

Optimality and Natural Selection in Markets

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Received July 10, 1998; final version received September 5, 2000;
published online May 31, 2002

We ask if natural selection in markets favors profit-maximizing firms and, if so, is there a difference between the predictions of models which assume all firms are profit maximizers and the predictions of models which explicitly take account of population dynamics in the market. We show that *market selection* favors profit maximizing firms, but we also show that the long-run behavior of evolutionary market models is nonetheless not consistent with equilibrium models based on the profit-maximization hypothesis. Dynamic equilibrium paths with market selection are not Pareto optimal, nor even asymptotically optimal. The discrepancy arises because the dynamics created by firm evolution causes prices to vary over time and the resulting dynamical system need not have stable steady states. © 2002 Elsevier Science (USA)

Key Words: market selection hypothesis; profit maximization.

1. INTRODUCTION

The axiom that firms maximize profit is crucial to most of neoclassical equilibrium and welfare analysis. Various claims have been made defending the empirical validity of this axiom and its use in modelling the long-run behavior of markets. The most prominent defense of this axiom is the *market selection hypothesis*, which justifies the use of neoclassical equilibrium models with profit-maximizing firms to describe long-run market behavior by claiming that dynamic market forces continually weed out non-maximizing firms. This claim has two parts: First, that market forces select against firms that do not maximize profits; second, that the long-run behavior of this dynamic market process can be described by conventional market equilibrium. In this paper we will investigate both of these claims.

The first claim, that profit-maximizing firms drive firms with different decision rules from the market, is strongly identified with Milton Friedman [7, p. 22], who wrote:

¹ This research was supported by the National Science Foundation.

Whenever this determinant (of business behavior) happens to lead to behavior consistent with rational and informed maximization of returns, the business will prosper and acquire resources with which to expand; whenever it does not the business will tend to lose resources and can be kept in existence only by the addition of resources from the outside. The process of natural selection thus helps to validate the hypothesis (of profit maximization) or, rather, given natural selection, acceptance of the hypothesis can be based largely on the judgment that it summarizes appropriately the conditions for survival.

Alchian [1] made similar arguments, as did Enke [6] who wrote:² “In these instances the economist can make aggregate predictions *as if* each and every firm knew how to secure maximum long-run profits.” The intuition offered by Alchian, Enke and Friedman is that eventually capital reallocation will drive out firms that do not maximize profits.

The first claim comes in two varieties. One is that non-maximizing firms will make losses and be driven out of the market because they cannot continue to fund their operations out of retained earnings. This story is most closely associated with Winter [11, 12] and Nelson and Winter [9], who claimed that the retained earnings of profit maximizers will grow fastest and consequently that these firms will come to dominate the market. The second story is that non-maximizing firms will fail to attract sufficient investment capital to fund their operations. Here profitability is a signal which attracts investors’ funds. The dynamics implied by each of these stories is distinct, and so we will examine them both.

The market selection hypothesis has long had its critics. Nelson and Winter [9, p. 158] understood that the coevolution of firm behavior and the economic environment resulting from a complete model of the dynamic process could pose problems for the evolutionary defense of profit maximization. They observed that among the “... less obvious snags for evolutionary arguments that aim to provide a prop for orthodoxy ...” is “... that the relative profitability ranking of decision rules may not be invariant with respect to market conditions.” In other words, the first claim of the market selection hypothesis, that non-maximizing firms are weeded out, may fail.

Koopmans [8, p. 140] took issue with the second claim of the market selection hypothesis, that market equilibrium theory describes the long-run behavior of markets whose dynamic behavior is governed by the selection process. He characterized and criticized the argument as follows:

Here a postulate about individual behavior is made more plausible by reference to the adverse effect of, and hence penalty for, departures from the postulated behavior.... But if this is the basis for our belief in profit maximization, then we should postulate that basis itself and not the profit maximization which it implies in certain circumstances.

² The italics are present in the original.

Our analysis of the market selection hypothesis shows Koopmans to be correct in his concern about the connection between equilibrium analysis and long-run market behavior. We first construct a sequential-equilibrium, market-clearing model in which retained earnings determines the scales of firm operation. In its focus on retained earnings, this model is in the spirit of the Nelson-Winter analysis. In Sections 2 and 3 we lay out the basic model and show that under reasonable conditions, the claim that only profit-maximizing firms survive in the long-run is true for the retained earnings dynamic. However, this reassuring result does not imply that the long-run outcomes of the retained earnings dynamic can be described as competitive equilibria of economies with only profit maximizing firms. We examine this in Sections 4 and 5 and show that limit behavior of the retained earnings dynamic can fail to be Pareto optimal. Optimality can fail because the retained earnings dynamic need not allocate retained earnings correctly over firms using differing technologies. On the ω -limit set of the retained earnings dynamic, only profit maximizers have positive retained earnings. Further, at any steady state, retained earnings are apportioned across firms so that they operate at their competitive equilibrium levels. So steady states of the retained earnings dynamic are competitive equilibria and are Pareto optimal. But the retained earnings dynamic need not converge to these steady states. As retained earnings change over time, output levels change, and thus prices change, causing the relative profitability of firms to change over time. We provide examples showing that if prices are sensitive enough to output levels, then the limit behavior of the retained earnings dynamic can include cycles and even strange attractors. Along these non-converging paths, firms are not producing at their competitive output levels. Consumer efficiency fails, and we even provide an example in which producer efficiency fails, that is, the economy operates inside the production possibility frontier.

The retained earnings model imposes a drastic market failure—the non-existence of capital markets. In Section 6 we ask if the capital markets justification for profit maximization is correct. To do this we add a capital market to our model. If all investors have rational expectations, then the addition of the market makes the market structure dynamically complete. In a complete markets equilibrium, no investor invests in non-maximizing firms and so these firms never operate. This result could be interpreted as providing support for the market selection hypothesis, but we find it wanting. In such an equilibrium, no real selection occurs. The market gets it right from the outset.

In models with capital markets, the objects of selection are investors' decision rules rather than firms' decision rules. Firms rise and fall together with, and on account of, the particular investors who favor them. In order to ever have non-maximizing firms raise financial capital, and thus operate,

investors must have differing expectations about firms' profitability. But if expectations are heterogeneous, then markets are incomplete. In such an economy there can be initial investment in non-maximizing firms. The interesting question is, will this investment disappear, causing the non-maximizing firms to shut down? In a series of examples we show that the answer can be "no." Investment in non-maximizing firms will disappear only if those investors with incorrect expectations about firms' profitability disappear. We show that this need not happen. In fact, in incomplete markets economies, the wealth of irrational investors may grow, rather than decline to zero, and these investors may keep non-maximizing firms in existence by continually investing in them. We conclude that the capital markets version of the market selection hypothesis is even less successful than the retained earnings version.

