Supporting Online Material for
The evolution of giving, sharing, and lotteries

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1 Allocations and Payoffs

As stated in the main paper, the primary fitness payoff to an individual is a function of the fraction of the resource consumed, v, and the returns to consumption, x. This payoff is given by:

$$\pi(v) = v^x$$

An individual also benefits secondarily from any increase in his partner's fitness payoff, to an extent governed by the degree of interdependence, s. Given that the partner consumes whatever the focal individual does not $(1 - v)$, the total payoff for the focal player of any given allocation is:

$$\pi_{focal}(v) = v^x + s(1 - v)^x \quad (1)$$

Since the partner obtains a fraction $1 - v$ of the resource, the partner's payoff for a given allocation to the focal individual is:

$$\pi_{partner}(v) = (1 - v)^x + sv^x \quad (2)$$

When should the focal player prefer to give an extra unit of the resource to the partner rather than keeping it for himself? This means asking when an allocation of i units to the focal gives a better fitness payoff than an allocation of j units, where $i < j$. This will be the case when:

$$i^x + s(1-i)^x > j^x + s(1-j)^x$$

Rearranging:

$$s[(1-i)^x - (1-j)^x] > j^x - i^x \tag{3}$$

Since the benefit to the partner of an extra unit of resource is the increase in the partner's personal payoff $(1-i)^x - (1-j)^x$, and the cost to the focal is the decrease in his personal payoff $j^x - i^x$, we can rewrite Inequality (3) as $sb > c$. This inequality states the general condition which must be satisfied for the focal to be selected to transfer a unit of resource to the partner if no other costs are present. It is intuitive, since sb represents the focal's secondary payoff from a payoff of b to the partner, and thus the inequality amounts to the requirement that the focal's secondary gain must exceed his primary loss if he is to benefit from transferring a unit of resource to the partner.

The fitness payoffs for the focal and the partner (from (1) and (2)) under different allocations of the resource, and different parameter settings, are plotted in figure 1 of the main paper. When returns are diminishing ($x < 1$) and the two players have a stake in one another ($s > 0$; the subplot in the top row, second column, and the subplot in the top row, third column) a player's payoff reaches a maximum when he allocates less than all of the resource to himself ($0 < v < 1$).

To find the allocation which maximizes a player's payoff when returns are diminishing ($x < 1$), we differentiate Equation (1) with respect to the fraction allocated to him, which gives us:

$$\frac{d\pi}{dv} = xv^{x-1} - sx(1-v)^{x-1}$$

or:

$$\frac{d\pi}{dv} = x\left(v^{x-1} - s(1-v)^{x-1}\right) \tag{4}$$

Equation (4) equals zero either when $x = 0$ or when

$$v^{x-1} - s(1-v)^{x-1} = 0 \tag{5}$$

Equation (5) can be rewritten as:

$$s = \left(\frac{v}{1-v}\right)^{x-1}$$

Solving for v:

$$s^{\frac{1}{x-1}} = \frac{v}{1-v}$$

Resulting in:

$$\hat{v} = \frac{s^{\frac{1}{x-1}}}{1 + s^{\frac{1}{x-1}}} \tag{6}$$

Equation (6) is plotted in Supplementary Figure 1. $\hat{v}$ represents the share of the resource that a player would optimally allocate to himself if he can completely and costlessly control the allocation. When returns are linear or accelerating ($x \geq 1$) or there is no interdependence ($s = 0$) a player prefers all of the resource for himself ($\hat{v} = 1$). When returns are diminishing ($x < 1$) and the two players have a stake in one another ($s > 0$) a player prefers less than all of the resource ($\hat{v} < 1$).

