A Case Study of Real Time Implementation of Extended Kalman Filter in Dual Core DSP for The On-line Estimation of Induction Machine Parameters

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Abstract—This paper proposed the estimation of the magnetizing inductance and rotor resistance of the induction motors (IM) based on the extended Kalman-filter (EKF) in a real time emulator. So far, the performance of estimation for EKF in real time emulator has not been researched in case of motor parameters variation. Furthermore, inaccurate state space model of IM are applied within EKF in the published literature. This paper proposed the on-line estimation of the IM parameters with helping of the modified state space model minimizing the dependency of the state matrix indexes to state vector. To analyze the process of algorithm in F28377D emulator, the induction machine is modeled and solved by applying the numerical solutions. The analytically analyzation of the EKF algorithm and the real time emulator results are presented in this paper.

Keywords— On-line estimation, extended Kalman-filter, induction motors parameters, induction motor derives

I. INTRODUCTION

In induction motor derives based on the field-oriented control FOC, the rotor flux \( \lambda_r \) of the IM needs to be aligned with d-axis of synchronize frame. This is achieved by applying the correct slip angle \( \theta_{sl} \). The accuracy of the calculation for \( \theta_{sl} \) depends on the applied magnetizing inductance \( L_m \) and rotor resistance \( r_r \) [1]. The accurate slip calculation helps the FOC to have the best performance in controlling the air gap flux. The inaccurate \( r_r \) and \( L_m \) causes the non-zero flux in q axis \( \lambda_{qr} \) which reduce the output torque of the IM. Therefore, the appropriate estimation algorithm that could apply the correct \( L_m \) and \( r_r \) within the slip calculation mechanism, is the trend subject for tractions applications [2]. The EKF can be selected as one of accurate method for motor parameters estimation as the result of considering the effect of measured and process noises [3].

To apply the actual motor parameters in each interrupt service routine of microcontroller, the estimation of parameter is required [4]. These parameters are affected by variation of motor temperature, fundamental frequency and desired d-q axis currents [5]. The modification in desired current causes the variation in magnetizing current which is responsible for inductance saturation. Therefore, the motor parameters needs to be determined by using the data sheet of the machine, online estimation or offline determination techniques and the high frequency signal injection methods. The online estimation technique is considered as the more effective solution in parameter extraction methods because the motor parameters becomes update as the drive is operating. It should be noticed that the accuracy in online estimation and offline determination depends on the sample rate and the resolution of measurement devices [6, 7].

The online estimation of \( r_r \) based on the EKF has been presented in [4, 8]. However, the deviation of \( L_m \) has not been considered. So, the variation of \( L_m \) is reflected on estimated \( r_r \) which causes the inaccurate estimation. This problem tried to be solved with applying the offline determined \( L_m \) in EKF used to estimator \( r_r \) [6]. Although this improves the performance of the estimation algorithm, however the perfect estimation is not guaranteed. In [9], the adjusted EKF loops are achieved based on the quadratic programming techniques to constraint the estimated parameters in case of high transient variation in estimation process. The other approach of parameter estimation is unscented Kalman filter (UKF) where the minimal selected samples of non-linear system is used [10]. So, the single point linearization of the non-linear system with helping of Jacobean matrix is not required and the continues non-linear dynamic equations are directly utilized in the algorithm. This causes that the accurate mean and covariance are achieved in UKF by analyzing the Guassian random variables which are close to second order of Taylor series. However, as described in [11], the accuracy of the estimated parameters and processing time is not significantly improved by applying UKF as the alternative of EKF. In [11], the initialization of covariance matrixes are improved at cost of long computation process. This leads to add the computation time to conventional EKF which already has long
processing time. As the alternative approach of designing the covariance matrixes, the process and measurements noise covariance matrixes are defined by helping of filtering methods [12]. In this method, the average of three captured voltage and current signals is defined as the main signal where the difference with captured signals is considered as the noise. This noise is tried to be filtered by the low pass filter which is not the suitable option for the white Gaussian noise included of all low and high frequencies.