2. THE MODEL

This section describes the model and the concept of intertemporal equilibrium without markets for financial capital. Time is discrete and is indexed by $t = 1, 2, \dots$. At each date, the economy has J commodities. Date t prices for these commodities are non-negative vectors $p_t \in \mathbf{R}_+^J$. The set of commodities $\{1, \dots, J\}$ is partitioned into two sets: inputs which are used in production, **Inp**, and outputs which are produced and consumed, **Con**. (The symbols **Inp** and **Con** will represent both the respective sets and their cardinality.) There are two types of infinitely lived consumers: workers and capitalists. Workers are indexed by $i = 1, \dots, I$ and have stationary endowments $e^i \in \mathbf{R}_+^{\text{Inp}}$ in each period. We assume that the aggregate endowment is strictly positive, $\sum_{i=1}^I e^i \gg 0$. Capitalists are indexed by $h = 1, \dots, H$ and own firms. The consumption set for both types is $C = \mathbf{R}_+^{\text{Con}}$. All consumers have perfect foresight.

Worker i has utility function $U_i(c) = \sum_{t=1}^{\infty} \beta_i^t u_i(c_t)$ on infinite consumption streams $c = (c_1, c_2, \dots)$. Capitalist h 's utility function on consumption streams is $U_h(c) = \sum_{t=1}^{\infty} \beta_h^t u_h(c_t)$. The discount factors, β_i and β_h , are non-negative and less than one. The one-period reward functions $u_i(\cdot)$ and $u_h(\cdot)$ from C to \mathbf{R}_+ are strictly concave, C^1 , and differentially strictly monotonic. In addition, we make the usual assumption that indifference curves do not transversally cut the boundary of the consumption set.

Axiom I. For every consumer (capitalist or worker) any consumption good j and any sequence $\{c^n\}_{n=1}^{\infty}$ of consumption bundles such that for good j , $c_j^n \rightarrow 0$, $D_j u(c^n) \rightarrow +\infty$.

Capitalist h owns firm h .³ As in optimal growth models, firms turn today's inputs into outputs available tomorrow. The technology for firm h is described by a production possibility set $T^h \subset \mathbf{R}_+^J$. The sets T^h are closed convex cones; that is, technology is convex and exhibits constant returns to scale. For firm h , any input-output vector $\omega^h \in T^h$ can be written $\omega^h = (\omega^{h-}, \omega^{h+})$, where $\omega^{h-} \in \mathbf{R}^{\text{Imp}}$ is the vector of inputs and $\omega^{h+} \in \mathbf{R}^{\text{Con}}$ is the vector of outputs. Our dynamics are driven by the assumption that production takes time. Inputs ω_t^{h-} available at date t are used to produce outputs ω_{t+1}^{h+} at date $t+1$. For a given price vector p , we let p^+ and p^- denote the vectors of prices for outputs and inputs, respectively. So $p_t^- \omega_t^{h-}$ is the cost of firm h 's date t inputs and $p_{t+1}^+ \omega_{t+1}^{h+}$ is the revenue from firm h 's date $t+1$ outputs.

2.1. Constrained Equilibrium

In an economy with rational investors there is no evolution unless the set of available intertemporal contracts is constrained. Otherwise, a complete markets equilibrium would begin at time 1 and develop over time. Rational investors would simply not invest in less profitable enterprises. We impose two constraints on the set of available intertemporal contracts. First, we assume that workers have no opportunities for lending or borrowing across different dates. Second, we assume that capitalists can transfer resources over time only through investment in their firm. In each period, capitalists receive their firm's revenue. They decide how much to spend on current consumption and how much to invest in their firm to generate tomorrow's revenues. We assume that the firm's input purchases must be financed with this investment of financial capital. Thus, we have a cash-in-advance constraint on firms.⁴ Specifically, retained earnings, or financial capital, is used to purchase inputs at date t . These inputs generate output, and thus revenue, at date $t+1$. The portion of this revenue that is retained in the firm becomes its new financial capital. The economy is initialized by endowing each firm with a stock of output, $\omega_1^{h+} > 0$, in the first period.

We test for the emergence of profit maximization, and therefore we need to allow for a variety of behaviors by firms. The behavior of firm h is described by a *decision rule*

$$(\omega_t^{h-}, \omega_{t+1}^{h+}) \in d^h(p_t, p_{t+1}, y_t^h),$$

³ We do not consider multiple owners of firms. What is important for our analysis is that the owner(s) of a firm want it to maximize profits. With a single owner, perfect competition, and a deterministic world this is clear. With multiple owners we would also need to consider the mechanism determining payouts.

⁴ Within our model, this cash-in-advance constraint is necessary to have financial capital play any role. There may be other interesting ways to model the evolution of firms, such as durable and irreversible investment in physical capital.

where p_t and p_{t+1} are the prices at dates t and $t+1$, respectively, and y_t^h is the amount firm h has available to spend on inputs at date t . We refer to d^{h-} and to d^{h+} for the projection of d^h onto inputs and outputs, respectively. Decision rules are assumed to satisfy four properties:

1. Production must be feasible: $d^h(p, q, y) \subset T^h$.
2. The firm's cash-in-advance constraint must be met: $p^- \cdot d^- = y$ for all $d^- \in d^{h-}(p, q, y)$.
3. For given arguments, all outputs produce the same revenue: $q^+ \cdot d^+ = q^+ \cdot d^{+'}$ for all $d^+, d^{+'} \in d^{h+}(p, q, y)$. Thus we can define a revenue function $R^h(p, q, y) = q^+ \cdot d^+$ for $d^+ \in d^{h+}(p, q, y)$.
4. The decision rule is upper hemi-continuous.

One such rule is constrained profit maximization:

$$\begin{aligned} \max_{(\omega^{h-}, \omega^{h+})} \quad & q^+ \cdot \omega^{h+} - p^- \cdot \omega^{h-} \\ \text{s.t.} \quad & (\omega^{h-}, \omega^{h+}) \in T^h \\ & p^- \cdot \omega^{h-} = y^h. \end{aligned}$$

We denote this special decision rule by $D^h(p, q, y)$. Note that it is equivalent to revenue maximization subject to the operating capital constraint.