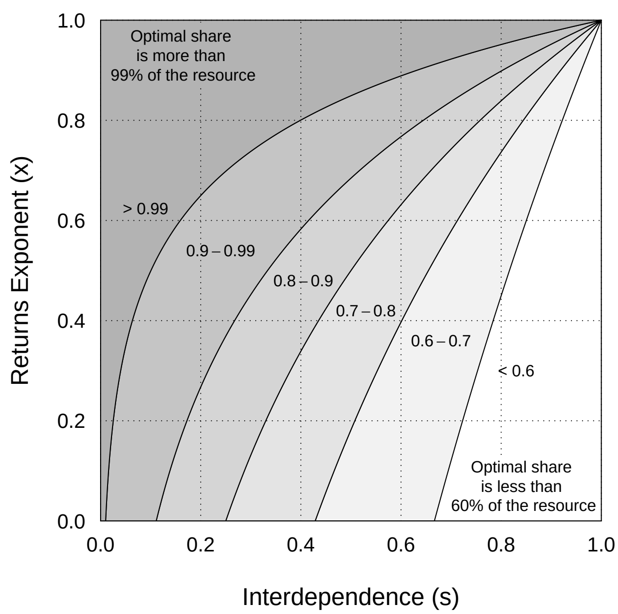

Figure 1: Optimal share sought as a function of returns and interdependence. Numbers within the shaded regions depict the range of fractions of the resource a player would prefer to allocate to himself, $\hat{v}$.

With diminishing returns and interdependence, we can use the "optimal" allocation, given by Equation (6), to compute the payoff a player would gain if he controls the allocation, and the payoff he would gain if he concedes control of the allocation to his partner. These payoffs are respectively given by:

$$\pi(\hat{v}) = \hat{v}^x + s(1-\hat{v})^x \tag{7}$$

and:

$$\pi(1-\hat{v}) = (1-\hat{v})^x + s\hat{v}^x \tag{8}$$

The difference between (7) and (8), which we label $B_{own-cede}$, is given by:

$$B_{own-cede} = (1-s)\hat{v}^x + (s-1)(1-\hat{v})^x$$

This can be rewritten as:

$$B_{own-cede} = (1-s)\Big(\hat{v}^x - (1-\hat{v})^x\Big) \tag{9}$$

$B_{own-cede}$ represents the net benefit from controlling the resource over ceding control to the other party (i.e., the payoff difference to a player between controlling the resource completely or letting his partner control it). Equation (9) is plotted in figure 2 of the main paper.

From Equation (9) we can see:

- The benefit of controlling the allocation decreases as interdependence increases.
- The benefit of controlling the allocation increases as returns to consumption increase.

2 Costs

We consider two types of cost: an ownership cost, o and a conflict cost, c. The ownership cost represents the cost of staking a claim to the allocation of the resource, and monitoring whether this claim is being respected. The conflict cost is contingent on the other player's behaviour; if the other player also attempts ownership, then a conflict erupts and it is costly for both players to settle it. Note here the similarities and differences with Maynard Smith's (1982) *HAWK–DOVE* model. In that model, there is no cost of making an ownership claim (no equivalent of our o). There is a cost of conflict c, but this is only paid by the loser of the conflict, whereas our conflict cost is paid by both parties. This latter difference is unimportant. The consequential difference between our model and the *HAWK–DOVE* model as regards costs is the introduction of o, and this model reduces to the *HAWK–DOVE* model in the case where $s = 0$, $x = 1$, and $o = 0$.

If the focal player pays the ownership cost o and his partner does not, the focal player controls the allocation and takes $\hat{v}$ for himself, leaving $(1 - \hat{v})$ for the other. If neither player pays the ownership cost, both players begin to consume the resource, and we assume that each player will, on average, consume half of the resource. If both players pay the ownership cost, there is a conflict, which imposes a further cost c on both players. The conflict is decided in favour of one player or the other with equal probability. Note that when costs are paid, they affect both the payoff of the player paying them, and the payoff of the other player, scaled by s.

These costs should be thought of as fractions of the maximum value of the resource. If a player consumes all of the resource, the payoff is 1, regardless of the returns on consumption. Thus, a value of $o = 0.1$ and $c = 0.2$ implies that the cost of claiming ownership of the resource is 10% of the value of the resource and the cost of a conflict over the allocation is 20% of the value of the resource.

3 Strategies and Interaction Payoffs

We consider three behavioural strategies.

- *SHARE* does not attempt to control the resource allocation, and consequently never pays the ownership cost o. If the other player attempts to control the resource, an individual playing *SHARE* cedes the resource and consumes the remainder left him, $(1 - \hat{v})$. When two *SHAREs* meet, since neither claims ownership, they end up consuming half the resource each. The *SHARE* captures the empirically-observed relational model of *Communal Sharing*, in that *SHAREs* neither attempt to own the resource, nor control the other party's access it (Fiske, 1991).
- *DOMINATE* attempts to control the resource allocation, always paying the ownership cost o. If the other player does not attempt to control the resource, an individual playing *DOMINATE* consumes a fraction $\hat{v}$ of the resource, leaving $1 - \hat{v}$ to the other player. If the other player attempts to control the resource too, paying the ownership cost, a conflict erupts. In the event of a conflict, both players must expend the conflict cost, c. In half of these disputes, an individual playing *DOMINATE* succeeds in controlling the allocation, garnering a fraction $\hat{v}$ of the resource for himself; in the other half, he loses control of the resource and consumes only $1 - \hat{v}$. This strategy represents the attempt to exert private property rights.

