A Probabilistic Adaptive Model Predictive Power Pinch Analysis (PoPA) Energy Management Approach to Uncertainty

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Abstract

This paper proposes a probabilistic power pinch analysis approach based on Monte Carlo simulation for energy management of hybrid energy systems uncertainty. The systems power grand composite curve is formulated with the chance constraint method to consider load stochasticity. In a predictive control horizon the power grand composite curve is shaped based on the pinch analysis approach. The robust power management strategy (PMS) effected in a control horizon is inferred from the likelihood of a bounded predicted PGCC, violating the pinch. Furthermore, the response of the system using the PMS of the proposed method is evaluated against the day ahead and adaptive power pinch strategy.

1 Introduction

The significant impact of greenhouse gas emission on the ecosystem has continued to encourage the deployment of renewable energy sources (RES) such as wind turbines and solar panels, for distributed generation (DG). Additionally, the standalone RES serves as a good alternative for rural electrification [1]. However, due to the high variability of RES, the hybrid energy storage has been incorporated in the DG network to enhance reliability. Consequently, energy produced by fossil fuels is reduced by the RES penetration [2]. The combination of DG and hybrid energy storage systems (HESS) transforms the traditional power network from “fit and forget” to a more dynamical and multi-purpose system. Energy can also flow in multiple directions and in various forms (like electrical and thermal) [3]. Therefore energy management strategies (EMS) with active control of the HESS are desirable but introduce significant challenges.

Power pinch analysis (PoPA) [4] is a graphical method which enables the systematic identification of energy recovery opportunities in hybrid RES. It considers power demand and supply requirements with respect to time in the form of the Power grand composite curves (PGCC) to identify inflection points (called pinches) where power demands must be satisfied by external, non-renewable energy sources. The identification of pinch points allows the development of EMS which support efficient internal energy recovery so that the use of non-renewable energy can be avoided. The PGCC has mainly been used to identify energy needs in deterministic scenarios to enhance reliability and optimal operation of HESS. An EMS, using the PoPA graphical tool, was proposed in an isolated hybrid micro-grid to identify the minimum energy targets required for storage and consequently the needed outsourced electricity to cater for energy deficit in a hybrid system [4]. The operation cost and emission impact of diesel generator was reduced after integration with renewable energy from wind turbines and photovoltaic panels (PVs) using PoPA in [5]. In [2] PoPA was presented as an enhanced graphical tool, exploiting the demand and supply PGCC of an isolated hybrid micro-grid in a novel way. In addition, load shifting has been employed using PoPA in [6]. The PGCC was used in the form of a cascaded demand-supply table and as a graphical tool to identify peak demand thereafter, time shifted to off-peak period, in a bid to enhance grid reliability. More recently, [7] proposed a chance constrained optimisation in a Pinch Analysis framework for RES sizing to meet a predefined reliability to uncertainty, thereafter the sequential Monte Carlo simulation was used to verify the approach.

In [8], the shaping of the PGCC was implemented online, for the first time, using the day ahead rolling horizon model predictive approach to compute the open loop control sequence offline for the activation of the energy assets. The EMS which consequently shapes the system’s PGCC in order to avoid violating the pinch point is pre-determined in the predictive horizon and implemented on the system online. However, the open loop approach becomes insufficient in handling forecast error due to uncertainty which causes a mismatch between the estimated and real load and weather data profile.

The PoPA therefore benefits the operator as it serves as a minimalist conservative technique for planning in advance, demand and power supply schedule using the PGCC. However,
weather intermittency as well as the stochastic consumers load usage pattern, poses risk on both the reliability and infrastructure sustainability of the assets. Therefore incorporating the chance constraint method in the EMS for energy recovery is essential.

The problem of weather and load profile uncertainty therefore needs more emphasis as deterministic models are often employed with the PoPA which consequently impacts on reliability as minimum constraints are only considered.

This paper presents two adaptations of the original method presented in [8]. Firstly, the prediction horizon is adapted in a receding model predictive framework based on the prediction error. Secondly, a probabilistic method; Monte Carlo simulation is proposed for investigating the robustness against load/weather uncertainty.