Equilibrium in this economy is a sequence of prices, consumption bundles, and production plans such that consumers maximize utility subject to various constraints and such that the allocation is feasible. For each worker, the constraints are the single-period budget constraints. For each capitalist, the constraints are budget constraints involving the allocation of resources between consumption and production and the decision rule. We call equilibrium with behavior as described above *constrained equilibrium*. Formally,

DEFINITION 2.1. A *constrained equilibrium* is a sequence $(p_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H)_{t=1}^\infty$ with $p_t^* \in \mathbf{R}_+^J \setminus \{0\}$ such that

- For all workers i , $\{x_t^{i*}\}_{t=1}^\infty$ solves

$$\begin{aligned} \max_{\{x_t\}} \quad & \sum_t \beta_t^i u_i(x_t) \\ \text{s.t.} \quad & p_t^{+*} \cdot x_t - p_t^{-*} \cdot e^i \leq 0 \quad \text{for all } t, \\ & x_t \in C \quad \text{for all } t. \end{aligned}$$

- For all capitalists h , $\{x_t^{h*}, \omega_t^{h*}\}_{t=1}^\infty$ solves

$$\begin{aligned} & \max_{\{x_t, w_t\}} \sum_{t=1}^{\infty} \beta_h^t u_h(x_t) \\ & \text{s.t.} \quad p_t^{+*} \cdot x_t - p_t^{*+} \cdot \omega_t^{h*+} + p_t^{*-} \cdot \omega_t^{h*-} \leq 0, \\ & \quad (\omega_t^{h*-}, \omega_{t+1}^{h*+}) \in d^h(p_t^*, p_{t+1}^*, p_t^{*-} \cdot \omega_t^{h*-}) \quad \text{for all } t, \\ & \quad \text{and } x_t \in C \quad \text{for all } t. \end{aligned}$$

- At each date t , $\sum_i x_t^{i*} + \sum_h x_t^{h*} - \sum_h \omega_t^{h*+} = 0$ and $\sum_h \omega_t^{h*-} - \sum_i e^i = 0$, where $(w_1^{h*+})_{h=1}^H$ and $(e^i)_{i=1}^I$ are given.

Constrained equilibria have an important recursive property which is an immediate consequence of the definition.

LEMMA 2.1. *If $(p_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H)_{t=1}^\infty$ is a constrained equilibrium, so is $(p_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H)_{t=1}^\infty$ for any T .*

2.2. Competitive Equilibrium

In a competitive equilibrium, workers maximize utility subject to a single budget constraint, capitalists face a single budget constraint and can choose any technologically feasible input–output vectors, and markets clear at each date.

DEFINITION 2.2. A *competitive equilibrium* is a sequence $(q_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H)_{t=1}^\infty$ with $q_t^* \in \mathbf{R}_+^J / \{0\}$ such that

- For all workers i , $\{x_t^{i*}\}_{t=1}^\infty$ solves

$$\begin{aligned} & \max_{\{x_t\}} \sum_t \beta_i^t u_i(x_t) \\ & \text{s.t.} \quad \sum_{t=1}^{\infty} q_t^{*+} \cdot x_t - q_t^{*-} \cdot e^i \leq 0, \\ & \quad x_t \in C \quad \text{for all } t. \end{aligned}$$

- For all capitalists h , $\{x_t^{h*}, \omega_t^{h*}\}_{t=1}^\infty$ solves

$$\begin{aligned} & \max_{\{x_t, w_t\}} \sum_{t=1}^{\infty} \beta_h^t u_h(x_t) \\ & \text{s.t.} \quad \sum_{t=1}^{\infty} q_t^{*+} \cdot x_t - q_t^{*+} \cdot \omega_t^{h*+} + q_t^{*-} \cdot \omega_t^{h*-} \leq 0 \\ & \quad x_t \in C \quad \text{for all } t. \\ & \quad (\omega_t^{h*-}, \omega_{t+1}^{h*+}) \in T^h \quad \text{for all } t. \end{aligned}$$

• At each date t , $\sum_i x_t^{i*} + \sum_h x_t^{h*} - \sum_h \omega_t^{h*+} = 0$ and $\sum_h \omega_t^{h*-} - \sum_i e^i = 0$, where $(w_1^{h*+})_{h=1}^H$ and $(e^i)_{i=1}^I$ are given.

The competitive equilibrium described here is equivalent to a competitive equilibrium in a private-ownership economy in which each capitalist owns all of his or her own firm and maximizes profit. Given the linear production sets, this in turn implies that each firm earns zero profits in equilibrium.

The definition of competitive equilibrium presupposes the existence of a market structure sufficient to transfer wealth across dates and firms. Constrained equilibrium presupposes that the market structure is inadequate for this task. In fact, constrained equilibrium simply modifies the definition of competitive equilibrium with incomplete markets—in this case, missing capital markets—to include the possibility that some firms may not maximize profits. Our interest is in whether the dynamics induced by constrained equilibrium eventually compensates for the lack of complete markets.

3. SELECTION FOR PROFIT MAXIMIZERS

We first ask if constrained equilibrium dynamics selects for profit-maximizing firms. More precisely, we ask if less profitable firms are driven out of the market by more profitable firms. We begin with a general result about the fate of two capitalists with differing firm decision rules, utility functions, and discount factors. The key to the result is the relationship between the capitalists' discounted marginal rates of return on investment. Along any constrained equilibrium path, each capitalist sets his or her marginal rate of substitution between expenditure on consumption at dates t and $t+1$ equal to his or her discounted marginal rate of return on investment at date t . More generally, the marginal rate of substitution between expenditure on consumption at dates 1 and T is equal to the product of discounted marginal rates of return on investment from date 1 to date T . Suppose capitalist h has over time a uniformly larger discounted rate of return on investment than does capitalist k ; that is, h faces a more attractive investment opportunity at each date than does k . Then h 's marginal rate substitution (MRS) between consumption at dates 1 and T must grow exponentially relative to k 's MRS. Consumption is bounded above, so marginal rates of substitution are bounded above. Thus k 's MRS must converge to 0. That is, the marginal utility of income must diverge for k . So k 's consumption and the financial capital of the firm owned by k must converge to 0. Of course, this path is k 's optimal choice, but nonetheless k is being driven out of the market by h .

To develop this intuition we use the Euler equation for interior optima to describe each capitalist's optimal savings plan given the firm's (perhaps inoptimal) decision rule. For this approach to work we need to ensure that the Euler equation is well defined and that there are no boundary optima (which is to say that the interior Euler equation is a necessary condition for optimization). In order to guarantee these two properties we need some conditions on the revenue functions $R^h(p, q, y)$.

Axiom R. Along the equilibrium path $(p_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H)_{t=1}^\infty$, for every firm h ,

1. The partial derivative $R_y^h(p_t^*, p_{t+1}^*, p_t^{*-} \cdot \omega_t^{h*-})$ exists.
2. $\liminf_{y \rightarrow 0} R_y^h(p_t^*, p_{t+1}^*, y) \neq 0$.

The first condition is that the marginal rate of return on investment is well defined. The second assumption, along with our Inada condition on utility functions (Assumption I), rules out boundary solutions with zero investment in finite time.

