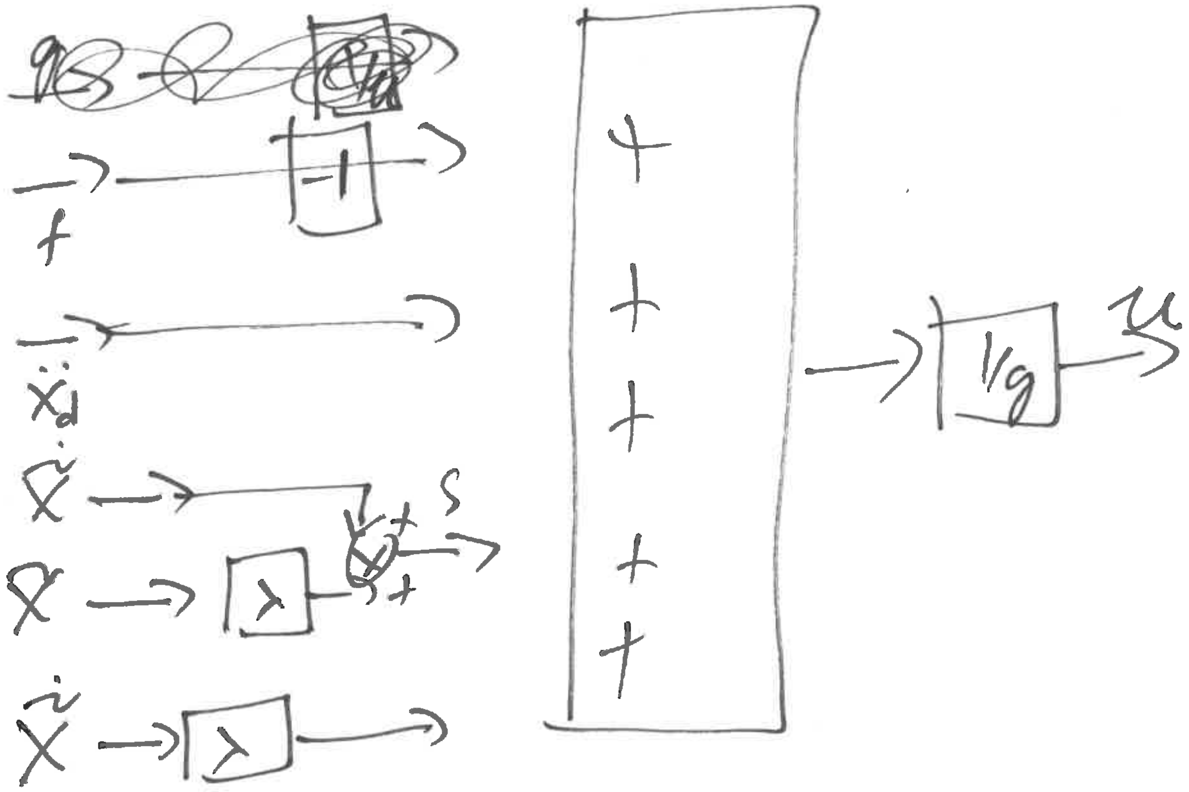
$\dot{s} = -k\cdot \mathrm{sign}(s)$ $\quad\quad$ $s = \dot{\tilde{x}} + \lambda\cdot\tilde{x}$

$\Downarrow$

$f + g\cdot u - \ddot{x}_d + \lambda\dot{\tilde{x}} = -k\cdot \mathrm{sign}(s)$

$$u = \frac{1}{g}\left(-f + \ddot{x}_d - \lambda\dot{\tilde{x}} - k\,\mathrm{sign}(s)\right)$$



$\ddot{x} = f + g\cdot u$

$= \cancel{f} + \cancel{g}\left(\frac{1}{\cancel{g}}\cdot\left(\cancel{-f} + \ddot{x}_d - \lambda\dot{\tilde{x}} - k\,\mathrm{sign}(s)\right)\right)$

$\ddot{\tilde{x}} + \lambda\dot{\tilde{x}} + k\,\mathrm{sign}(s) = 0$

$$\ddot{x} = \underbrace{f(\ ) + g(\ )}_{\text{unknowns.}} \cdot u \rightarrow \text{real sys.}$$

$$\ddot{x} = \underbrace{\hat{f} + \hat{g}}_{\text{perf. known.}}\ u. \rightarrow \text{our model}$$

$$|f - \hat{f}| \le F \quad , \quad |g - \hat{g}| \le G.$$

$$\text{Assume} \quad g = \hat{g} = 1$$

$$\frac{dV}{dt} = s \cdot \dot{s} \qquad u = -\hat{f} + \ddot{x}_d - \lambda \dot{\tilde{x}} - k \cdot \text{sign}(s)$$

$$\dot{s} = \ddot{\tilde{x}} + \lambda \dot{\tilde{x}} = \ddot{x} - \ddot{x}_d + \lambda \dot{\tilde{x}}$$

$$\boxed{\dot{s} = -s}$$

$$= f + u - \ddot{x}_d + \lambda \dot{\tilde{x}}$$

$$\cancel{= f + -\hat{f} + \ddot{x}_d - \lambda \dot{\tilde{x}}}$$

$$\cancel{\dot{s} = -ks}$$

$$\dot{S} = -k \cdot \text{sign}(s) \Rightarrow \cancel{f - \hat{f} + \ddot{x}_d - \lambda \dot{\tilde{x}} = -k\,\text{sign}(s)}$$

$$= f - \hat{f} - k\,\text{sig}(s). \le F - k\,\text{sig}(s)$$

$\dot{s} = F - k\,\mathrm{sign}(s).$

$$\frac{dV}{dt} = s \cdot \dot{s} = s \cdot (F - k\,\mathrm{sign}(s)) < 0$$

$\cdot\ s > 0$ $\quad s \cdot (F - k\,1) < 0$

$F - k < 0$

or $\boxed{k > F}$

$\cdot\ s < 0$ $\quad F - k(-1) > 0$

$F + k > 0$

$k > -F$

$F > -F$

so if $k > F \Rightarrow \dfrac{dV}{dt} < 0$