Nonlinear Behavior of Self-excited Induction Generator Feeding an Inductive Load

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***Abstract*-- The nonlinear behavior of a self-excited, smooth air gap, cage induction generator feeding an inductive load is analyzed in this paper, allowing for the effects of machine saturation. The self autonomous system is shown to exhibit a transition from a periodic orbit to a quasi-periodic orbit through a Neimark bifurcation.**

***Index Terms*-- Bifurcation theory, induction generator, induction machine, inductive load, nonlinear dynamics**

I. INTRODUCTION

INDUCTION generators are widely used in conjunction with small hydro or wind turbine to produce electric power, mainly due to their low cost, compared with synchronous machines. The generator in such applications is usually connected directly to the ac supply network which also provides the necessary reactive power for the production of the machine rotating magnetic flux. This need for reactive power limits the use of the induction machine as a stand alone generator for remote applications where a supply connection is not available. To overcome this problem the reactive power can be supplied from a capacitor bank connected across the stator terminals, allowing the machine to work as a Self-Excited Induction Generator (SEIG) in the absence of a supply connection.

State space methods [1]-[4] have to be used to model and study the dynamics of these systems. The states of the system may be the machine currents, fluxes or a combination of these [5], [6]. The model must also include components that represent the magnetic nonlinearities (mainly cross-saturation phenomena) of the machine [7] as the machine is normally working with values of magnetic flux density near the saturation level. Hence the overall model of the system will be highly nonlinear and time varying. The dynamical analysis of the system is further complicated by the use of capacitor bank which provides the reactive power to the generator. This paper studies the dynamical behavior of self-excited induction generators and shows that it is possible to have bifurcation phenomena which force the system to change its desired stable response. The bifurcation that causes this loss of stability is shown to be a Neimark bifurcation.

The machine nonlinear model is presented and described and the operation of the self-excited generator on no-load, and when feeding a purely resistive load, is examined to show that the system exhibits a normal period one orbit. When linear inductive components are included in the load the machine undergoes a transition from a stable period one orbit to a quasi-periodic through a Neimark bifurcation.

II. MODELING OF THE SATURATED INDUCTION MACHINE

The mathematical model of the induction machine uses four states (currents and/or fluxes) and is linear time-varying rotor speed depended. If the chosen states are the stator and rotor currents expressed at a Stationary Reference Frame (SRF) the model is:

$$\mathbf{U} = \mathbf{RI} + \mathbf{L_1}\frac{d\mathbf{I}}{dt} + \omega_r \mathbf{L_2} \tag{1}$$

where **U** is the vector with the stator and rotor voltages, **I** is the vector with the stator and rotor currents, **R** is the resistive matrix, ω_r is the rotor speed and $\mathbf{L}_1$, $\mathbf{L}_2$ are inductive matrices. To model the nonlinearity the last two matrices have to change and to be a function of the magnetizing current instead of being constant. Hence the 5 matrices of the magnetically nonlinear system are:

$$\mathbf{U} = \begin{bmatrix} u_{sD} & u_{sQ} & u_{rd} & u_{rq} \end{bmatrix}^T,\ \mathbf{I} = \begin{bmatrix} i_{sD} & i_{sQ} & i_{rd} & i_{rq} \end{bmatrix}^T,$$

$$\mathbf{R} = diag(R_s, R_s, R_r, R_r),$$

$$\mathbf{L_1} = \begin{bmatrix} L_{sd} & L_{dq} & L_{md} & L_{dq} \\ L_{dq} & L_{sq} & L_{dq} & L_{mq} \\ L_{md} & L_{dq} & L_{rd} & L_{dq} \\ L_{dq} & L_{mq} & L_{dq} & L_{rq} \end{bmatrix},\ \mathbf{L_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & L_m & 0 & L_r \\ -L_m & 0 & -L_r & 0 \end{bmatrix}$$

where L_m is the magnetizing inductance: $L_m = \left|\bar{\psi}_m\right| / \left|\bar{i}_m\right|$

The cross-saturation inductance (L_{dq}) is [7]:

$$L_{dq} = \frac{i_{md} i_{mq}}{i_m} \times \frac{dL_m}{d\left|\bar{i}_m\right|} \tag{2}$$

this equation can be simplified to:

$$L_{dq} = \frac{i_{md} i_{mq}}{i_m} \times \frac{L - L_m}{\left|\bar{i}_m\right|} \tag{3}$$

where $L = d\left|\bar{\psi}_m\right| / d\left|\bar{i}_m\right|$ is the dynamic inductance.