For a given constrained equilibrium $(p_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H)_{t=1}^\infty$, define

$$R_{yt}^h = R_y^h(p_t^*, p_{t+1}^*, p_t^{*-} \cdot \omega_t^{h*-}).$$

THEOREM 3.1. *Suppose that Assumptions I and R hold. In any constrained equilibrium, $(p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H)_{t=1}^\infty$, and for any capitalists h and k with discount factors β_h and β_k , if $\prod_{\tau=1}^{t-1} (\beta_k R_{y\tau}^k) / (\beta_h R_{y\tau}^h) \rightarrow 0$ as $t \rightarrow \infty$ then $\lim_t y_t^k / (\sum_l y_t^l) = 0$ and $\lim_t c_t^k = 0$.*

Theorem 3.1 provides a general characterization of the market selection process. Which capitalists and firms survive depends on discount factors and marginal rates of return, but not on one-period utility functions. The only feature of utility functions that matters for the result is that marginal utility of consumption diverges as consumption goes to zero.

Theorem 3.1 also has implications for the survival of constrained profit maximizers. Along any equilibrium path, let $r_t^h = R(p_t^*, p_{t+1}^*, y_t^h) / y_t^h$ denote average return on investment in period t . Note that for constrained profit maximizing firms, $R_y^h(p, q, y)$ exists and is independent of y . Thus for a constrained maximizer, the marginal rate of return on investment, $R_{y_t}^h$, equals the average return on investment, r_t^h , for all t . To ensure that a maximizer drives out a non-maximizer with the same discount factor we need to rule out increasing returns to investment by the non-maximizer. The following concavity assumption does this.

Axiom C. For every firm h and prices (p, q) , the revenue function $R^h(p, q, y)$ is concave in y .

With this assumption, the selection criterion can be restated in terms of average rates of return.

COROLLARY 3.1. *Suppose that Assumptions I, R, and C hold. For any capitalists h and k , if capitalist h maximizes constrained profits and*

$$\prod_{\tau=1}^{t-1} \frac{\beta_k r_{\tau}^k}{\beta_h r_{\tau}^h} \rightarrow 0,$$

then the conclusions of Theorem 3.1 still hold.

If two capitalists have a common discount factor and one maximizes constrained profits while the other does not, then Corollary 3.1 implies that the maximizer will drive out the boundedly rational firm if the maximizer has a consistently greater rate of return on investment. This "selection for profit maximizers" result can also be stated directly in terms of revenue functions.

COROLLARY 3.2. *Suppose that Assumptions I, R, and C hold. Consider two capitalists h and k with $\beta_h = \beta_k$ and suppose that capitalist h maximizes constrained profits. In any constrained equilibrium $(p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H)_{t=1}^{\infty}$, the conclusions of Theorem 3.1 hold if:*

1. $R^h(p_t, p_{t+1}, y) \geq R^k(p_t, p_{t+1}, y)$ for all t and y , and
2. *There is an $\alpha > 1$ such that for all y , $R^h(p_t, p_{t+1}, y) > \alpha R^k(p_t, p_{t+1}, y)$ infinitely often.*

Suppose that capitalists h and k have a common discount factor, produce common outputs from common inputs, and maximize constrained profits, but that k 's technology is dominated by h 's technology. Then Corollary 3.2 implies that either k is driven out of the market or the regions in which k 's technology is dominated by h 's technology are not used infinitely often. Similarly, if h and k have identical technologies, but k does not maximize constrained profits, then k is driven out of the market if he or she infinitely often does strictly worse than a constrained profit maximizer.

Corollaries 3.1 and 3.2 apply even if the two firms are in different industries, have different technologies available to them, or are owned by capitalists with different utility functions. All that matters is rates of return and discount factors. Any firm with a consistently lower rate of return will be driven out by a firm with a higher rate of return as long as their owners have a common discount factor. Of course, rates of return are endogenous so this does not mean that all inefficient firms necessarily disappear (even in the case in which all capitalists have a common discount factor). For example, suppose that there are two firms, each the sole producer of some

good, operated by two capitalists with a common discount factor. Then, regardless of their decision rules, neither firm can be driven out of the market. To see this note that if one firm was to disappear then the price of the good that it produces would diverge (by the Inada condition on utility) and thus its rate of return on investment would similarly diverge.

Because we have endogenized dividend rates, discount factors matter. If firms' decision rules and their owners' discount factors are correlated in some funny way, then higher discount factors can compensate for inferior decision rules. The important role of discount factors in driving market selection is demonstrated in the following corollary, which shows that if two profit-maximizing capitalists have not too dissimilar long-run rates of return, then discount factors alone determine who survives.

COROLLARY 3.3. *Suppose Assumptions I and R hold. If capitalists h and k are profit maximizers, if*

$$\limsup_t \prod_{\tau=1}^{t-1} \frac{r_{\tau}^k}{r_{\tau}^h} < \infty,$$

and if $\beta_k/\beta_h < 1$, then the conclusions of Theorem 3.1 hold.

From Theorem 3.1 one might suspect that if one firm's decision rule is less efficient than the "aggregate decision rule" of the other firms in the market, then the inefficient firm will be driven out, and the production side of the economy would operate efficiently in the limit. Example 5.2 shows this hypothesis to be false.

4. DYNAMICS

In the previous section, we gave conditions guaranteeing that among all firms, the market survivors will be profit maximizers. More generally, we saw that the market selects from among the firms with a given technology those firms which are most profitable. The question that we turn to now is whether the financial capital dynamic also ensures that each industry operates efficiently. The questions of interest are: If several firms produce several goods from common inputs with differing technologies, does the market select for those firms which are most efficient? In particular, does the economy eventually operate on the production possibility frontier and does it eventually achieve a Pareto optimal allocation? If a new firm enters an industry with an efficient technology (one that expands the production possibility frontier in a relevant direction) will this firm flourish (or is it possible that it will be driven out by the retained earnings dynamic)?

The answers to these questions are “no” if there are no profit-maximizing firms or if the profit-maximizing firms belong to capitalists with low discount factors. To see this, consider two capitalists with the same technology and differing discount factors. If the capitalist with the low discount factor owns the profit-maximizing firm, and if the other firm has a sufficiently high (although not maximal) rate of return, then according to Theorem 3.1 the profit-maximizing firm would disappear. In the limit, the economy would not be operating on its production possibility frontier. To rule this phenomenon out, we assume for the remainder of the paper:

Axiom D. (i) All consumers have a common discount factor β .

(ii) All technologies come from a set of available technologies $\{T_k\}_{k=1}^K$. For each available technology T_k there is at least one capitalist h_k who maximizes constrained profit using technology T_k .

To answer the questions about producer efficiency we need to analyze the dynamics induced by constrained equilibrium in more detail. In particular, the relationship between constrained and competitive equilibria is important. We begin with a characterization of competitive equilibria. Under rather general conditions, competitive equilibria will have turnpike properties. In our economy the turnpike is reached in one step.