In [7], the parameters added to the state space model of IM as the new states are not defined accurately. This means that some indexes in the Jacobian matrix are not constant when the new states (estimated parameters) are changing. However, those indexes are considered as the fixed value in [7]. In this paper, the new states of $\frac{1}{\tau_m}$ and $\frac{L_r^2}{L_r}$ are considered as the estimated parameters. This selection of parameters not only helps to estimate $L_m$ and $r_c$ but also keep some index variables constant in Jacobian matrix irrespective to variations in estimated parameters. Based on the defined state space matrix of the IM, the performance of the algorithm on the real time experimenter board F28377D has been analyzed. This is achieved by modeling the IM in CPU2 of the microcontroller and solving the differential equations of the model based on the numerical solutions. The paper is organized as follow. The mathematical model of IM based on the state space is developed in section II. Section III describes the implementation of EKF and IM model in F28377D. Section IV presents the real time result of the applied algorithm on the IM modeled in CPU2. Finally, the work is concluded in section V.

II. MATHEMATICAL MODEL DEVELOPMENT

A. State Space Model of Induction Machine

In stationary frame, the state space model of IM can be described as:

$$\frac{d}{dt} \begin{bmatrix} i_n^e \\ i_c^e \\ \lambda_n^e \\ \lambda_c^e \end{bmatrix} = \begin{bmatrix} a & 0 & c & b \\ 0 & a & -b & c \\ \frac{L_c}{\tau_c} & 0 & -1 & -w_r \\ 0 & \frac{L_m}{\tau_r} & \frac{1}{\tau_r} & w_r \end{bmatrix} \begin{bmatrix} i_n^e \\ i_c^e \\ \lambda_n^e \\ \lambda_c^e \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma L} & 0 \\ \frac{1}{\sigma L} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_n^e \\ V_c^e \end{bmatrix}$$

(1)

In (1), $i_{dq}$, $\lambda_{dq}$ and $V_{dq}$ are description of stator current, rotor flux and stator voltage in the d-q axis of the stationary frame respectively. As described in (2), the stator currents are considered as the output signal which can be measured by the current sensors. The discrete time varying model of the IM in sampling interval of $t_s$ and discrete sequence of $k$ can be described as:

$$x(k+1) = F(k)x(k) + G(k)u(k)$$
$$y(k) = Hz(k)$$

The process noise $w(k)$ and measurement noise $v(k)$ with constant scaler variance of $Q$ and $R$ needs to be added to the discrete model. This helps to make the IM model more close to practical nature. This stochastic state space model is solved by applying the Gaussian distribution where the state error covariance matrix $P(k)$ and the output error covariance matrix $S(k)$ needs to be calculated. The answers of Gaussian distribution is defined based on the distribution mean vectors of state and output $(\hat{x}, \hat{y})$ depended on the probability of the below distribution:

$$N(\hat{x}(k), P(k))$$
$$N(\hat{y}(k), S(k))$$

B. Kalman-Filter

The estimated parameters in Kalman filter algorithm are defined by $(\hat{x}, \hat{y})$. So, the state prediction of Kalman-filter algorithm can be calculated by:

$$\hat{x}(k+1/1) = F(k)\hat{x}(k/k) + G(k)u(k)$$
$$P(k+1/1) = F(k)P(k/k)F(k)^T + Q$$

and the state correction equations are:

$$K(k+1) = P(k+1/k)H(k+1)[H(k)P(k+1/k)H(k+1)^T + R]^{-1}$$
$$\hat{x}(k+1/k+1) = \hat{x}(k+1/k) + K(k+1)[y(k+1) - H(k+1)\hat{x}(k+1/k)]$$
$$P(k+1/k+1) = P(k+1/k) - K(k+1)H(k+1)P(k+1/k)$$

(9)

(10)

As shown in (9), the Kalman gain is defined by $k(k)$. It should be noticed that $\hat{x}(k/k)$ and $\hat{x}(1/k+1)$ are the estimation of the state in discrete sequence of $k$ which is based on data available up to and including $k$ and $k + 1$ sequence respectively. It should be noticed that the state estimation in state correction equations are based on general SISO linear difference equations.

$$y(k) + \sum_{i=1}^{n} a_i y(k-i) = \sum_{i=1}^{n} b_i u(k-i)$$

(12)

As shown in (12), the model parameters, outputs and inputs vectors are defined by $\theta^T$, $y(k)$ and $\theta(k)$ respectevtely. By
assuming that the output parameters and input variables can be measured, then the model parameters can be estimated by:

\[ \hat{\theta}(k) = \hat{\theta}(k-1) + L(k)[y(k) - \theta'(k-1)\mathcal{O}(k)] \]  

(13)

The gain matrix which is computed in each iteration is defined by \( L(k) \). So this method of the system parameters identification is used in Kalman-filter by reformulate the parameters as states to estimate the unmeasurable states of Induction Machine. As explained before, the measurement noise covariance matrix is considered as result of uncertainty of stator current measurement. Selection of aforementioned value for \( R \) matrix in (9) is the result of the measurement error. To find the accurate value of \( Q \) matrix in practical experiment is based on trial and error and check the captured data.

