Table of useful formulae
EEE 3008 & 8005

1. The transformation matrix for a translation vector $\mathbf{r}(r_x, r_y, r_z)$ is $\mathrm{Trans}(\mathrm{r_x}, \mathrm{r_y}, \mathrm{r_z}) = \begin{bmatrix} 1 & 0 & 0 & r_x \\ 0 & 1 & 0 & r_y \\ 0 & 0 & 1 & r_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2. The transformation matrix for rotation along the y-axis is $\mathrm{Rot}(y,\phi) = \begin{bmatrix} \cos(\phi) & 0 & \sin(\phi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3. The transformation matrix for rotation along the x-axis is $\mathrm{Rot}(x,\phi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) & 0 \\ 0 & \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4. The transformation matrix for rotation along the z-axis is $\mathrm{Rot}(z,\phi) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 & 0 \\ \sin(\phi) & \cos(\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5. $$\left({}^{R}\mathbf{T}_N\right)^{-1} = \begin{bmatrix} x_X & y_X & z_X & p_X \\ x_Y & y_Y & z_Y & p_Y \\ x_Z & y_Z & z_Z & p_Z \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} x_X & x_Y & x_Z & -x_X p_X - x_Y p_Y - x_Z p_Z \\ y_X & y_y & y_Z & -y_X p_X - y_Y p_Y - y_Z p_Z \\ z_X & z_Y & z_Z & -z_X p_X - z_Y p_Y - z_Z p_Z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. Orientation of and effector by using sines and cosines:

$$\theta = -\sin^{-1}(x_z), \quad \psi = \sin^{-1}\left(\frac{y_z}{\cos(\theta)}\right), \quad \phi = \sin^{-1}\left(\frac{x_y}{\cos(\theta)}\right)$$

7. Orientation of and effector by using ATAN2: $\phi = \mathrm{atan2}(\mathrm{x_y}, \mathrm{x_x})$, $\theta = \mathrm{atan2}(-x_z, x_x c\phi + x_y s\phi)$, $\psi = \mathrm{atan2}(y_z, z_z)$

8. $$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b), \cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b), \sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$

9. Total velocity: $\dfrac{d\,{}^{R}\overline{p}_{n+1}(t)}{dt} = \dfrac{d\,{}^{R}\overline{p}_n(t)}{dt} + {}^{R}\Omega_n \times {}^{R}Rot_n\, {}^{n}\overline{p}_{n+1} + {}^{R}Rot_n\, {}^{n}\overline{u}_{n+1}$

10. Propagation from link to link

a) Revolute joint:
$$\left.\begin{aligned} {}^{n+1}\overline{u}_{n+1} &= {}^{n+1}R_n\left({}^{n}\overline{u}_n + {}^{n}\Omega_n \times {}^{n}\overline{p}_{n+1}\right) \\ {}^{n+1}\Omega_{n+1} &= {}^{n+1}R_n\, {}^{n}\Omega_n + {}^{n+1}\begin{bmatrix} 0 & 0 & \dot{\theta}_{n+1} \end{bmatrix}^T \end{aligned}\right\}$$

b) Prismatic joint:
$$\left.\begin{aligned} {}^{n+1}\overline{u}_{n+1} &= {}^{n+1}R_n\left({}^{n}\overline{u}_n + {}^{n}\Omega_n \times {}^{n}\overline{p}_{n+1}\right) + {}^{n+1}\begin{bmatrix} 0 & 0 & \dot{d}_{n+1} \end{bmatrix}^T \\ {}^{n+1}\Omega_{n+1} &= {}^{n+1}R_n\, {}^{n}\Omega_n \end{aligned}\right\}$$