THEOREM 4.1. *If Assumption D holds, then every competitive equilibrium consumption path is stationary from period 2 on. That is, if $(q_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H)_{t=1}^\infty$ is a competitive equilibrium, then for each i and h , respectively, there are consumption bundles x^i and x^h such that $x_t^i = x^i$, $x_t^h = x^h$ for all $t \geq 2$.*

Non-stationary competitive equilibrium production paths are possible because of our assumption of constant returns to scale. But the proof of Theorem 4.1 shows that every competitive consumption path can be supported by a competitive equilibrium in which production plans are stationary. We call such equilibria *stationary competitive equilibria*.

To describe the relationship between competitive and constrained equilibria it will be convenient to be able to normalize constrained equilibrium prices. We assume homogeneity of firms' decision rules in order to do this.

DEFINITION 4.1. A decision rule $d(p, q, y)$ is *homogeneous* if, for all positive scalars α and β and all prices, price expectations, and revenues p , q , and y , $d(\alpha p, \beta q, \alpha y) = d(p, q, y)$.

The constrained profit-maximizing decision rule exhibits *homogeneity*. If input prices and financial capital are rescaled so as to leave the firm's budget set unchanged, and output prices are rescaled so that relative prices

of outputs do not change, then optimal production plans do not change. Perhaps we could avoid assuming homogeneity of decision rules for non-maximizers, but homogeneity is in all likelihood required for demonstrating the existence of constrained equilibria and in any case firms with non-homogeneous decision rules are not maximizing constrained profits and will be driven out of the market.

Axiom H. Each firm's decision rule is homogeneous.

In a standard competitive equilibrium, a consequence of the 0-degree homogeneity of demand and supply in prices is that the aggregate price level is indeterminate. Constrained equilibrium exhibits more price-level indeterminacy because consumers and firms are not free to take advantage of arbitrary relative intertemporal prices. In an economy with homogeneous decision rules, constrained equilibrium determines relative prices only among commodities available at the same date. The price level is indeterminate, period by period.

LEMMA 4.1. *Suppose firm decision rules are homogeneous (Assumption H). If*

$$(p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H)_{t \geq 1}$$

is a constrained equilibrium and $(\lambda_t)_{t \geq 1}$ is a sequence of strictly positive scalars, then

$$(\lambda_t p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H)_{t \geq 1}$$

is also a constrained equilibrium.

Thus we are free to normalize prices period by period. A convenient normalization, which we often use, is to set aggregate financial capital to one in each period, $\sum_h y_t^h = 1$, for all t .

The important question is, do constrained equilibria converge to a competitive equilibrium? Competitive equilibrium allocations are stationary. So if a constrained equilibrium converges to a competitive equilibrium the limit allocation is stationary. We say that a constrained equilibrium path is stationary if consumption paths are stationary and if each firm's share of total factor costs remains constant over time.

DEFINITION 4.2. A constrained equilibrium $(p_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H)_{t=1}^\infty$ is *stationary* if there exists consumption bundles x^i and x^h for each worker and capitalist, respectively, and production plans w^h such that the following properties hold for all $t \geq 1$: For all workers $x_t^i = x^i$, for all capitalists $x_t^h = x^h$, and for all firms $\omega_t^h = \omega^h$. A stationary constrained equilibrium is

interior if for each technology k , $\omega^{hk} \neq 0$. Finally, a stationary constrained equilibrium is *locally stable* if for any initial outputs of the firms $(\omega_1^{h+})_{h=1}^H$ sufficiently close to $(\omega^{h+})_{h=1}^H$, there is a constrained equilibrium path such that workers' consumptions converge to the respective x^i , capitalists' consumptions converge to the respective x^h , and production plans converge to the respective ω^h .

We first show that any stationary constrained equilibrium with at least one active constrained-profit-maximizer per technology is competitive. From Corollary 3.2 it follows that all active firms maximize constrained profit and thus maximize the gross rate of return they earn on retained earnings. At a stationary constrained equilibrium, real retained earnings must be constant, so the real rate of return must be one. Thus, in a stationary constrained equilibrium, each active firm is maximizing (unconstrained) profit. If the equilibrium is interior there is at least one active firm for each technology. Technologies exhibit constant returns to scale so the actual number of active firms per technology is irrelevant.

THEOREM 4.2. *Suppose Assumptions I, R, C, D, and H hold. The allocation resulting from any stationary and interior constrained equilibrium is a competitive allocation.*

Not all stationary constrained equilibria are competitive. Suppose that there are two technologies, each used by exactly one capitalist, and that one of the capitalists is endowed with 0 initial output. This capitalist's firm can never grow, so the constrained equilibrium is stationary, but unless the non-producing firm's technology is redundant, this equilibrium is not Pareto optimal and therefore not competitive. Suppose, however, that the constrained equilibrium path is initially interior and converges to a stationary state in which some firm has zero financial capital and is thus inactive. This firm must be making losses along the way since it once had positive financial capital. If prices were continuous in financial capital stocks it would follow that the firm would make a loss if it operated at the limit prices. Thus the financial capital constraint would not be binding on such a firm. To ensure the needed continuity we place an assumption on workers' endowments that guarantees uniqueness of prices. With this assumption we show that every locally stable, stationary constrained equilibrium is competitive.

It is convenient to have every constrained equilibrium supported by a price vector that is unique up to the renormalization described by Lemma 4.1. As a consequence of our assumptions on preferences, this already holds for consumption goods' prices because each consumption bundle is supported by a unique budget line. We could use similar smoothness, curvature, and boundary assumptions on production to guarantee the

uniqueness of supporting input prices, but our examples in Section 5 all involve piecewise linear production. The following nondegeneracy (ND) assumption has the same effect.

Axiom ND. The matrix of worker endowments $\begin{pmatrix} e^1 \\ \vdots \\ e^I \end{pmatrix}$ has rank equal to

\mathbf{Inp} , the number of inputs.

Given the workers' consumption bundles $(x^i)_{i=1}^I$ and consumption goods' prices p^+ , the workers' budget constraints must solve

$$p^- \begin{pmatrix} e^1 \\ \vdots \\ e^I \end{pmatrix}^T = p^+ \begin{pmatrix} x^1 \\ \vdots \\ x^I \end{pmatrix}^T.$$

Assumption ND implies that for each $(x^i)_{i=1}^I$ and p^+ there is a unique p^- which allows all the budget equations to be met.

THEOREM 4.3. *Suppose Assumptions I, R, C, D, H, and ND hold. The allocation resulting from any stationary, locally stable constrained equilibrium is a competitive allocation.*

5. STABILITY

Theorems 4.2 and 4.3 show that interior or locally stable stationary constrained equilibrium allocations are competitive. These results lend credence to the argument that market selection leads to Pareto optimality. Nonetheless, in this section we show that the conclusion is not correct. Market selection fails to generate Pareto optimal allocations because the financial capital dynamic need not converge and because non-steady-state allocations can be far from Pareto optimal allocations. We demonstrate this failure with a sequence of examples.