2 Methodological Framework

2.1 Hybrid Energy Storage Systems Modelling

As a case study, the HRES comprises of renewable energy generation via solar panels (PV), fuel cell (FC) and electrolyser (EL), battery (BAT), water (WT) and hydrogen tank (HT), with a backup diesel generator (DSL). The HRES parameters are the same as [3] for consistency. The state of the accumulator SOAcc is expressed mathematically as follows;

\[
SOAcc_{i,m}^n(t) = SOAcc_i(t-1) + \sum_{j=1}^{n} \Delta \delta \mathcal{C}_{ij}(t) - \sum_{j=1}^{n} \Delta \delta \mathcal{C}_{ij}(t) \times \Delta t
\]

The flow of electrical energy or material is defined as follows:

\[
F_{x,y}^i(t) = \epsilon_i(t) \cdot \delta Q_i^j(t), \quad i \in \{x,y\}
\]

where, \(\epsilon_i(t)\) is a binary variable of the converter’s state, \(\delta\) is used for varying the magnitude of energy or material \(Q_i^j(t)\) converted by the \(i^{th}\) unit. In addition, the existence of an edge, represented by the binary variable \(\epsilon_i(t)\) \((0,1)\), is inferred from the state of the storages SOAcc\((t)\) and subscript \(l\) refers to storage system. And \(m, n\) superscripts refer to the actual and estimated value of the HESS.

\[
\epsilon_i(t) = L(\epsilon_i^{\text{Avl}}(t), \epsilon_i^{\text{Red}}(t), \epsilon_i^{\text{Gen}}(t))
\]

2.2 Power Pinch for Hybrid Energy Storage Management

The BAT acts as the primary storage and the excess energy in the system is utilised by the EL to produce hydrogen while the BAT is charged (state-of-charge \(SOAcc_{BAT} > 90\%\)). The FC in turn charges the battery using the stored hydrogen when \(SOAcc_{BAT}\) is below the minimum set limit (30%). The energy management system works to keep \(SOAcc_{BAT}\) of the battery within the acceptable region the \((30\% < SOAcc_{BAT} < 90\%)\) to ensure reliability and optimum operation of the HESS.

The main principles of the PoPA concept, as applied in power generation systems, are illustrated in Figure 1. An accumulator (e.g. hydrogen tank) stores excess energy from a converter (e.g. PV) due to saturation of another converter (e.g. battery’s state of charge \(SOAcc_{BAT} \geq Up\)). Similarly, another converter (e.g. Fuel cell) utilises the stored hydrogen to maintain the \(SOAcc_{BAT}\) from falling below a lower limit \((SOAcc_{BAT} < Lo)\). When maintenance of the system within these limits becomes impossible, they are violated. This is all shown through a plot of \(SOAcc_{BAT}\) with respect to time (continuous black line in Figure 1) which is called the Power Grand Composite Curve (PGCC) of the system. In the case of the \(Lo\) limit, the PGCC indicates the amount of minimum outsourced electricity supply (MOES) required (e.g. through non-renewable resources) in order to move and keep the curve above it. In the case of the \(Up\) limit, the PGCC indicates the amount of minimum absorbed energy (MAE) that needs to be dumped, which is also undesirable. By shifting the entire PGCC up or down (red dotted-line in Figure 1), the point in time where the PGCC touches the \(Lo\) or \(Up\) lines is called the Pinch point. The new, shifted curve indicates the energy storage targets at each instance in order to operate within the desired limits. The initiation or termination of the appropriate converters allows the generation of an overall PMS that best matches the shifted curve. However, realising the PoPA successfully via Day-ahead (DA) strategy requires accurate load and weather data which is often not the case due to uncertainty. The effect of the uncertainty \(\Delta H_{1,2}\) causes a mismatch between the actual and predicted \(SOAcc_{BAT}\) parameters as shown in Figure 2. Therefore violation of the upper and lower pinch may occur.
2.3 Monte Carlo Simulation for RES

The Monte Carlo simulation (MCS) has mostly been utilised to obtain probabilistic insight in order to negate uncertainty via robust sizing [7], energy reserve planning [9] and peak load shaving [10], as well as economic risk analysis [11] in power systems. Furthermore, Monte Carlo simulation is favourable for low capacity planning in low voltage grid to which the simultaneity factor often employed in high-medium reserve planning becomes less accurate.

