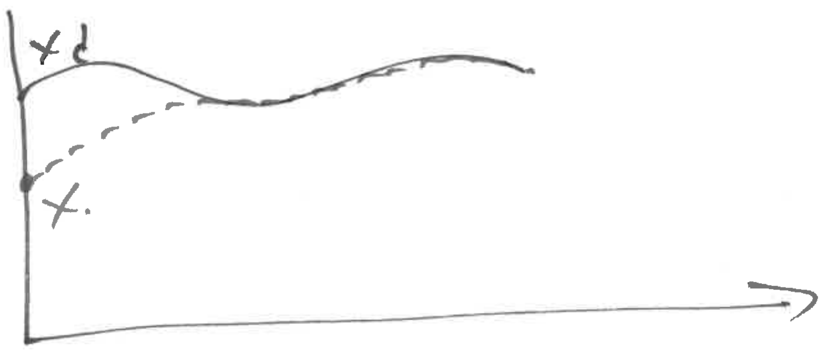
Revision

$$\ddot{x} = f(\dot{x},x,t) + g(\dot{x},x,t)\cdot u$$

$\downarrow$ $u=?$

stable ODE $\tilde{x} = x - x_d$



$$s = \dot{\tilde{x}} + \lambda \tilde{x}, \quad \lambda > 0$$

Def a L.F. $V(s) > 0$, $\dot{V}(s) < 0$

$u = ?$ $\quad \dot{V} < 0$ when $V(s) = \frac{1}{2} s^2$

then $\dot{V} = s \cdot \dot{s}$, $\dot{s} = -s$ $\qquad \boxed{\dot{s} = -k \cdot s}$

$u = ?$: $\quad \dot{s} = -s$

$$\frac{d}{dt}(\dot{\tilde{x}} + \lambda\tilde{x}) = -(\dot{\tilde{x}} + \lambda\tilde{x}) \Rightarrow \dots$$

$$u = \frac{1}{g}\left(-f + \ddot{x}_d - (1+\lambda)\cdot\dot{\tilde{x}} - \lambda\tilde{x}\right)$$

$$\Rightarrow \ddot{\tilde{x}} + (1+\lambda)\cdot\dot{\tilde{x}} + \lambda\tilde{x} = 0 \Rightarrow r_1 = -1$$

$$r_2 = -\lambda$$

$\ddot{x} = f + g \cdot u$ system

$\ddot{x} = \hat{f} + \hat{g} \cdot u$ model

then $u = \frac{1}{\hat{g}} \cdot (-\hat{f} + \ddot{x}_d - (1+\lambda) \cdot \dot{\tilde{x}} - \lambda \tilde{x})$

cases when $\dot{s} \neq -s \Rightarrow \dot{V} \nless 0$

$\Rightarrow$ ODE of s : unstable

$V(s) = \frac{1}{2} s^2$

$u = ?$ $\quad \dot{V} = s \cdot \dot{s} < 0$

$u = ?$ $\quad \cancel{\dot{s} = -s} \rightarrow \dot{s} = -k \cdot \text{sign}(s)$

$\Rightarrow \frac{dV}{dt} = -k \cdot |s| < 0$

$\frac{d}{dt}(\dot{\tilde{x}} + \lambda \tilde{x}) = -k \ \text{sign}(s) \Rightarrow$

$u = -f + \ddot{x}_d - \lambda \dot{\tilde{x}} - k \cdot \text{sign}(s)$

More Robust

Assume $g=1$, $\hat{g}=1$

$\ddot{x}=f+u$

$\ddot{x}=\hat{f}+u$

$\frac{dV}{dt} \measuredangle 0$

$s=\dot{\tilde{x}}+\lambda\tilde{x}$

$\dot{s}=\ddot{\tilde{x}}+\lambda\dot{\tilde{x}}$, $u=-\hat{f}+\ddot{x}_d-\lambda\dot{\tilde{x}}-k\cdot sign(s)$