All examples in this section share a common structure. There are two consumption goods x and y and a single input good z . We assume that all firms are constrained profit maximizers. All consumers' utility functions on infinite consumption paths are of the form:⁵

$$u(x, y) = \sum_{t=1}^{\infty} \beta^t \log(x_t^\rho + y_t^\rho)^{1/\rho}.$$

⁵ In Section 2 we assumed that the range of utility functions was \mathbf{R}_+ . This lower bound on utility was used only in the proof of Theorem 1. We have verified the conclusion of Theorem 1 directly for all of the examples in this section and the following section.

Both β and $\rho < 1$ are common to all consumers. Consequently, demand at each date aggregates. Furthermore, one-period indirect utility for a capitalist with income z is $\log z + \phi(p)$, where ϕ depends upon the parameter ρ . Thus the intertemporal decision problems for capitalists are particularly simple. The solutions all require that capitalists invest a constant fraction β of their revenues in input purchases.

The first example shows that even in a standard economy with a unique competitive equilibrium, financial capital stocks need not converge. We find a limit cycle of retained earnings, and a corresponding limit cycle of constrained equilibrium allocations, none of which are competitive equilibria.

EXAMPLE 5.1. There are two capitalists and one worker. The worker is endowed at each date with one unit of good z which is used by the firms to produce x and y . Firm one produces 1 unit of x and 0.1 units of y at date $t+1$ for every unit of z that it purchases at date t ; firm 2 produces 0.001 units of x and 1 unit of y at date $t+1$ for every unit of z that it purchases at date t .

For any ρ this economy has a unique competitive equilibrium with constant relative prices

$$p_z^* = 1 \quad p_x^* = 0.90009, \quad \text{and} \quad p_y^* = 0.9991$$

and quantities which depend on ρ .

In any constrained equilibrium, capitalists invest fraction β of their revenues in their firm and spend the remaining fraction on consumption. Workers consume the entire value of their endowment. The demands by any consumer for goods x and y are

$$x = \frac{I}{p_x^{1-r}(p_x^r + p_y^r)} \quad \text{and} \quad y = \frac{I}{p_y^{1-r}(p_x^r + p_y^r)},$$

where $r = \rho/(\rho - 1)$ and I is the consumer's expenditure on consumption. These demands aggregate, and so at any date t ,

$$\frac{p_{xt}}{p_{yt}} = \left(\frac{x_t}{y_t} \right)^{\rho-1}. \quad (1)$$

Prices are normalized at each date so that expenditures on inputs always equal 1. Thus at date t , $R_t^1 + R_t^2 = 1$ and so firm h purchases share R_t^h of inputs. Consequently production by firm h is

$$(x_t^h, y_t^h) = \begin{cases} R_t^1(1, 0.1) & \text{if } h = 1, \\ R_t^2(0.001, 1) & \text{if } h = 2. \end{cases}$$

Recalling that fraction β of the date t revenues from the sale of these outputs will be retained in the firm to purchase more input at date $t+1$ we have

$$R_{t+1}^h = \begin{cases} \beta(p_{xt} + 0.1p_{yt}) R_t^h & \text{if } h = 1, \\ \beta(0.001p_{xt} + p_{yt}) R_t^h & \text{if } h = 2. \end{cases}$$

Total expenditure on date t consumption has to equal the total wealth of the capitalists, for what the capitalists do not consume directly they transfer to the workers in return for inputs, and the workers spend this payment on consumption goods. With our normalization, total capitalist wealth must equal $1/\beta$, and so the aggregate budget constraint is

$$p_{xt}x_t + p_{yt}y_t = \frac{1}{\beta}.$$

It follows from Eq. (1) that

$$p_{yt} = \frac{1}{\beta} \frac{y_t^{\rho-1}}{x_t^\rho + y_t^\rho} \quad \text{and} \quad p_{xt} = \frac{1}{\beta} \frac{x_t^{\rho-1}}{x_t^\rho + y_t^\rho}.$$

Consequently, the financial capital dynamic is

$$\begin{aligned} x_t &= R_t^1 + 0.001(1 - R_t^1) \\ y_t &= 0.1R_t^1 + (1 - R_t^1) \\ R_{t+1}^1 &= R_t^1 \left[\frac{x_t^{\rho-1} + 0.1y_t^{\rho-1}}{x_t^\rho + y_t^\rho} \right]. \end{aligned}$$

We know from Theorem 4.2 that for each ρ this difference equation system has exactly one interior steady state and that this steady state characterizes a stationary constrained equilibrium whose allocation is competitive. If this steady state is locally stable then for any $0 < R_1^1 < 1$ the sequence of constrained equilibria converges to the competitive equilibrium. Otherwise, more complex limit behavior must occur. Calculation shows that at the steady state:

$$\left| \frac{dR_{t+1}^1}{dR_t^1} \right| \leq 1 \quad \text{as } \rho \geq -1.49.$$

So as long as the consumption goods are not too strongly complementary the competitive equilibrium is locally stable. But if ρ is sufficiently small, less than -1.49 , then the unique competitive equilibrium is unstable. Figure 1 illustrates the map from R_t^1 to R_{t+1}^1 for $\rho = -3$. The instability of

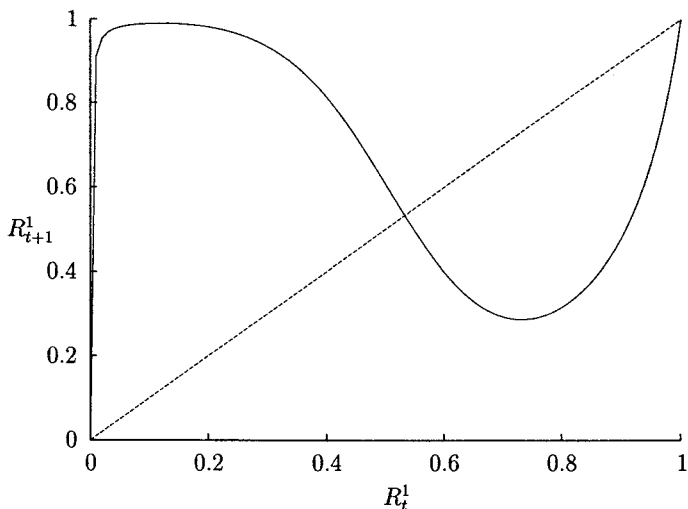


FIG. 1. Firm 1's retained earnings.

the steady state arises because if firm 1's purchasing power is too large, then the output of good x exceeds its competitive equilibrium level. Indifference curves are so curved around the competitive equilibrium consumption that the additional output of good x and corresponding reduced output of good y reduces the market clearing relative price of good x so much that firm 1 experiences a large loss. Thus firm 1's retained earnings fall below the equilibrium level, while the opposite holds for firm 2 which predominantly produces y . This causes the opposite response in the next period, with firm 2 overproducing and good y in excess of its competitive level. When the goods are sufficiently complementary this cycle of profits and losses produces cycles in the levels of financial capital that do not damp out. Figure 2 illustrates the limit behavior of financial capital stocks as a function of ρ . The data for this figure were generated by iterating the map describing the evolution of firm 1's retained earnings starting from an initial financial capital for firm 1 of $R_1^1 = 0.5$.⁶ For each value of ρ the equilibrium equation system was iterated until either it was evident that a stable cycle had been reached or it had been iterated 80,000 times. For $\rho > -1.49$ the purchasing power of firm 1 converges to its steady state value and the limit allocation is competitive. For $-2.22 < \rho < -1.49$ a two-cycle emerges; for $-2.44 < \rho < -2.22$ a four-cycle emerges; and so on, generating a period-doubling cascade. For sufficiently negative values of ρ this map displays chaotic behavior with the limit purchasing power of firm 1 varying from about 0.2 to almost 1. A three-cycle emerges at $\rho \approx -3.24$.