3 Adaptive Power Pinch with Monte Carlo Simulation for Energy Management

The chance constraint sizing approach presented in [7], for minimum solar panel array area utilised in the PoPA framework, primarily targeted reliability of the deterministic load demand management being met as well as the battery being charged. Furthermore, energy management of BAT in the event that the battery becomes fully charged and the utilisation of the excess energy were not discussed. Thus, this paper presents an adaptation of the works of [4] and [7] by defining the robust adaptive energy management algorithm in a probabilistic chance constrained framework. Furthermore, the excess energy in the system, represented by overcharging the BAT ($SOAcc_{BAT}^{m} > 90\%$) and energy recovered as well as over discharging the BAT ($SOAcc_{BAT}^{m} < 30\%$) is considered in the chance constraints evaluated with the Monte Carlo simulation.

The sequential MC sampling is performed iteratively in the prediction horizon to determine the likelihood of the PGCC, violating constraint (5) - (8). The pinch set points are expressed probabilistically using the chance constraint. Therefore, two PGCCs form a (upper and lower) closed bound within which the uncertainty is defined. Consequently, the PMS which infers the optimal control sequence to keep the system within the desired operating limits is effected in advance at the beginning of the receding control horizon, while incorporating robustness to uncertainty, as follows:

$$SOAcc_{m}^{mijk}(k + 1)^{tr} = \sum_{k=1}^{T} \left[ \sum_{j=1}^{n} f(e_{i}(k), SOAcc_{m}^{mijk}(k), U_{c}(k)) \right]$$

(4)

$$\epsilon_{\text{ FC gen}}(k) + \epsilon_{\text{ EL gen}}(k) \leq 1$$

(5)

$$SOAcc_{i}^{n}(k_{o}) \equiv SOAcc_{i}^{m}(T - 1)$$

(6)

$$S_{LO_{i}} \leq SOAcc_{i}^{m}(k) \leq S_{UP_{i}}$$

(7)

$$\mathcal{F}_{\text{min}} \leq \mathcal{F}_{\text{BAT-Conv}} \leq \mathcal{F}_{\text{max}} \quad \text{Conv} \in \{\text{FC, EL}\}$$

(8)

Moreover, if $\Delta H_{1,2}$ is greater than the error threshold $\xi$ at any sampling instance, the PoPA is repeated in the predictive horizon in order to determine the optimal control sequence from that instant up until time $T$. The threshold error $\xi$ is set at 5% to reduce the computational cost. The state of charge of the battery in the model for PGCC re-computation is updated as follows:

$$\Delta H(k)[k] = [y^{m}(k) - y^{n}(k)[k - 1]]$$

(9)

where, $y^{m}(k)$ is the output state measured at time $k$, and superscripts $m, n$ refers to the real and the estimated state of charge respectively.

The Monte Carlo simulation is performed on the distribution $f(e_{i}(k), SOAcc_{m}^{mijk}(k), U_{c}(k))$ over $j \in [1:n]$ number of random demand load samples, each time the PGCC is recomputed in the prediction horizon. Furthermore, the probabilistic minimum energy target required to satisfy the load demand with a given reliability index for robustness is presented as follows:

The cumulative distribution function of the random variable $SOAcc_{m}^{mijk}$ violating the lower limit is constrained by a chance factor, $\alpha_{1}$:

$$\text{F}(SOAcc_{m}^{mijk}) = \text{Pr}[SOAcc_{m}^{mijk}(k) \geq S_{LO_{i}}] \geq \alpha_{1}, \quad \alpha_{1} \in [0,1]$$

![Figure 3: Probability distribution function of the dependent random variable SOAcc_{m}^{mijk}(%)](image-url)
Therefore the probability of violating the lower operating limit for BAT is constrained as follows:

\[ Pr[SOAcc\text{\textsubscript{BAT}}^m(k) \leq S_{\text{min}}] \geq 1 - \alpha \]  \hspace{1cm} (12)

Similarly, the chance of violating the upper pinch limit is as follows:

\[ Pr[SOAcc\text{\textsubscript{BAT}}^m(k) \geq S_{\text{max}}] \leq \alpha_2 \hspace{1cm} (13) \]

\[ Pr[SOAcc\text{\textsubscript{BAT}}^m(k) \leq S_{\text{max}}] \geq 1 - \alpha_2 \hspace{1cm} (14) \]

\[ Pr[SOAcc\text{\textsubscript{BAT}}^m(k) \geq S_{\text{max}}] \geq 1 - \alpha_2 \hspace{1cm} (15) \]