- *LOTTERY* plays either the *SHARE* strategy or the *DOMINATE* strategy with equal likelihood, using some freely available cue to decide which (for example, when arriving first, plays DOMINATE, and arriving second, plays SHARE). The consequence of this convention is that two individuals playing *LOTTERY* avoid any disputes and never have to pay the conflict cost c.

The *DOMINATE* and *SHARE* strategies considered in this model are analagous to *HAWKS* and *DOVES* in Maynard Smith's (1982) canonical model of resource conflict, whilst the *LOTTERY* strategy is a homologue of the *BOURGEOIS* strategy, in that it uses an uncorrelated asymmetry as a convention to avoid disputes.

Supplementary Table 1 defines the payoffs for the row player for all possible interaction pairs.

Table 1: Interaction Payoffs

Strategy	*SHARE*	*DOMINATE*	*LOTTERY*
SHARE	$\pi_{S,S}$	$\pi_{S,D}$	$\pi_{S,L}$
DOMINATE	$\pi_{D,S}$	$\pi_{D,D}$	$\pi_{D,L}$
LOTTERY	$\pi_{L,S}$	$\pi_{L,D}$	$\pi_{L,L}$

We now derive each of the nine possible interaction payoffs listed in Supplementary Table 1.

- The payoff for playing *SHARE* against another *SHARE* is the payoff for consuming half the resource, namely:

$$\pi_{S,S} = \pi(0.5) = 0.5^x + 0.5s^x$$

  Rearranging:

$$\pi_{S,S} = \pi(0.5) = (1 + s)0.5^x$$

- When *SHARE* meets *DOMINATE*, the payoff is:

$$\pi_{S,D} = \pi(1 - \hat{v}) - so$$

- When *SHARE* meets *LOTTERY*, the *LOTTERY* attempts to control the resource half the time and otherwise doesn't attempt control, resulting in the payoff:

$$\pi_{S,L} = 0.5\pi_{S,D} + 0.5\pi_{S,S}$$

- When *DOMINATE* meets *SHARE*, the payoff is:

$$\pi_{D,S} = \pi(\hat{v}) - o$$

- When *DOMINATE* meets *DOMINATE*, there is always a dispute, with both players paying both the ownership cost, o, and the conflict cost, c. Each combatant will, on average, control the resource allocation half the time, resulting in the payoff:

$$\pi_{D,D} = 0.5\Big(\pi(\hat{v}) + \pi(1-\hat{v})\Big) - (1+s)(o+c)$$

  Substituting (7) and (8) into this expression, we have:

$$\pi_{D,D} = 0.5\Big(\hat{v}^x + s(1-\hat{v})^x + (1-\hat{v})^x + s\hat{v}^x\Big) - (1+s)(o+c)$$

  This simplifies to:

$$\pi_{D,D} = (1+s)\Big[0.5\Big(\hat{v}^x + (1-\hat{v})^x\Big) - (o+c)\Big]$$

- When *DOMINATE* meets *LOTTERY*, the *LOTTERY* attempts to control the resource half the time and cedes the control otherwise, resulting in the payoff:

$$\pi_{D,L} = 0.5\pi_{D,D} + 0.5\pi_{D,S}$$

- When *LOTTERY* meets *SHARE*, the *LOTTERY* attempts to control the resource half the time and otherwise doesn't attempt control, resulting in the payoff:

$$\pi_{L,S} = 0.5\pi_{D,S} + 0.5\pi_{S,S}$$

- When *LOTTERY* meets *DOMINATE*, the *LOTTERY* attempts to control the resource half the time and cedes the control otherwise, resulting in the payoff:

$$\pi_{L,D} = 0.5\pi_{D,D} + 0.5\pi_{S,D}$$

- When *LOTTERY* meets *LOTTERY*, we assume that the two individuals coordinate without conflict; during each turn, one player controls the resource and the other cedes control, resulting in the payoff:

$$\pi_{L,L} = 0.5\pi_{D,S} + 0.5\pi_{S,D}$$

4 Evolutionary Dynamics

We can now calculate the evolutionary dynamics following Maynard Smith (1982). For each parametric combination, we want to find the evolutionarily stable strategies (ESSs). There are eight such possibilities:

1. No strategy is an ESS, resulting in a three-way polymorphism.
2. There is a polymorphic ESS between *SHARE* and *DOMINATE*.
3. There is a polymorphic ESS between *SHARE* and *LOTTERY*.
4. There is a polymorphic ESS between *LOTTERY* and *DOMINATE*.
5. *SHARE* is the only ESS.
6. *DOMINATE* is the only ESS.
7. *LOTTERY* is the only ESS.
8. Both *SHARE* and *DOMINATE* are ESSs.
9. Both *SHARE* and *LOTTERY* are ESSs.
10. Both *DOMINATE* and *LOTTERY* are ESSs.
11. All three strategies are ESSs.

Using the interaction payoffs listed in previous section, we derive the following results:

If *LOTTERY* is an ESS, then neither *SHARE* nor *DOMINATE* are ESSs. To see why, we find the conditions when *LOTTERY* is an ESS. In order for *LOTTERY* to be an ESS, *LOTTERY* must be an ESS against both *SHARE* and *DOMINATE*. We first find when *LOTTERY* is an ESS against *SHARE*.

$$\begin{aligned} \pi_{L,L} &> \pi_{S,L} \\ 0.5\pi_{D,S} + 0.5\pi_{S,D} &> 0.5\pi_{S,D} + 0.5\pi_{S,S} \\ \pi_{D,S} &> \pi_{S,S} \end{aligned} \tag{10}$$

Next, we find when *LOTTERY* is an ESS against *DOMINATE*.

$$\begin{aligned} \pi_{L,L} &> \pi_{D,L} \\ 0.5\pi_{D,S} + 0.5\pi_{S,D} &> 0.5\pi_{D,D} + 0.5\pi_{D,S} \\ \pi_{S,D} &> \pi_{D,D} \end{aligned} \tag{11}$$

From these two results, we can see that *LOTTERY* is an ESS if *DOMINATE* can invade a population of *SHARE* (10) and *SHARE* can invade a population of *DOMINATE* (11). So, if *LOTTERY* is an ESS, then neither *SHARE* nor *DOMINATE* are ESSs, eliminating possibilities 9, 10, and 11 from the list above.

We can also eliminate possibility 1 (i.e., no strategy is an ESS). For there to be no ESS, each strategy can invade a population of comprised of one of the other two strategies. With three strategies, there are six such inequalities which must be simultaneously satisfied. Inequalities (10) and (11) are two of these six. However, when (10) and (11) are satisfied, *LOTTERY* is an ESS against both *SHARE* and *DOMINATE*.

***SHARE* and *LOTTERY* cannot be part of a polymorphic ESS.** For both of the strategies to be part of a polymorphic ESS, each would have to be able to invade a population of the other. For *SHARE* to invade a population of *LOTTERY* requires:

$$\begin{aligned} \pi_{S,L} &> \pi_{L,L} \\ 0.5\pi_{S,D} + 0.5\pi_{S,S} &> 0.5\pi_{D,S} + 0.5\pi_{S,D} \\ \pi_{S,S} &> \pi_{D,S} \end{aligned} \tag{12}$$

And for *LOTTERY* to invade a population of *SHARE* requires:

$$\begin{aligned} \pi_{L,S} &> \pi_{S,S} \\ 0.5\pi_{D,S} + 0.5\pi_{S,S} &> \pi_{S,S} \\ \pi_{D,S} &> \pi_{S,S} \end{aligned} \tag{13}$$

Inequalities (12) and (13) cannot be simultaneously satisfied, so *SHARE* and *LOTTERY* cannot exist in a polymorphic ESS, thereby eliminating possibility 3 from the list above.

***DOMINATE* and *LOTTERY* cannot be part of a polymorphic ESS.** For both of the strategies to be part of a polymorphic ESS, each would have to be able to invade a population of the other. For *DOMINATE* to invade a population of *LOTTERY* requires:

$$\begin{aligned} \pi_{D,L} &> \pi_{L,L} \\ 0.5\pi_{D,D} + 0.5\pi_{D,S} &> 0.5\pi_{D,S} + 0.5\pi_{S,D} \\ \pi_{D,D} &> \pi_{S,D} \end{aligned} \tag{14}$$

And for *LOTTERY* to invade a population of *DOMINATE* requires:

$$\begin{aligned} \pi_{L,D} &> \pi_{D,D} \\ 0.5\pi_{D,D} + 0.5\pi_{S,D} &> \pi_{D,D} \\ \pi_{S,D} &> \pi_{D,D} \end{aligned} \tag{15}$$

Inequalities (14) and (15) cannot be simultaneously satisfied, so *DOMINATE* and *LOTTERY* cannot exist in a polymorphic ESS, thereby eliminating possibility 4 from the list above.