$=\ddot{x}-\ddot{x}_d+\lambda\dot{\tilde{x}}$

$=f+u-\ddot{x}_d+\lambda\dot{\tilde{x}}$

$\dot{s}=f-\hat{f}+\cancel{\ddot{x}_d}-\cancel{\lambda\dot{\tilde{x}}}-k\cdot sign(s)\ \cancel{-\ddot{x}_d}+\cancel{\lambda\dot{\tilde{x}}}$

$=f-\hat{f}-k\cdot sign(s)$

$\dot{V}=s\cdot\dot{s}=s\cdot(f-\hat{f}-k\cdot sign(s))$

- $f=\hat{f} \Rightarrow \dot{V}=-k|s|<0$
- $f>\hat{f}$ $\left\{ \begin{array}{l} |f-\hat{f}|<F \\ f-\hat{f}<F \end{array} \right\}$

$\dot{V}=s\cdot(f-\hat{f}-k\, sign(s))<s\cdot(F-k\, sign(s))$

$s\cdot(F-k\cdot sign(s))<0 \Rightarrow$

- $s>0 \Rightarrow F-k\cdot 1<0 \Rightarrow k>F$
- $s<0$

$(F-(-1)\cdot k)>0 \Rightarrow F+k>0$

$k>-F$

$$\ddot{x} = f(\dot{x}, x, t) + u \cdot g(\dot{x}, x, t)$$

$$g = 1.$$

$$\ddot{x} = \underbrace{f(\dot{x}, x, t)}_{\substack{\text{known} \\ \text{+ unknown parameters}}} + u.$$

$$f(\dot{x}, x, t) = f_1(\ )\cdot p_1 + f_2(\ )\, p_2 + \dots$$

eg $\ddot{x} + A\dot{x} + Bx = u. \Rightarrow f = \underbrace{-A}_{p_1}\underbrace{\dot{x}}_{f_1}\underbrace{-B}_{p_2}\underbrace{x}_{f_2}$

$$f = \underbrace{(\dot{x}^2 + \cos x)}_{f_1} \cdot \underbrace{7}_{p_1} + \underbrace{x \cdot |x|}_{f_2} \cdot \underbrace{(-\pi)}_{p_2} + \underbrace{\cos t}_{p_3} \cdot \underbrace{\cos x}_{f_3} \cdot 3$$

$$f = f_1 \cdot p_1 + f_2 \cdot p_2 + \dots$$

f_1, f_2: KNOWN Functions of $x, \dot{x} \dots$

p_1, p_2: unknown constants

$\ddot{x} = f(\quad) + u.$

$s = \dot{\tilde{x}} + \lambda\tilde{x} \qquad V(s) = \frac{1}{2}s^2 \qquad \dot{V} = s \cdot \dot{s}$

$\dot{s} = \dots = f + u - \ddot{x}_d + \lambda\dot{\tilde{x}}$

$= u + f_1 p_1 + f_2 p_2 + \dots - \ddot{x}_d + \lambda\dot{\tilde{x}}$

$= u + F \cdot P \quad , F = [f_1 \quad f_2 \dots \quad \ddot{x}_d \quad \dot{\tilde{x}}]$

$P = [p_1 \quad p_2 \dots \dots -1 \quad \lambda]^T$

I proved $\quad \dot{s} = u + F \cdot P$

$\dot{V} = s \cdot \dot{s} = s(u + F \cdot P). \Big\} \Rightarrow$

If P = known $\quad u = -F \cdot P - s\,k$

$\dot{V} = -k \cdot s^2$

$\hat{P}$ = estimation

P = unknown

$\Rightarrow u = -F \cdot \hat{P} - ks$

$\dot{s} = -F \cdot \hat{P} - ks + FP$

$= -ks + F \cdot \tilde{P} \quad , \tilde{P} = P - \hat{P}$

$\dot{V} = s \cdot \dot{s} = -ks^2 + Fs \cdot \tilde{P}$

If only $\tilde{P} \to 0$ cond then $s \to 0$

or $\tilde{x} \to 0$

$V_1^{(s)} > 0$, $\dot{V}_1 < 0 \Rightarrow s \to 0$

$V_2(\tilde{p}) > 0$, $\dot{V}_2 < 0 \Rightarrow \tilde{p} \to 0$

I want $\tilde{p}_1 \to 0$, $\tilde{p}_2 \to 0$ .....