Therefore, the desired operating range for \( SOAcc\text{\textsubscript{BAT}}^m(k) \) with respect to the chance constraint is expressed with the inverse cumulative distribution function (CDF) in equation (16):

\[ F^{-1}_{SOAcc\text{\textsubscript{BAT}}^m}(\alpha_1) \leq F(SOAcc\text{\textsubscript{BAT}}^m) \leq F^{-1}_{SOAcc\text{\textsubscript{BAT}}^m}(1 - \alpha_2) \]

Similarly, the Probability density function equivalent of the CDF for the desired operating region is shown in Figure 3 and presented in equations (16)-(17). The shaded portion of Figure 3, represents the desired operating region while the lower and upper tails.

\[ F_{\text{max}} \int_{F_{\text{min}}} f(SOAcc) \text{ } d(SOAcc) = F(S_{\text{max}}) - F(S_{\text{min}}) \]  \hspace{1cm} (17)

The decision variable \( U_c \) for the activation of the fuel cell and electrolyser based on the pinch analysis as a consequence of violating equations (2) and (3) respectively is as follows;

\[ U_c = \begin{cases} F_{\text{BAT} \rightarrow FC} & F^{-1}_{SOAcc\text{\textsubscript{BAT}}^m}(\alpha_1) < S_{\text{min}} \\ F_{\text{BAT} \rightarrow EL} & F^{-1}_{SOAcc\text{\textsubscript{BAT}}^m}(1 - \alpha_2) > S_{\text{max}} \end{cases} \]  \hspace{1cm} (18)

where, \( F^{-1}_{SOAcc\text{\textsubscript{BAT}}^m}(\alpha_{1,2}) \) is the inverse cumulative distribution of the randomly distributed variable \( SOAcc\text{\textsubscript{BAT}}^m(k) \). Additionally, the MOES and MAE necessary for maintaining the lower and upper pinch points respectively are obtained as follows in equations (19) and (20) respectively:

\[ \text{MOES} = (S_{\text{min}} - F^{-1}_{SOAcc\text{\textsubscript{BAT}}^m}(\alpha_1)) \]  \hspace{1cm} (19)

\[ \text{MAE} = (F^{-1}_{SOAcc\text{\textsubscript{BAT}}^m}(1 - \alpha_2) - S_{\text{max}}) \]  \hspace{1cm} (20)

The PMS decision making variable \( U_c \) in conjunction with the magnitude of energy flow determined in equations (16) and (17), satisfy the lower and upper pinch points with regards to the chance constraint equations (9) - (11) in an adaptive receding horizon model predictive framework. Furthermore, the power management control sequence obtained with the sequential Monte Carlo simulation of the model is therefore effected in the control horizon taking the overall risk of violating the utility pinch constraints into consideration.

### 4 Load demand and Weather Data

The historical household load demand profile with peak load of 1.5 KW and solar irradiance data corresponding to 54.9783° N, 1,6178° W, are obtained from [13] and [14] respectively. The load profile data set consists of the aggregated power demand of uncontrollable appliances at each hourly time interval representing consumer’s usage pattern. The historical load profile data set, \( A(j,k) \) collected over 365 days, at each hourly time step \( k \), such that \( j=1, 2, 3\ldots365 \) is partitioned into disjointed groups of \( A(j,k) = \{A_1, A_2, A_3, A_4\} \) [12]. Each group of load demand data set corresponds to the consumer’s power usage pattern correlating to the four seasons [9]. Therefore, from the consumer’s historical energy consumption profile as shown in Figure 4, a probability distribution is deduced. In the sequential Monte Carlo simulation, the load demands in each cluster (for each \( j \) at time step \( k \)) are assumed to be normally distributed [10]. Furthermore, in order to validate the proposed approach, the actual load is randomly selected with uniform probability from the load demand distribution corresponding to the time instance \( k \). The Day-Ahead PoPA and Adaptive strategies utilising the average load for each season are compared against the proposed probabilistic PoPA method.

![Figure 4: Load demand profile showing energy consumption pattern variability during winter.](image)

### 5 Uncertainty Analysis of the HRES

The proposed method utilising the chance constrained power pinch for energy management is simulated in MATLAB based on 1000 samples randomly generated from a uniform distribution \( A(j,k) \).