If *SHARE* is an ESS against *DOMINATE* ($\pi_{S,S} > \pi_{D,S}$), then *SHARE* is also an ESS against *LOTTERY* ($\pi_{S,S} > \pi_{T,S}$). This follows because *LOTTERY* alternates between *SHARE* and *DOMINATE* when playing *SHARE*. Thus, on half the interactions, a *LOTTERY* will match the payoff of a *SHARE*; on the other half, a *LOTTERY* will have a lower payoff.

If *DOMINATE* is an ESS against *SHARE* ($\pi_{D,D} > \pi_{S,D}$), then *DOMINATE* is also an ESS against *LOTTERY* ($\pi_{D,D} > \pi_{T,D}$). This follows because *LOTTERY* alternates between *SHARE* and *DOMINATE* when playing *DOMINATE*. Thus, on half the interactions, a *LOTTERY* will match the payoff of a *DOMINATE*; on the other half, a *LOTTERY* will have a lower payoff.

The preceding analyses pare down the list of possible evolutionary outcomes to:

- There is a polymorphic ESS between *SHARE* and *DOMINATE*.
- *SHARE* is the only ESS.
- *DOMINATE* is the only ESS.
- *LOTTERY* is the only ESS.
- Both *SHARE* and *DOMINATE* are ESSs.

Before we find the conditions for these evolutionary outcomes, we examine the *SHARE–DOMINATE* polymorphic ESS more closely.

What is the distribution of *SHARE* and *DOMINATE* at the polymorphic ESS? Let $\hat{p}$ be the fraction of *DOMINATE* at the polymorphic ESS. At this polymorphic ESS, the payoff of *SHARE* and *DOMINATE* will be the same, an individual will interact with a partner playing the *DOMINATE* with probability $\hat{p}$, and interact with a partner playing the *SHARE* with probability $1 - \hat{p}$:

$$\hat{p}\pi_{D,D} + (1 - \hat{p})\pi_{D,S} = \hat{p}\pi_{S,D} + (1 - \hat{p})\pi_{S,S}$$

Solving for $\hat{p}$:

$$\hat{p} = \frac{\pi_{S,S} - \pi_{D,S}}{\pi_{D,D} - \pi_{S,D} - \pi_{D,S} + \pi_{S,S}} \tag{16}$$

Can *LOTTERY* invade this polymorphic ESS? Inequalities (10) and (11) show us that *LOTTERY* is an ESS whenever *SHARE* can invade *DOMINATE* and vice versa (i.e., when there is a polymorphic ESS between the two strategies). Suppose that the population is at the *SHARE–DOMINATE* polymorphic ESS. We can ask whether *LOTTERY* can invade. For this to happen, the payoff of a mutant *LOTTERY* must be higher than the payoff of the residents, comprised of a mix of *SHARE* and *DOMINATE*. At the polymorphic equilibrium, the payoff of *SHARE* and *DOMINATE* will be same, so we can compare the payoff of a mutant *LOTTERY* with the payoff of either the *SHARE* or *DOMINATE* strategies. Here, we compare the payoff of a mutant *LOTTERY* against a *SHARE*:

$$\hat{p}\pi_{L,D} + (1 - \hat{p})\pi_{L,S} > \hat{p}\pi_{S,D} + (1 - \hat{p})\pi_{S,S}$$

Solving for $\hat{p}$:

$$\hat{p} > \frac{\pi_{S,S} - \pi_{D,S}}{\pi_{D,D} - \pi_{S,D} - \pi_{D,S} + \pi_{S,S}} \tag{17}$$

From Equation (16), we see that the right-hand side of Inequality (17) is equal to $\hat{p}$. Making this substitution, Inequality (17) becomes $\hat{p} > \hat{p}$, a condition which cannot be satisfied; *LOTTERY* cannot invade a population of *SHARE* and *DOMINATE*.