C. Extended Kalman-filter (EKF)

The parameters, that needs to be estimated, are added to the state vector in (1). In order to estimate the value of the \( r_r \) and \( L_m \), the arguments of \( \frac{1}{L_m} \) and \( \frac{L_m}{L_r}r_r \) are added to the state vector in this paper. The reason of adding these time varying states is that the all indexes in Jacobian matrix can be defined by these parameters and the constant variables such as \( \sigma L_z, \frac{1}{\sigma L_m} \) and \( \frac{L_m}{\sigma L_{slr}} \), when \( r_r \) and \( L_m \) varying. The new state space model is the non-linear model because of existing the multiplication of new states in the state matrix. The non-linear state space model is solved by the EKF algorithm as the non-linear state estimator. The new state vector should be defined as:

\[ \theta(k+1) = \theta(k) + n(k) \]  

(14)

\( n(k) \) is defined as the random disturbance in (16). Then, the extended state space model can be derived as follow:

\[ x(k+1) = F(\theta(k),k)x(k) + G(\theta(k),k)u(k) + w(k) \]  

(15)

By considering the parameters as the added state, the modified state vector can be expressed as:

\[ z(k) = \begin{bmatrix} x(k) \\ \theta(k) \end{bmatrix} \]  

(16)

Therefore, the augmented state model is presented as:

\[ z(k+1) = f(z(k),u(k)) + w'(k) \]  

(17)

where:

\[ f(z(k),u(k)) = \begin{bmatrix} F(\theta(k),k)x(k) + G(\theta(k),k)u(k) \\ \theta(k) \end{bmatrix} \]  

\[ w'(k) = \begin{bmatrix} w(k) \\ n(k) \end{bmatrix} \]  

(18)

The output vector, which is not depend on \( \theta(k) \) can be explained as follow:

\[ y(k) = Hz(k) + v(k) \]  

(19)

In KF and EKF, the mean value of the white Gaussian noise is assumed to be zero. So, the covariance of the measurement and process noises are as follow:

\[ E\left\{v(k)v(j)^\top\right\} = R\delta_{kj} \]  

(20)

\[ E\left\{ww'\right\} = \begin{bmatrix} Q_{ww} & 0 \\ 0 & Q_{ww} \end{bmatrix} \delta_{kj} \]  

(21)

The covariance of natural state uncertainty and covariance of parameters disturbance vector are symbolled by \( Q_{ww} \) and \( Q_{mm} \). As the result of no correlation between the natural and parameters states, \( Q_{nw} = Q_{wm} = 0 \). The initial state error covariance matrix can be defined as follow:

\[ P(0) = \begin{bmatrix} P_{ww} & P_{wm} \\ P_{mw} & P_{mm} \end{bmatrix} \]  

(22)

The quality of the prior information of the states, \( P_{ww} \) and \( P_{mm} \) needs to be defined correctly.

D. State predication and state correction in EKF

To solve the EKF estimating the states and parameters of IM, the state prediction and state correction equations need to be implemented. The state prediction equation of EKF can be calculated by:

\[ f(z(k),u(k)) = \begin{bmatrix} F(\theta(k),k)x(k) + G(\theta(k),k)u(k) \\ \theta(k) \end{bmatrix} \]  

(23)

\[ \begin{bmatrix} i_{ds}^e(k+1) \\ i_{qs}^e(k+1) \\ \lambda_{ds}^e(k+1) \\ \lambda_{qs}^e(k+1) \\ \frac{1}{L_m} \end{bmatrix} + \begin{bmatrix} A & 0 & B & 0 & 0 \\ 0 & A & -C & B & 0 \\ D & 0 & E & G & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{ds}^e(k) \\ i_{qs}^e(k) \\ \lambda_{ds}^e(k) \\ \lambda_{qs}^e(k) \\ \frac{1}{L_m} \end{bmatrix} \]  

\[ + \frac{\tau_s}{\sigma L_s} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]  

