## Denavit and Hartenberg Method

Define:

1. $1_{n-1}$ : Link length: The distance from $\mathrm{z}_{\mathrm{n}-1}$ to $\mathrm{z}_{\mathrm{n}}$ measured along the $\mathrm{x}_{\mathrm{n}-1}$ axis.
2. $a_{n-1}$ : Link twist: The angle between $z_{n-1}$ and $z_{n}$ measured about the $x_{n-1}$ axis.
3. $d_{n}$ : Link offset: The distance from $\mathrm{x}_{\mathrm{n}-1}$ to $\mathrm{x}_{\mathrm{n}}$ measured along the $\mathrm{z}_{\mathrm{n}}$ axis.
4. $\theta_{\mathrm{n}}$ : Link angle: The angle between $\mathrm{x}_{\mathrm{n}-1}$ and $\mathrm{x}_{\mathrm{n}}$ measured about the $\mathrm{z}_{\mathrm{n}}$.

## Rules:

1. Identify the joints (including their variables) and links (including the robot base). We start from $\mathrm{n}=1$ for the robot base, i.e. reference frame $\mathrm{n}-1=0$.
2. Identify the joint axes; assign the z axes such as they coincide with the joint axes.
3. Identify the common normal between $\mathrm{z}_{\mathrm{n}-1}$ and $\mathrm{z}_{\mathrm{n}}$. At the point of the intersection of the common normal of $n-1$ and $n$, i.e. $1_{n-1}$, and joint axis $n-1$ assign the origin of frame $\{n-1\}$.
4. Assign the $\mathrm{x}_{\mathrm{n}-1}$ axis to point along the common normal of $\mathrm{n}-1$ and n .
5. Find the $y_{n-1}$ by the right-hand rule.
6. Assign frame $\{0\}$ in such a way that it coincides with $\{1\}$ when the joint variable is zero.
7. Last frame ( n ):
a. Prismatic: Assign $\mathrm{x}_{\mathrm{n}}$ such as $\theta_{n}=0$, and place the origin at the intersection of $x_{n-1}$ and $z_{n}$ when $d_{n}=0$.
b. Revolute: Assign $\mathrm{x}_{\mathrm{n}}$ such as $\mathrm{x}_{\mathrm{n}}$ is parallel to $\mathrm{x}_{\mathrm{n}-1}$ when $\theta_{n}=0$, and place the origin such as $\mathrm{d}_{\mathrm{n}}=0$.
Special cases:
8. If the two joint axes $\left(\mathrm{z}_{\mathrm{n}-1}\right.$ and $\left.\mathrm{z}_{\mathrm{n}}\right)$ intersect, then the origin of $\{\mathrm{n}-1\}$ is located at the intersection and the $\mathrm{x}_{\mathrm{n}-1}$ axis is orthogonal to the plane that the joint axes create.
9. If the two joint axes are parallel, then assign the origin in such a way that it is going to make $\mathrm{d}_{\mathrm{n}}$ zero.


$$
{ }^{n-1} T_{n}=\operatorname{Rot}\left(x, a_{n-1}\right) \operatorname{Trans}\left(l_{n-1}, 0,0\right) \operatorname{Rot}\left(z, \theta_{n}\right) \operatorname{Trans}\left(0,0, d_{n}\right) \text { and }{ }^{R} T_{H}={ }^{0} T_{1}^{1} T_{2}{ }^{2} T_{3} \cdots{ }^{n-1} T_{H}
$$