The chance constraints factors were both set to 1% during the simulation. Therefore, state of charge of the battery has a 98% probability of operating within the optimal region (30% \( \leq SOAcc\text{\textsubscript{BAT}}^m \leq 90\% \)). As shown in Figure 5, the systems PGCC is bounded, by both the probabilistic lower and upper PGCC. The response of the system over a period of 72 hrs is shown in Figure 5. The red and blue lines in Figure 5, are the lower and upper predicted PGCC based on the chance constraint. The yellow dashed line represents the actual response of the system. The PGCC upper pinch violation at 40th hr accurately predicts the pinch during the first 72 hrs hence the EL is activated.
We compare three algorithms. The Day Ahead (DA) algorithm where uncertainty is not considered, the RHMP- PoPA algorithm where the prediction error is corrected and the MC-PoPA algorithm where uncertainty is considered. From Table 1, the proposed method has a total of 319 upper pinch violations compared to the DA and RHMP-PoPA with 603, 653. However, it has the most significant number of violations, 1374 and 1208 respectively, in both lower pinch zones (SOA< 20% and 20% < SOA< 30%). Consequently, the RHMP-PoPA has 1176 with the DA-PoPA having the least violations of 1094. The activation of the DSL accordingly correlates with the lower pinch violation, thus the proposed method activated the DSL more frequently, 576 times. This however, is not significant compared to the 561 times the DSL was activated utilising the RHMP-PoPA. Furthermore, the DA-PoPA despite having the lowest DSL activation of 325 times, does not truly reflect positive gains. As gains largely results from over-charging (upper pinch violation) and the stochastic dynamics of the actual load demand profile. Furthermore, the DA-PoPA exhibits lower participation in the energy recovery the FC and EL are activated 79 and 205 times compared the MC-PoPA (194 and 882 times) and RHMP (213 and 724 times). Evidently, the low frequency of activation is due to the day-ahead targeting as the strategy does not account for energy mismatch caused by uncertainty within the horizon.

**Conclusion**

An adaptive power pinch analysis based on Monte Carlo simulation for energy management of hybrid energy storage systems, to uncertainty, has been proposed. The stochastic analysis evidently showed the proposed method performed better in clipping the PGCC from violating the upper pinch. The proposed method was compared to the day-ahead and adaptive PoPA which utilised the average load. However, the day-ahead PoPA which had the most upper violation consequently had a lowest lower pinch violation and DSL utilisation. The adaptive PoPA had a marginally better lower pinch violation compared to the proposed approach. Regardless of the better lower pinch violation the DA-PoPA is arguably sufficient only if the variance in energy target for the day is negligible and does not account for uncertainty. Hence, future work would investigate sensitivity of the methods with respect to several classes of load demand profiles.

<table>
<thead>
<tr>
<th>Operational Parameter</th>
<th>DA-PoPA</th>
<th>RHMP-PoPA</th>
<th>MC-PoPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC attempt (cycles/year)</td>
<td>250</td>
<td>1687</td>
<td>2147</td>
</tr>
<tr>
<td>FC start-stop (cycles/year)</td>
<td>79</td>
<td>194</td>
<td>213</td>
</tr>
<tr>
<td>EL attempts (cycles/year)</td>
<td>205</td>
<td>882</td>
<td>724</td>
</tr>
<tr>
<td>EL start-stop (cycles/year)</td>
<td>205</td>
<td>882</td>
<td>717</td>
</tr>
<tr>
<td>PV start-stop (cycles/year)</td>
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<td>8031</td>
<td>8343</td>
</tr>
<tr>
<td>DSL start-stop (cycles/year)</td>
<td>325</td>
<td>561</td>
<td>576</td>
</tr>
<tr>
<td>Lower Pinch violation (counts/year)</td>
<td>1094</td>
<td>1176</td>
<td>1374</td>
</tr>
<tr>
<td>Upper Pinch violation (counts/year)</td>
<td>998</td>
<td>1025</td>
<td>1208</td>
</tr>
<tr>
<td>Upper Pinch violation (counts/year)</td>
<td>764</td>
<td>729</td>
<td>417</td>
</tr>
<tr>
<td>Upper Pinch violation (counts/year)</td>
<td>653</td>
<td>603</td>
<td>319</td>
</tr>
</tbody>
</table>

Table 1: Operational indices for 8760 hrs.

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