In fact, the payoff of a *LOTTERY* is the same as the payoff of residents of a *SHARE–DOMINATE* equilibrium. The same situation occurs with the *BOURGEOIS* strategy against a population of *HAWKS* and *DOVES* (Maynard Smith, 1982). In order to transition from the *SHARE–DOMINATE* polymorphic ESS to the *LOTTERY* ESS, some kind of assortment is required, like kin-biased interaction, which increases the probability of mutant *LOTTERIES* interacting with one another above chance. With such assortment, selection will result in the *LOTTERY* ESS.

To prove this, we introduce a new model parameter, r, meant to represent non-random assortment, which could be generated through kin-biased interactions, for example. Again, we let $\hat{p}$ represent the frequency of *DOMINATE* at the polymorphic equilibrium between *DOMINATE* and *SHARE*. When considering rare mutants playing *LOTTERY*, the frequency of *LOTTERY* is approximately zero and so the frequency of SHARE will be approximately $1 - \hat{p}$.

As the overall frequency of *LOTTERY* is close to zero, the average payoff of *DOMINATE* and *SHARE* will be dominated by interactions with others playing *DOMINATE* and *SHARE*. As such, we can assign the probability of either a *DOMINATE* or *SHARE* interacting with a *LOTTERY* to be approximately zero. Additionally, we can assign the probability of *LOTTERY* interacting with another *LOTTERY* above and beyond r, the non-random assortment parameter, to be approximately zero.

With these assumptions, we can define the probabilities of forming different types of pairs. We denote these probabilities with $Pr(i|j)$ which represents the probability of interacting with a partner playing strategy i given the focal individual plays strategy j.

$$\begin{aligned}
Pr(D|D) &= r + (1-r)\hat{p} \\
Pr(S|D) &= (1-r)(1-\hat{p}) \\
Pr(L|D) &\approx 0 \\
\\
Pr(D|S) &= (1-r)\hat{p} \\
Pr(S|S) &= r + (1-r)(1-\hat{p}) \\
Pr(L|S) &\approx 0 \\
\\
Pr(D|L) &= (1-r)\hat{p} \\
Pr(S|L) &= (1-r)(1-\hat{p}) \\
Pr(L|L) &\approx r
\end{aligned} \tag{18}$$

In order to derive the equilibrium distribution of *DOMINATE* and *SHARE*, we find when their expected payoffs are equal:

$$Pr(D|D)\pi_{D,D} + Pr(S|D)\pi_{D,S} = Pr(D|S)\pi_{S,D} + Pr(S|S)\pi_{S,S}$$

Substituting the interaction probabilities into (18), and solving for $\hat{p}$, we have:

$$\hat{p} = \frac{\pi_{S,S} - (1-r)\pi_{D,S} - r\pi_{D,D}}{(1-r)\Big(\pi_{D,D} - \pi_{D,S} - \pi_{S,D} + \pi_{S,S}\Big)} \tag{19}$$

Note, when $r = 0$, Equation (19) reduces to Equation (16).

Next, to determine whether *LOTTERY* can invade with non-random assortment, we find when the payoff of a mutant playing *LOTTERY* is higher than the payoff of a resident. (Note, at the polymorphic equilibrium, the payoff of *SHARE* and *DOMINATE* will be same, so we can compare the payoff of a mutant *LOTTERY* with the payoff of either the *SHARE* or *DOMINATE* strategies. Here, we compare the payoff of a mutant *LOTTERY* against a *SHARE*.)

$$Pr(D|L)\pi_{L,D} + Pr(S|L)\pi_{L,S} + Pr(L|L)\pi_{L,L} > Pr(D|S)\pi_{S,D} + Pr(S|S)\pi_{S,S}$$

Substituting in the interaction probabilities from (18), and solving for $\hat{p}$, we have:

$$\hat{p} > \frac{\pi_{S,S}(1+r) - \pi_{D,S} - r\pi_{S,D}}{(1-r)\Big(\pi_{D,D} - \pi_{D,S} - \pi_{S,D} + \pi_{S,S}\Big)} \tag{20}$$

Note, when $r = 0$, Inequality (20) reduces to Inequality (17).