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The nonlinear curves of the magnetizing and dynamic inductance are taken from [7] and are shown in Fig. 1.

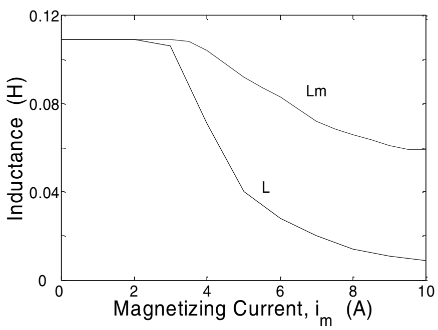

Fig. 1. The saturated magnetizing inductance curve L_m and the dynamic inductance curve L.

The direct and quadrature axis saturated inductances are

$$L_{md} = L_m + \frac{i_{md}}{i_{mq}} L_{dq}\ ,\ L_{mq} = L_m + \frac{i_{mq}}{i_{md}} L_{dq} \tag{4}$$

The stator and rotor dq axis inductance are as following

$$\begin{aligned} L_{sd} &= L_{sl} + L_{md}\,; \quad L_{sq} = L_{sl} + L_{mq} \\ L_{rd} &= L_{rl} + L_{md}\,; \quad L_{rq} = L_{rl} + L_{mq} \end{aligned} \tag{5}$$

where L_{sl} and L_{rl} are the unsaturated stator and rotor leakage inductance, respectively. The mechanical equation between the prime mover and the electrical torque is

$$T_e + T_m = J\frac{d\omega}{dt} \tag{6}$$

where

$$T_e = \frac{3}{2}\frac{P}{2}\left(\psi_{rq} i_{rd} - \psi_{rd} i_{rq}\right) \tag{7}$$

III. The Machine Parameters and The Procedures of Running The SEIG

By using the equations that were presented in section II a 4-pole start connected IG of 1.5kW, with a capacitor bank (135µF per phase) was simulated. The rated voltage and current of the machine were 220/380V and 7/4A respectively and the rated frequency was 50Hz. The stator and rotor resistances were 0.6Ω and 0.83Ω respectively while the stator and rotor impedances were 1.8Ω/phase and 1.8Ω/phase respectively. The prime mover was represented by a dc machine rotating at 1500rev/min. To represent the effect of the capacitor bank and the various loads that were applied the following dq equivalent circuit was used [8]:

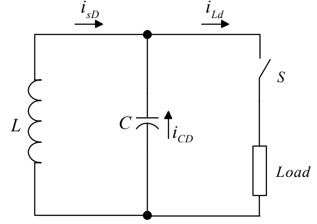

Fig. 2. Stator direct component without load, i_{CD} is the capacitor current and i_{Ld} is the load current.

A. The Initial Self Excitation of the Induction Machine with No Load

As the machine is working under no load the switch S remains open and hence the d-q voltages are:

$$u_{CD} = -u_{sD} = -\frac{1}{C}\int i_{sD}\,dt \tag{8}$$

$$u_{CQ} = -u_{sQ} = -\frac{1}{C}\int i_{sQ}\,dt \tag{9}$$

By using the mathematical model which is presented in section II and by using (8) and (9) it is possible to simulate the behavior of the IG which is driven by a dc machine at a constant speed of 1500 rev/min under no load. From that test it can be seen that as the stator voltage increases (entering the saturation area) so does the magnetizing current and hence a big drop of the magnetizing inductance is observed, Fig.4, which agrees with the curves shown in Fig.1.

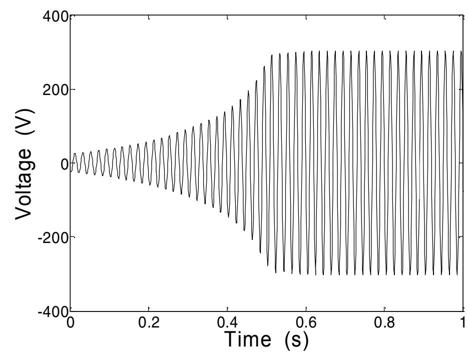

Fig. 3. Stator line to line voltage builds up without load.