$$V(s) = \frac{1}{2}s^2 + \frac{1}{2}\tilde{p}_1^2 + \frac{1}{2}\tilde{p}_2^2 + \ldots > 0$$

or

$$V(s) = \frac{1}{2}s^2 + h_1 \cdot \frac{1}{2}\tilde{p}_1^2 + \frac{1}{2}h_2\tilde{p}_2^2 + \ldots..$$

$$= \frac{1}{2}s^2 + \frac{1}{2}\tilde{p}^T \cdot H \cdot \tilde{p} \qquad H = \begin{bmatrix} h_1 & 0 & 0 & 0 \\ \cdot & \cdot & h_2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & h \end{bmatrix}$$

$$\dot{V} = \frac{d}{dt}\left(\qquad \right)$$

$$\frac{d}{dt}\left(\frac{1}{2}\tilde{p}^T H \tilde{p}\right) = \underbrace{\frac{1}{2}\dot{\tilde{p}}^T H \tilde{p}} + \frac{1}{2}\tilde{p}^T \cdot H \cdot \dot{\tilde{p}}$$

Assume dim of $\tilde{p}, \dot{\tilde{p}}$ is 9×1

H 9×9

$\tilde{p}^T, \dot{\tilde{p}}^T$ 1×9

$(1 \times 9) \times (9 \times 9) \times (9 \times 1)$

1×1

$3^T = 3$

$-5^T = 5$

$$(\dot{\tilde{p}}^T \cdot H \tilde{p})^T = \dot{\tilde{p}}^T \cdot H \cdot \tilde{p}$$

$$\tilde{p}^T \cdot H \cdot \dot{\tilde{p}} = \dot{\tilde{p}}^T H \tilde{p}$$

$$\frac{d}{dt}\left(\frac{1}{2}\tilde{p}^T H \tilde{p}\right) = \frac{1}{2}\dot{\tilde{p}}^T H \tilde{p} + \frac{1}{2}\tilde{p}^T H \dot{\tilde{p}}$$

$$= \frac{1}{2}\dot{\tilde{p}}^T \cdot H \tilde{p} + \dot{\tilde{p}}^T H \tilde{p}\,\frac{1}{2}$$

$$= \dot{\tilde{p}}^T \cdot H \tilde{p} \quad \Rightarrow$$

$$\tilde{p} = p - \hat{p}$$

$$\dot{\tilde{p}} = \underbrace{\dot{p}}_{0} - \dot{\hat{p}} = -\dot{\hat{p}}$$

$$\frac{d}{dt}\left(\quad\right) = -\dot{\hat{p}}^T H \tilde{p}$$

$$V = \frac{1}{2}s^2 + \frac{1}{2}\tilde{p}^T H \tilde{p} \Rightarrow$$

$$\dot{V} = s \cdot \dot{s} - \dot{\hat{p}}^T H \tilde{p}$$

Also $\dot{s} = u + Fp$, $u = -F\hat{p} - ks$

$$\dot{s} = F\tilde{p} - ks$$

$$\dot{V} = -ks^2 + F\tilde{p} \cdot s - \dot{\hat{p}}^T H \tilde{p}$$

$$u = -F \cdot \hat{p} - ks$$

$$\dot{V} = -ks^2 + \underbrace{F\tilde{p} \cdot s - \dot{\hat{p}}^T H \tilde{p}}_{0}$$