If we now substitute the equilibrium fraction of *DOMINATE*, derived in Equation (19), for $\hat{p}$ in the left-hand side of Inequality (20), we have:

$$\frac{\pi_{S,S} - (1-r)\pi_{D,S} - r\pi_{D,D}}{(1-r)\Big(\pi_{D,D} - \pi_{D,S} - \pi_{S,D} + \pi_{S,S}\Big)} > \frac{\pi_{S,S}(1+r) - \pi_{D,S} - r\pi_{S,D}}{(1-r)\Big(\pi_{D,D} - \pi_{D,S} - \pi_{S,D} + \pi_{S,S}\Big)}$$

With some algebra and substitutions, this reduces to:

$$c > 0.5^x - 0.5\Big(\hat{v}^x - (1-\hat{v})^x\Big) \tag{21}$$

With linear returns ($x = 1$), a self-interested individual would prefer all of the resource for himself ($\hat{v} = 1$). Making these substitutions, condition (21) becomes $c > 0$. This means that, with *any* amount of assortment ($r > 0$), *LOTTERY* will invade a mix of *SHARE* and *DOMINATE* if there is *any* cost to resource conflict.

With accelerating returns ($x > 1$), a self-interested individual would again prefer all of the resource for himself ($\hat{v} = 1$). Substituting in these values, condition (21) becomes:

$$c > 0.5^x - 0.5$$

When returns accelerate ($x > 1$), the required cost of conflict is negative ($c < 0$). So, with accelerating returns, *LOTTERY* will always invade a mix of *SHARE* and *DOMINATE*, whatever the cost of conflict.

With diminishing returns ($x < 1$), the invasibility of *LOTTERY* is more complicated. Condition (21) is plotted below in Supplementary Figure 2, showing the minimum conflict cost (c) for *LOTTERY* to invade an equilibrium mix of *DOMINATE* and *SHARE* as a function of interdependence (s) and returns (x). As interdependence increases from zero to one, this minimum cost rapidly diminishes to zero. The threshold conflict cost reaches a maximum at returns intermediate between zero and one. So, when interdependence is near zero and the returns exponent is around 0.4, the minimum conflict cost reaches its maximum around 0.2 or 20% of the value of the resource if consumed completely by one person.

When the conflict cost is below the threshold value (i.e., Condition (21) is not satisfied), *LOTTERY* has the same payoff as residents of the mixed equilibrium; *LOTTERY* can only increase in frequency through drift, and when the frequency of *LOTTERY* is sufficiently high, selection will drive the population to the *LOTTERY* ESS. When the conflict cost is above the threshold, *LOTTERY* will invade and go to fixation when there is *any* non-random assortment ($r > 0$).

We now return to finding the conditions for the remaining evolutionary outcomes.

When is *DOMINATE* an ESS? As previously shown, when *DOMINATE* is an ESS over *SHARE*, *DOMINATE* is also an ESS over *LOTTERY*. The condition for *DOMINATE* to be an ESS over *SHARE* is given below.

$$\begin{aligned}
\pi_{D,D} &> \pi_{S,D} \\
0.5\Big(\pi(\hat{v}) + \pi(1-\hat{v})\Big) - (1+s)(o+c) &> \pi(1-\hat{v}) - so \\
0.5\Big(\pi(\hat{v}) - \pi(1-\hat{v})\Big) &> o + c(1+s) \\
0.5B_{own-cede} &> o + c(1+s) \qquad (22)
\end{aligned}$$

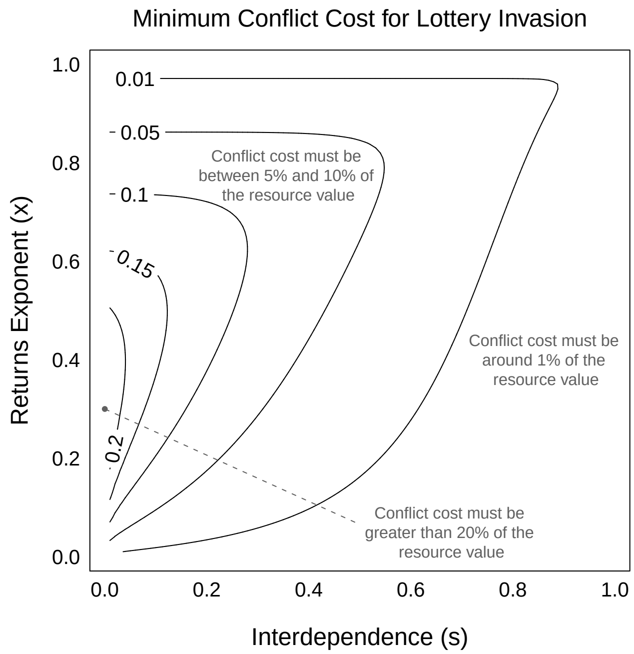

Figure 2: Minimum conflict cost for *LOTTERY* to invade an equilibrium mix of *DOMINATE* and *SHARE* as a function of interdependence (s) and returns exponent (x).