⁶ This picture is robust to the initial condition.

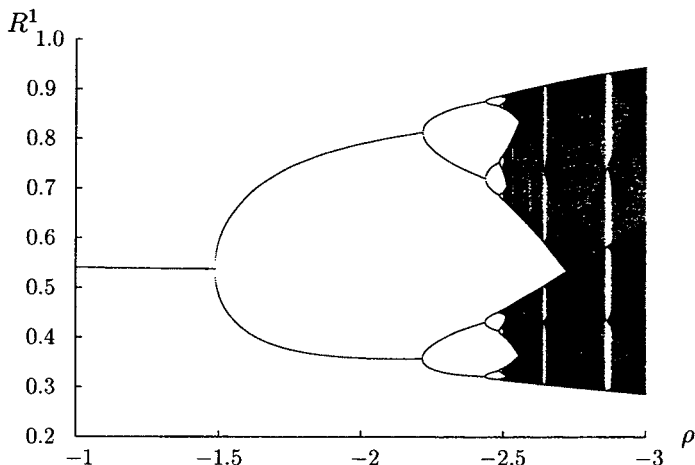


FIG. 2. Bifurcation plot—2 firm dynamics.

As a result of the instability of the steady state, the economy never achieves a Pareto optimal allocation.

A firm caught in a two-cycle is making a loss in one period followed by an offsetting profit in the next. If there was a market for financial capital, and if investors had perfect foresight, they would never put their capital today in firms that will have low returns tomorrow. We consider the ability of capital markets to resolve this problem in Section 6. For now we note that Example 1 demonstrates that for some economies the internal capital market induced by having profitable firms grow and unprofitable ones shrink is not sufficient to achieve Pareto optimality.

In Example 1, no Pareto optimal allocation is ever achieved. Nonetheless production does take place on the boundary of the economy's aggregate production possibility frontier. Pareto optimality fails only because the optimal mix of commodities is never produced. With two goods and two non-redundant firms, producer-efficient production always takes place. No matter how financial capital is allocated, the resulting allocation must be on the production possibility frontier. But with three or more firms even producer-efficiency can disappear. The following example shows how bad the dynamics can be. Here only inefficient firms survive. All efficient firms are driven out of the market.

EXAMPLE 5.2. Now there are four firms. From 1 unit of z , firm 1 can produce 1 unit of x and 0.1 units of y ; firm 2 can produce 0.05 units of x and 1 unit of y ; firm 3 can produce 0.9 units of x and 0.15 units of y ; and firm 4 can produce 0.3 units of x and 0.7 units of y . Calculation of the

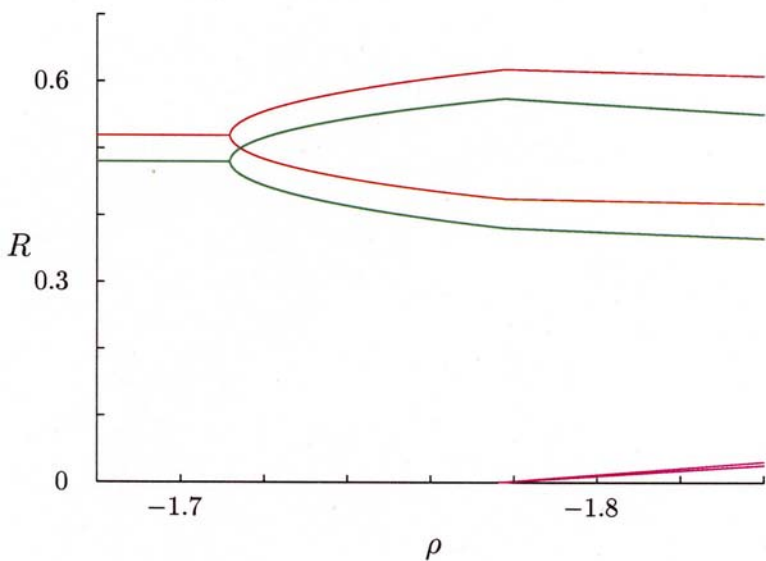
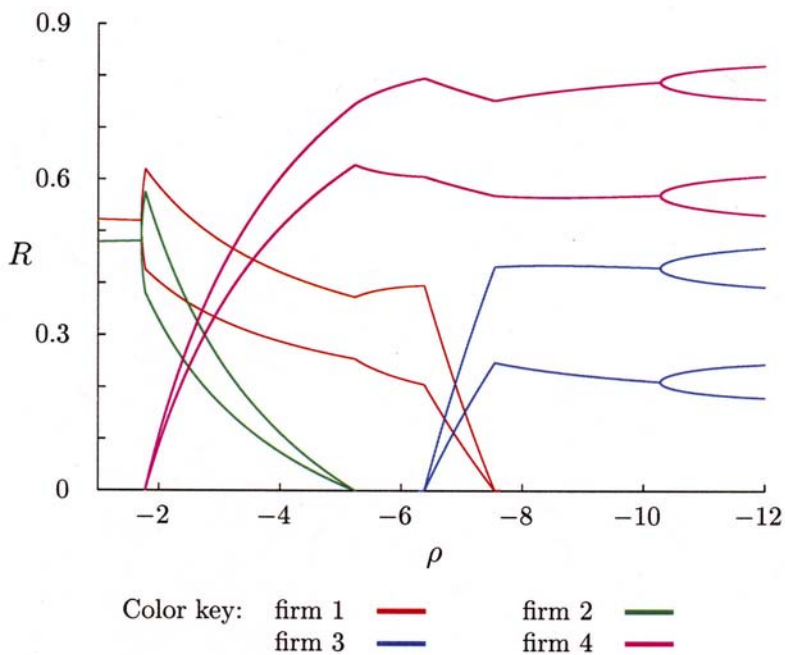


FIG. 3. Retained earnings in Example 5.2.