when $$sF\tilde{p} - \dot{\hat{p}}^T H \tilde{p} = 0$$

$$sF\cancel{\tilde{p}} = \dot{\hat{p}}^T H \cancel{\tilde{p}}$$

$$s \cdot F = \dot{\hat{p}}^T H.$$

$$\boxed{\dot{\hat{p}}^T = s \cdot F \cdot H^{-1}}$$

$$\hat{p}^T = \int sF \cdot H^{-1}\, dt \Rightarrow \hat{p} = \left(\int sFH^{-1} dt\right)^T$$

$$\downarrow$$

$$u = -F\hat{p} - ks$$

$$\ddot{x} = \underbrace{3\dot{x} + 2x}_{f} + u$$

$$u = -F\cdot\hat{P} - ks$$

$$s = \dot{\tilde{x}} + \lambda\tilde{x}$$

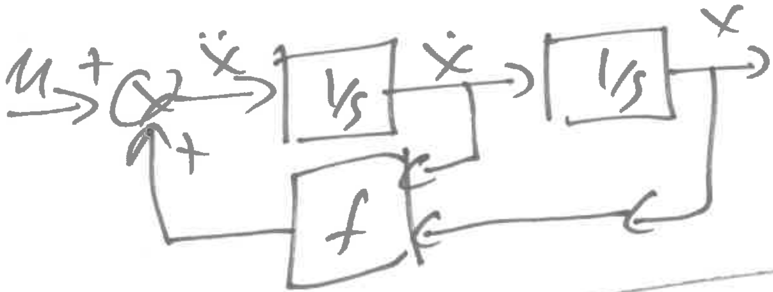
$$\tilde{x} = x - x_d \quad , \quad x_d = \dots$$

$$\hat{P} = \int sF\cdot\Gamma^{-1}\,dt$$

$f_1 = \dot{x}$ $\quad P_1 = 3$

$f_2 = x$ $\quad P_2 = 2$

$$F = [f_1 \quad f_2 \quad \ddot{x}_d \quad \dot{\tilde{x}}] = [\dot{x} \quad x \quad \ddot{x}_d \quad \dot{\tilde{x}}]$$



$x_d = \dots \longrightarrow$

$\dot{x}_d = \dots \longrightarrow$

$\ddot{x}_d = \dots \longrightarrow$

$\tilde{x} \longrightarrow$

$\dot{\tilde{x}} \longrightarrow$

$s \longrightarrow$

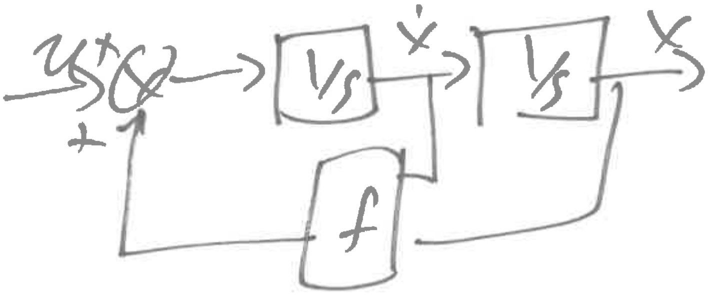
$\dot{x} \longrightarrow [1\ 0\ 0\ 0] \longrightarrow [\dot{x}\ 0\ 0\ 0]$