When is *SHARE* an ESS? As previously shown, when *SHARE* is an ESS over *DOMINATE*, *SHARE* is also an ESS over *LOTTERY*.

$$\begin{aligned} \pi_{S,S} &> \pi_{D,S} \\ \pi(0.5) &> \pi(\hat{v}) - o \\ \pi(\hat{v}) - \pi(0.5) &< o \end{aligned}$$

We re-label $\pi(\hat{v}) - \pi(0.5)$ as $B_{own-share}$, which represents the payoff difference between owning the resource (hence, controlling the allocation) and sharing it. (Note that this is not the same as $B_{own-cede}$, which is the payoff difference between owning the resource and the other player owning it). This results in:

$$B_{own-share} < o \tag{23}$$

That is, *SHARE* is an ESS when the difference between controlling the allocation of the resource and sharing it ($B_{own-share}$) is less than the cost of making an ownership claim, an intuitive result. Notice, the conflict cost doesn't enter into Inequality (23). In Maynard Smith's (1982) model, this kind of outcome (i.e., an evolutionarily stable population of *DOVES*) is not possible, since that model had no necessary cost of making an ownership claim. In the current model, *SHARE* does increasingly well as o becomes larger, and also as $B_{own-share}$ becomes smaller, which it does as interdependence increases and/or returns become more steeply diminishing.

When is there a polymorphic ESS between *SHARE* and *DOMINATE*? Or, when is *LOTTERY* an ESS? As previously shown, both of these outcomes happen under the same conditions.

In order for *SHARE* and *DOMINATE* to be a polymoprhic ESS, each strategy must be able to invade a population of the other.

There is a polymorphic ESS between *SHARE* and *DOMINATE* when *SHARE* can invade a population of *DOMINATE* and *DOMINATE* can invade a population *SHARE*. This situation occurs when neither Inequality (22) nor Inequality (23) are satisfied.

$$B_{own-share} > o > 0.5B_{own-cede} - c(1 + s) \tag{24}$$

When are both *SHARE* and *DOMINATE* ESSs? This occurs when both Inequalities (22) and (23) are satisfied.

$$0.5B_{own-cede} - c(1 + s) > o > B_{own-share} \tag{25}$$

This is an interesting case, which does not occur in Maynard Smith's *HAWK–DOVE* model. When condition (25) is satisfied, either *SHARE* or *DOMINATE* can be an ESS. The evolutionary outcome will be determined by path dependence; resource allocation can be based on domination or sharing. Note, even though either strategy can be an ESS, a population playing *SHARE* will always have higher average payoffs than a population playing *DOMINATE*. If there is any type of selection process which favors the equilibrium with the higher average payoff, allocations based on sharing should be more common than allocations based on domination.

Putting Conditions (22), (23), (24), and (25) together, Supplementary Table 2 shows when each of the four evolutionary outcomes result.

Table 2: Evolutionary Outcomes

	$0.5B_{own-cede} > o + c(1+s)$	$0.5B_{own-cede} < o + c(1+s)$
$B_{own-share} < o$	*DOMINATE* and *SHARE* ESS	*SHARE* ESS
$B_{own-share} > o$	*DOMINATE* ESS	*LOTTERY* ESS

The definitions of the parameters in Supplementary Table 2 are given below:

- $B_{own-cede} = \pi(\hat{v}) - \pi(1-\hat{v})$. $B_{own-cede}$ represents the difference in payoff between controlling the allocation of the resource $\pi(\hat{v})$ and ceding control of the allocation to the other player $\pi(1-\hat{v})$.
- $B_{own-share} = \pi(\hat{v}) - \pi(0.5)$. $B_{own-share}$ represents the difference in payoff between controlling the allocation of the resource $\pi(\hat{v})$ and sharing the resource qually with the other player $\pi(0.5)$.
- o represents the cost of claiming ownership of the resource.
- c represents the cost of a conflict, when both players claim ownership of the resource.
- s represents interdependence, the benefit that each player derives from having the other in the interaction environment.

The evolutionarily stable outcomes are plotted for a range of parameter values in figure 3 of the main paper.

References

Fiske, A.P. 1991. *Structures of Social Life: The Four Elementary Forms of Human Relations*. New York: The Free Press.

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