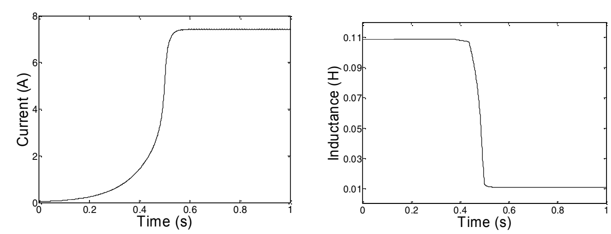

Fig. 4. Variation of magnetizing current (a) and magnetizing inductance (b) with voltage builds up without load.

B. The SEIG with a Resistive Load

If the contactor S closes and the IG is supplying a resistive load the extra equations needed are:

$$u_{CD} = -u_{Ld} = -Ri_{Ld} \tag{10}$$

$$i_{CD} = -C\frac{du_{Ld}}{dt} = -RC\frac{di_{Ld}}{dt} \tag{11}$$

$$i_{sD} = -i_{CD} + i_{Ld} \tag{12}$$

$$i_{sD} = RC\frac{di_{Ld}}{dt} + i_{Ld} \tag{13}$$

$$u_{CQ} = -u_{Lq} = -Ri_{Lq} \tag{14}$$

$$i_{sQ} = RC\frac{di_{Lq}}{dt} + i_{Lq} \tag{15}$$

By using these equations the IG was simulated and its response is shown in Figs. 5 & 6. Initially the IG is under no load and at 0.1s a resistive load of 27Ω is applied. It is clear from that figure that there is a drop at the output voltage as the system has to supply the extra load.

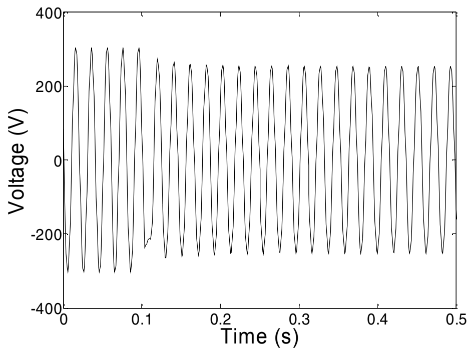

Fig. 5. Stator line to line voltage with a resistive load.

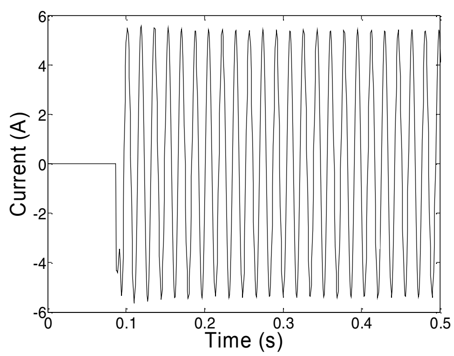

Fig. 6. Load current (i_{LD}) with a resistive load.

From these figures it can be seen that (regardless of the voltage drop) the solution curve in the state space follows a closed curve of period 1. At this point it has to be stated that in practice another capacitor is used in series with the resistive load which greatly decreases the voltage drop but from the dynamical point of view the behavior of the system remained qualitatively the same (i.e. the system exhibits a similar stable period one orbit) and hence due to space limitation it is not shown.

IV. The Nonlinear Behaviors of SEIG Feeding an Inductive Load

In this section a balanced three phase inductive load of 30Ω and 15mH per phase has been added to the system when the machine is driven at a constant speed of 1500rev/min. Therefore the equations describing this system are:

$$u_{CD} = -u_{Ld} = -Ri_{Ld} - L\frac{di_{Ld}}{dt} \tag{16}$$

$$i_{CD} = -C\frac{du_{Ld}}{dt} = -RC\frac{di_{Ld}}{dt} - LC\frac{d^2 i_{Ld}}{dt^2} \tag{17}$$

$$i_{sD} = -i_{CD} + i_{Ld} \tag{18}$$

$$i_{sD} = RC\frac{di_{Ld}}{dt} + LC\frac{d^2 i_{Ld}}{dt^2} + i_{Ld} \tag{19}$$

$$u_{CQ} = -u_{Lq} = -Ri_{Lq} - L\frac{di_{Lq}}{dt} \tag{20}$$

$$i_{sQ} = RC\frac{di_{Lq}}{dt} + LC\frac{d^2 i_{Lq}}{dt^2} + i_{Lq} \tag{21}$$

Thus the state equations of capacitor voltages of both axes are obtained by substituting (19) into (16) and (21) into (20).