$x \longrightarrow [0\ 1\ 0\ 0] \longrightarrow [0\ x\ 0\ 0]$

$\ddot{x}_d \longrightarrow [0\ 0\ 1\ 0] \longrightarrow [0\ 0\ \ddot{x}_d\ 0]$

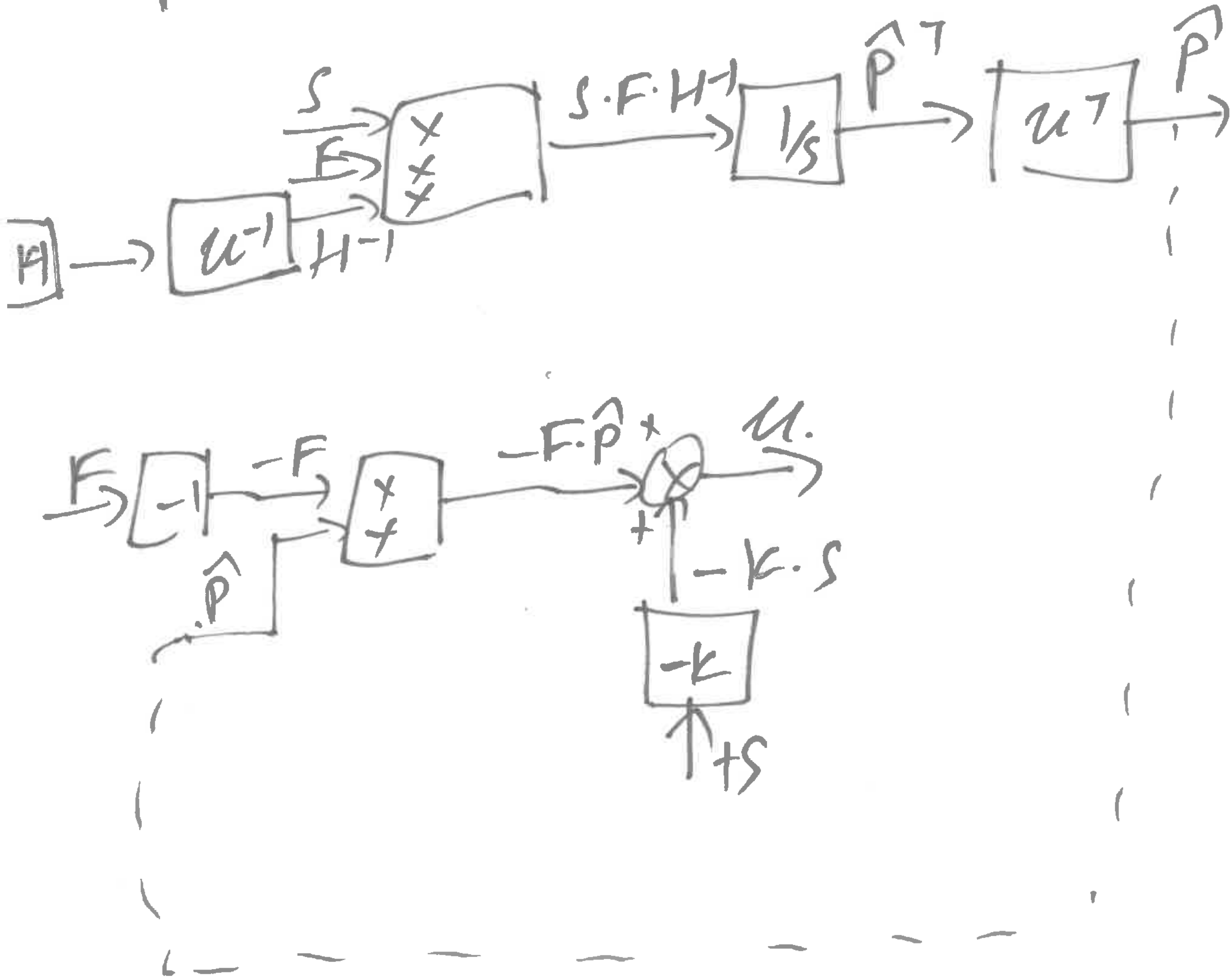
$\dot{\tilde{x}} \longrightarrow [0\ 0\ 0\ 1] \longrightarrow [0\ 0\ 0\ \dot{\tilde{x}}]$

$\longrightarrow \boxed{\begin{matrix}+\\+\\+\\+\end{matrix}} \longrightarrow [\dot{x} \quad x \quad \ddot{x}_d \quad \dot{\tilde{x}}] = F$



$$u = -\underline{F}\cdot\hat{p} - K\underline{s}$$

$$\hat{p}^T = \int s\cdot F\cdot H^{-1}\,dt$$



$$P = [P_1 \quad P_2 \quad -1 \quad \lambda]$$
$$= [3 \quad 2 \quad -1 \quad 1]$$