The variables used in (23) are defined as follow:
\[ A = 1 + t_s \left( \frac{-r_s}{\sigma L_s} + \frac{1 + \sigma}{\sigma L_m} z(6)z(5) \right) \]
\[ B = t_s \frac{L_m}{\sigma L_s L_r} (z(5)^2 z(6)) \]
\[ C = t_s \frac{L_m}{\sigma L_s L_r} w_r \]
\[ D = t_s (z(6)z(5)) \]
\[ E = 1 + t_s (-z(5)^2 z(6)) \]
\[ G = -t_s w_r \]

In above equation, the \( \frac{1}{L_m} \) and \( \frac{L_m^2}{L_r} r_r \) are considered as fifth and sixth states for state space model of Induction machine. \( w_r \) is the electrical speed which depends on the mechanical speed and number of motor poles. The state error covariance vector in state prediction and state prediction equations can be defined based on the partial derivation or Jacobian matrix as follow:

\[
F(k) = \frac{\partial f(\cdot)}{\partial z(k)} \]

\[
\rightarrow F(k) = \begin{bmatrix}
A & 0 & B & C & H & M \\
0 & A & -C & B & I & N \\
D & 0 & E & G & J & O \\
0 & D & -G & E & T & S \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

The defined variables in (24) are described as follow:

\[
H = t_s z(1) \left( \frac{-1 + \sigma}{\sigma L_s} z(6) + t_s z(3) t_s \frac{L_s}{\sigma L_s} (2 \times z(5)z(6)) \right) \\
I = t_s z(2) \left( \frac{-1 + \sigma}{\sigma L_s} z(6) + t_s z(4) t_s \frac{L_s}{\sigma L_s} (2 \times z(5)z(6)) \right) \\
J = t_s (z(6)z(1)) - t_s z(3) (2 \times z(5)z(6)) \\
T = t_s (z(6)z(2)) - t_s (z(4) (2 \times z(5)z(6)) \\
M = t_s z(1) \left( \frac{-1 + \sigma}{\sigma L_s} z(5) + t_s z(3) t_s \frac{L_s}{\sigma L_s} (z(5)z(5)) \right) \\
N = t_s z(2) \left( \frac{-1 + \sigma}{\sigma L_s} z(5) + t_s z(4) t_s \frac{L_s}{\sigma L_s} (z(5)z(5)) \right) \\
O = t_s (z(5)z(1)) - t_s z(3) (z(5)z(5)) \\
S = t_s (z(5)z(2)) - t_s (z(4) (z(5)z(5)) \\
\]

Substituting (24) in (9) to (12) helps to estimates the states and parameters in EKF. The initial state matrix has been considered as:

\[
\begin{bmatrix}
\hat{i}_d^s \\
\hat{i}_q^s \\
\hat{\lambda}_d^s \\
\hat{\lambda}_q^s \\
\frac{1}{L_m} \frac{L_m}{L_r} \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
3.6914 \\
0.6473 \\
\end{bmatrix}
\]

Also, the measurement and process noise covariance has been defined as follow:

\[
R = \begin{bmatrix}
0.1 & 0 \\
0.1 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}, \quad Q = \begin{bmatrix}
10 & 0 & 0 & 0 & 0 \\
0 & 10 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

### III. IMPLEMENTATION of EKF and IM MODEL in F28377D

As can be seen in (23), the stator voltage needs to be applied as the input signals. As shown in Fig. 1, the input voltage of the Space vector modulation SVM with the coefficient of \( \frac{2V_{dc}}{3} \) is considered to produce \( V_{d^s}, V_{q^s} \). The reason of considering this coefficient is that the compensation terms such as coupling and back EMF has not been added to the output of the PI controllers. So, the output of the current controllers are P.U. values and those can be modified in terms of the DC link voltage by having this coefficient. The other inputs of the EKF are \( i_{d^s}, i_{q^s} \) and \( w_r \). These signals are created in real time machine model which is consisted of five states including d-q axis of stator-rotor currents in stationary frame and speed created from torque model. As shown in Fig. 2, the three phase voltage of the machine is created based on the calculated modulation indexes \( m^f[0], m^f[1] \) and \( m^f[3] \) in Cpu1.Cla. The differential equations of IM model in real time processor is solved with Runge-Kutta (RK4) analysis which solves state space IM model with using the numerical solution method.