The response of the system was investigated for various values of the capacitance. For C=135μF (see Figs. 7 - 8) the response of the system is a period one closed orbit which indicates that the system operates within the desired specification. As this is a high order system it is not possible to plot all states and therefore only two representative states are shown in Fig. 8. All other combinations gave similar results and hence are not shown here.

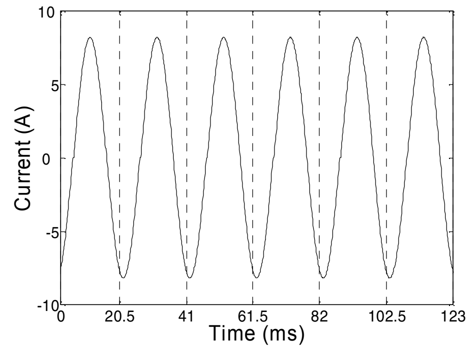

Fig. 7. Stator phase A current for C=135μF

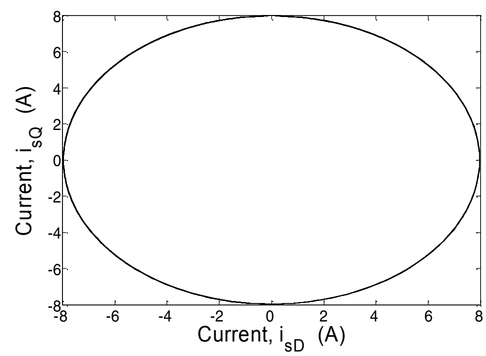

Fig. 8. Phase plane diagram for C=135μF

When the capacitance is increased to 156μF the response of the system changes to what initially appeared to be a period seven limit cycle (Fig. 9). A closer look reveals that the solution does not follow any periodic pattern but is instead a quasi-periodic behavior. By ignoring the initial transients the phase space was plotted using 5000 samples. Fig. 10 shows that the locus of the solution lies on a "toroid typed" manifold (difficult to visualize in such a high order).

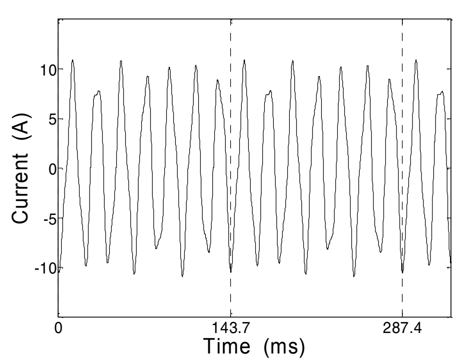

Fig. 9. Stator phase A current for C=156μF

Apart from this to prove that the system exhibits a quasi-periodic behavior it must be shown that the solution is dense on the torus, the Poincaré section is a closed orbit and also to show the bifurcation diagram. Other techniques can also be used like the Lyapunov exponent or the eigenvalues of the monodromy matrix of the period one orbit but in this paper only the first set is presented.

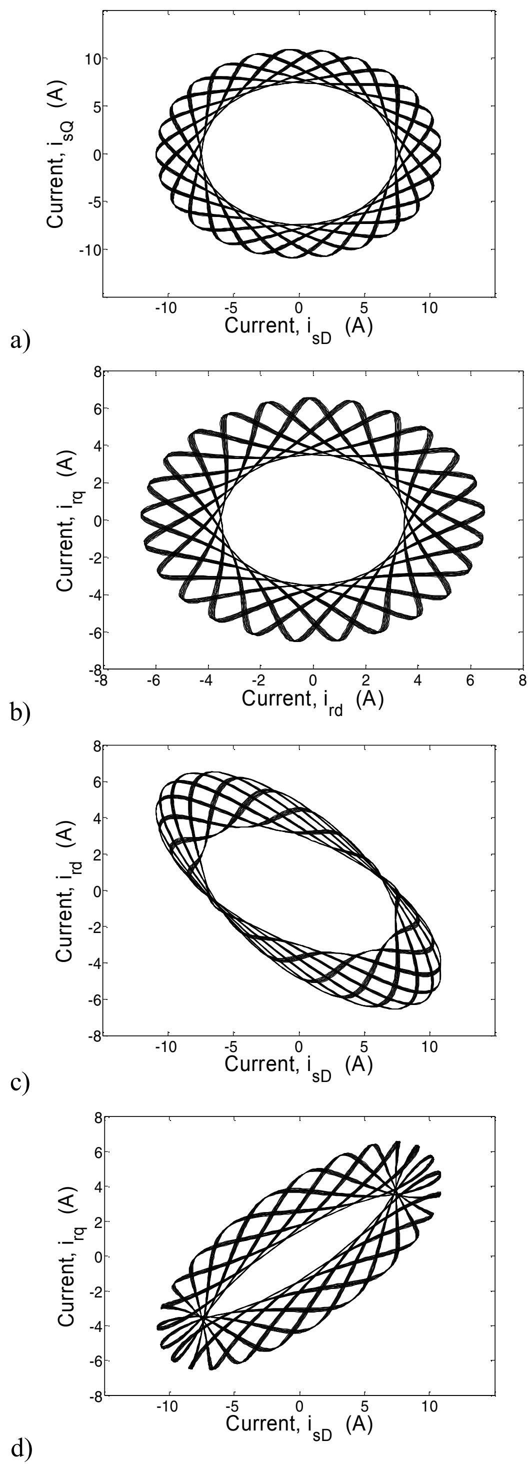

Fig. 10. Phase plane diagram for C=156μF (5000 sample points) (5000 sample points)

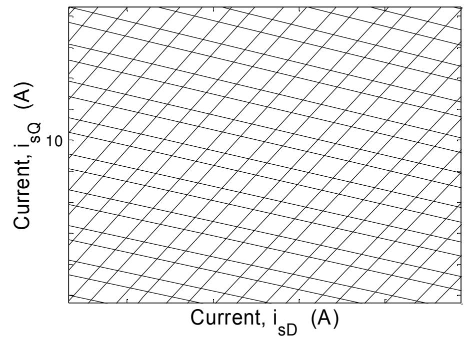

Fig. 11. Dense orbit in the torus

Fig. 11 shows the 20000 samples after the initial transient of one of previous tori and is clearly demonstrated that the orbit is dense on the torus. Furthermore, by sampling the state vector when the current i_{sD} is zero the Poincaré map of the system is derived and as it can be seen by Fig.12 it is a closed orbit which again proves the statement that the orbit is quasi-periodic.

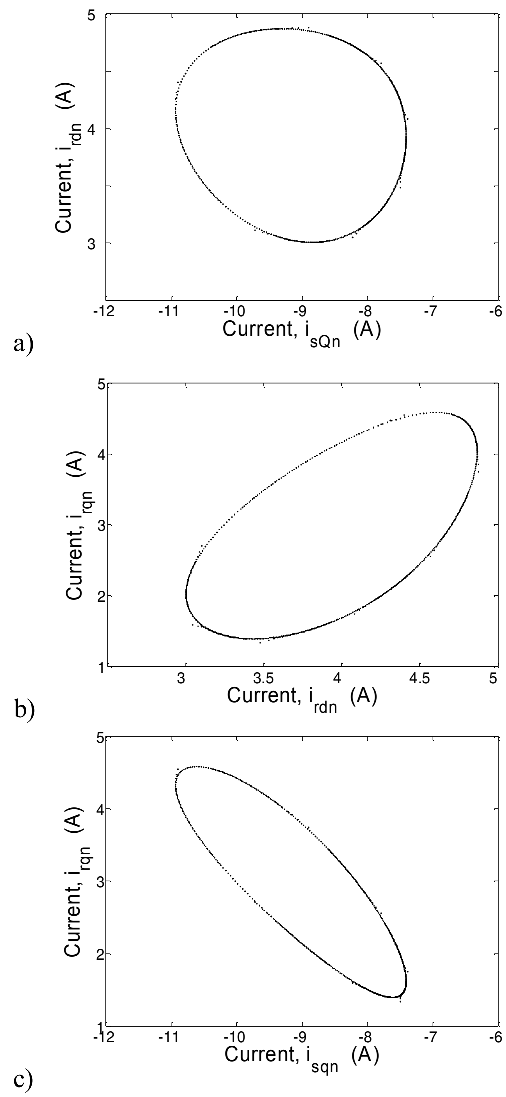

Fig. 12. Poincare section of stator q-axis versus rotor q-axis sample.