![Fig. 1. Imputes and outputs of proposed augmented states for EXF](image-url)
IV. REAL TIME EMULATION RESULTS

To validate the performance of the EKF, the PWM cycle period is selected at 10 KHz. This provides 100 microsecond (μs) interrupt which is suitable to run the EKF algorithm in cpu2. It should be noticed that the maximum modulation index calculated by SVM is 5000 for 10 KHz sampling frequency in F28377D with specified clock frequency. So, the input voltage of space vector modulation need to multiplied by the scale factor of $\frac{2}{3V_{dc}}$. In this test the mechanical speed of the machine is kept to 100 rad/sec. To keep the running time of solving the RK4 and EKF in Cpu2 within 100 us, the RK4 is solved for one time. To improve the accuracy of the numerical solver (RK4), two times of solving need to be considered. However, as the running time for EKF is already long, so it has been used for one time in this test. As sown in Fig. 3, the running time of EKF and RK4 in Cpu2 is 79us which is in range of 100us for 10KHz switching frequency.

To test the performance of the EKF, the machine model in cpu2 has been changed based on the half of the magnetizing inductance in especial time. It should be noticed to keep the field oriented algorithm, the slip speed needs to redefined based on the new magnetizing inductance. Otherwise, the value of produced torque goes unexpected as the d-axis of the rotor flux does not sit on the d-axis of synchronous frame.

Fig. 2. Implementation of extended Kalman-filter and induction motor emulator in F28377D

Then, the q-axis of rotor flux is not zero and it cause incorrect results in torque calculation. As shown in Fig. 4 and 5, the data are captured for 1000 samples. As each samples takes 100 us, so the timing period for captured data is 100 millisecond. As shown in Fig.4 a and b, the stator currents in stationary frame has 90 degree phase shift and the magnitude of rotor flux in Fig 4. c and d, are based on the applied reference currents $i_{d}^{e} = 2.3A$, $i_{q}^{e} = 3.98A$ and the specification of test IM in Table. 1 described in Appendix. The estimated parameters are also captured in Fig. 4 d which validate the performance of algorithm with proposed state space model.

Fig. 3. Running time of RK4 and EKF implemented in Cpu2 of F28377D
In experimental board, the induction machine models is changed based on the new magnetizing inductance which is half of the initial value. The captured transient results are shown in Fig. 5. As demonstrated in Fig. 5a, b, the d-q axis of stator currents are not changing as the result of following the desired stator currents. The d-q axis of rotor flux (Fig. 5c) start to decrease as the result of reduction in magnetizing inductance. The steady state results of rotor flux also are shown in Fig. 5d. As shown in Fig. 5e and f, the estimated parameters ($\frac{1}{L_m}$ and $\frac{L_m^2}{L_r}$) are close to expected values which validate the calculated results, real time implementation of EKF algorithm and IM model.

Fig. 4. Results of Estimated states in real-time emulator F28377D (a) d-axis stator current (b) q-axis stator current (c) d-q axis rotor flux (d) q-axis (d) ($\frac{1}{L_m}$ and $\frac{L_m^2}{L_r}$).
In this paper, the extended Kalman-filter based on the novel non-linear states space model of induction machine, included essential motor parameters, is implemented in real time emulator. To solve the proposed algorithm in F28377d experimental board, the induction machine is modeled and numerically solved. The performance of the estimation algorithm has been analyzed and validated in real time emulator where the magnetizing inductance modified to half of the initial value. The results confirm the reasonable degree of the accuracy for estimated states and parameters of IM. Furthermore, the results of estimation algorithm proves the feasibility of proposed algorithm in real time environments.

VI. APPENDIX

The specification of 4 Poles Induction machine is as follow:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_s)</td>
<td>2.291 Ω</td>
</tr>
<tr>
<td>(r_r)</td>
<td>2.5067 Ω</td>
</tr>
<tr>
<td>(V_{DC})</td>
<td>350 V</td>
</tr>
<tr>
<td>Current (peak)</td>
<td>4.6 A</td>
</tr>
<tr>
<td>(L_S)</td>
<td>0.2842 H</td>
</tr>
<tr>
<td>(L_m)</td>
<td>0.2709 H</td>
</tr>
<tr>
<td>(L_r)</td>
<td>0.2842 H</td>
</tr>
<tr>
<td>Base Speed</td>
<td>1500 rpm</td>
</tr>
</tbody>
</table>

Reference