The final test is to create the bifurcation diagram of the system which in this cases it was chosen to be the sampled value of the q-axis stator current when the d-axis stator current is zero, the bifurcation variable was the value of the capacitors used in the capacitor bank. Fig. 13 shows that diagram and it can be seen that the system looses its stability through a Neimark bifurcation and hence the system exhibits a quasi-periodic orbit [9].

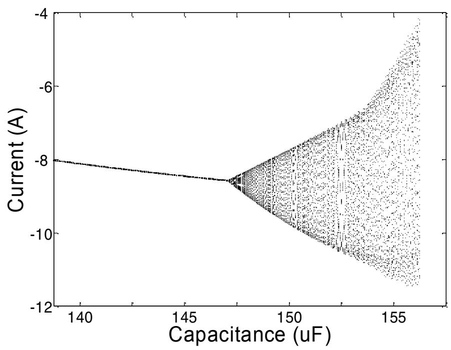

Fig. 13. Bifurcation diagram of stator q-axis current.

V. CONCLUSIONS

The performance of the self-excited induction generator with no load, resistive load and compensate capacitors is studied. The nonlinear model utilizing currents as state variables is then connected with an inductive load to the stator terminal. The nonlinear behaviors of the induction generator are investigated through a bifurcation diagrams, phase spaces and Poincaré sections while changing a control parameter, the self-excited capacitors. The results show that the autonomous dynamical system loses its stability from period one orbit moving to a quasi-periodic orbit as a result of small changes in the values of system parameters (in this case the self-excited capacitors). The practical experiments of the machine will be examined and compared with the simulation models in the future work.

VI. REFERENCES

[1] P. K. Shadhu Khan and J. K. Chatterjee, "Three-Phase Induction Generators: A Discussion on Performance," *Taylor & Francis Electric Machines and Power Systems,* vol. 27, pp. 813-832, August 1999.

[2] J. M. Elder, J. T. Boys, and J. L. Woodward," Self-excited induction machine as a small low-cost generator," *IEE Proceedings*, vol. 131, Pt. C, No. 2, pp. 33-41, March 1984.

[3] J. E. Brown, K. P. Kovacs, and P. Vas, "A Method of including the Effects of Main Flux Path Saturation in The Generalized Equations of A. C. Machines," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-102, No. 1, pp. 96-103, January 1983.

[4] P. Vas, K. E. Hallenius, and J. E. Brown, "Cross- Saturation Smooth-Air- Gap Electrical Machines," *IEEE Trans. Energy Conversion*, vol. 1, No. 1, pp. 103-113, March 1986.

[5] P. Vas, *Electrical Machines and Drives — A space-vector theory approach,* Oxford, Clarendon Press, 1992.

[6] F. A. Farret, B. Palle, and M. G. Simões, "State Space Modeling Parallel Self-excited Induction Generators for Wind Farm Simulation," *Industry Applications Conference, 2004. 39th IAS Annual Meeting. Conference Record of the 2004 IEEE*, vol. 4, pp. 2801-2807, October 2004.

[7] K. E. Hallenius, P. Vas, and J. E. Brown, "The Analysis of a Saturated Self-Excited Asynchronous Generator," *IEEE Trans. Energy Conversion*, vol. 6, No. 2, pp. 336-341, June 1991.

[8] M. G. Simões and F. A. Farret, *Renewable Energy Systems: Design and Analysis with Induction Generators*, CRC Press, 2004.

[9] S. Banerjee and G. C. Verghese, *Nonlinear Phenomena in Power Electronics*, New York, IEEE Press, 2001.

VII. BIOGRAPHIES

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