efficient frontier shows that the production processes used by firms 3 and 4 are dominated by combinations of those used by firms 1 and 2. Thus efficient production requires that only firms 1 and 2 operate. For any ρ there is a unique competitive equilibrium, and in this equilibrium firms 3 and 4 do not produce. This equilibrium corresponds to a steady state of the dynamic with only firms 1 and 2 having positive financial capital. For sufficiently small ρ the steady state is unstable. Figure 3 is a bifurcation plot which shows the limit behavior of capital stocks as a function of ρ . For ρ between 1 and -1.71 the economy converges to the competitive equilibrium, in which only firms 1 and 2 operate. The steady state is stable. At $\rho = -1.71$ the steady state crosses the stability threshold, and a stable two-cycle emerges. At this point the limit economy is producer- but not consumer-efficient. The economy operates on the production possibility frontier, defined by firms 1 and 2, but does not produce the optimal mix of consumer goods. Beginning at $\rho = -1.778$ (inefficient) firm 4 emerges in the limit, and producer efficiency fails as well. This region is blown up in Fig. 3a. At $\rho = -5.243$ firm 2 vanishes in the limit. Firm 3 makes its appearance at $\rho = -6.284$, and from $\rho = -7.64$ and beyond, only the two inefficient firms, 3 and 4, operate in the limit. Between $\rho = -1.71$ and $\rho = -10.281$ the limit is a stable two-cycle, but at $\rho = -10.281$ we see the beginnings of a period-doubling cascade with the two-cycles crossing the stability threshold and the emergence of a stable four-cycle. The kinks at -1.778 , -5.243 , -6.284 , and -7.63 are not computational artifacts. At these parameter values, where a firm just emerges or vanishes, the derivative of the retained earnings map is singular.

Nothing in this example requires the two firms not on the efficiency frontier to be constrained profit maximizers. If we presume that firms 1 and 2 maximize profits while firms 3 and 4 do not, the simulation results for $\rho < -1.778$ show that profit maximizers in general do not drive out profit maximizers, even when, collectively, their decision rules encompass those of the non-maximizing firms. The market does not coordinate firms 1 and 2 effectively to drive out firms 3 and 4. But if firms 1 and 2 were to merge, so that the capital allocation decision across them is internalized, then it will follow from Theorem 3.1 that firms 3 and 4 will vanish.

Example 5.2 is particularly disturbing when considering the entry of efficiency-enhancing firms. Consider the economy of Example 2 but in which only the two inefficient firms 3 and 4 exist and in which ρ is negative and large in magnitude, such that the financial capital dynamic for this economy has a stable four-cycle. Now suppose an entrepreneur discovers the technology of firm 1. This technology expands the aggregate production possibility set and would be used in any competitive equilibrium. If the

entrepreneur begins with little financial capital, he or she will lose it. Actually, simulations show that even if the entrepreneur begins with a large initial financial capital, say $R_1^1 = 1/3$, he or she will lose it. The inefficient firms drive out the efficient firm.

6. FINANCIAL MARKETS

Capitalists would like to invest only in those firms they believe to be most profitable. In the economy analyzed in the previous sections there is no market for financial capital, so capitalists do not have this opportunity. In this section, we add a market for investment in firms and a market for loans. This allows us to address the Friedman–Alchian capital market justification for profit maximization. We assume (for now) that all consumers have perfect foresight so that they make correct investment decisions.

Loans made by consumers at date t are denoted l_t^i and have a gross rate of return of g_{t+1} at date $t+1$. Loans are in zero net supply. The market clearing condition requires that the sum of loans across all workers and capitalists is zero. With access to consumption loans, both workers and capitalists can transfer income over time. To ensure that the present discounted value of each consumer's expenditures on consumption is no more than the present discounted value of his or her income, we require that, asymptotically, the present value of loans is non-negative.

Each capitalist has the additional opportunity to invest his or her savings in any firm he or she chooses. Firms use this investment as they used the investment of their owners in the previous model—as operating capital, to purchase inputs today in order to produce output and thus revenue tomorrow. This revenue is paid out to the investors, with each investor getting a share of the firm's revenue equal to the share of financial capital that he or she provided. Formally, at each date t , capitalist h decides how much to spend on current consumption, $p_t^+ \cdot x_t^h$; how much to loan out, l_t^h ; and how much to save for investment in firms, s_t^h . The capitalist invests fraction α_{kt}^h of s_t^h in firm k at date t . Firm k 's expenditures in period t are thus $\sum_h \alpha_{kt}^h s_t^h$. The rate of return between periods t and $t+1$ on this investment in firm k is $r_{kt}^* = p_{t+1}^{*+} \cdot \omega_{t+1}^{k*+} / p_t^{*-} \cdot \omega_t^{k*-}$. So capitalist h will have income $\sum_k \alpha_{kt}^h s_t^h r_{kt}^* + g_{t+1} l_t^h$ in period $t+1$. At the beginning of time, there are no outstanding loans, capitalists have endowments of the consumption goods, and workers have their constant endowments of inputs.

The definition of an equilibrium with financial markets is an extension of the definition of constrained equilibrium to include loans by consumers and investment by capitalists in other capitalist's firms.

DEFINITION 6.1. A rational expectations constrained financial equilibrium (RECFE) is a sequence

$$(p_t^*, g_t^*, (x_t^{i*}, l_t^{i*})_{i=1}^I, (x_t^{h*}, \alpha_t^{h*}, s_t^{h*}, l_t^{h*})_{h=1}^H, (w_t^{k*})_{k=1}^K)_{t=1}^\infty$$

with $p_t \in \mathbf{R}_+^J / \{0\}$ such that

- For all workers i , $\{x_t^{i*}, l_t^{i*}\}_{t=1}^\infty$ solves

$$\begin{aligned} \max_{\{x_t^i, l_t^i\}} \quad & \sum_t \beta_t^i u_i(x_t^i) \\ \text{s.t.} \quad & x_t^i \in C \quad \text{for all } t, \\ & \liminf_t \frac{l_t^i}{\prod_{\tau=1}^t g_\tau^*} \geq 0, \end{aligned}$$

$$\text{For all } t: p_t^{+*} \cdot x_t^i + l_t^i \leq m_t^i,$$

$$\text{where } m_t^i = p_t^{-*} \cdot e + g_t^* l_{t-1}^i \quad \text{and } l_0^i = 0.$$

- For all capitalists h , $\{x_t^{h*}, \alpha_t^{h*}, s_t^{h*}, l_t^{h*}\}_{t=1}^\infty$ solves

$$\begin{aligned} \max_{\{x_t^h, l_t^h, w_t^h, \alpha_t^h, s_t^h\}} \quad & \sum_t \beta_t^h u_h(x_t^h) \\ \text{s.t. for all } t: \quad & x_t^h \in C, \\ & p_t^{+*} \cdot x_t^h + l_t^h + s_t^h \leq m_t^h, \end{aligned}$$

$$m_t^h = \sum_k \alpha_{k,t-1}^h s_{t-1}^k r_{k,t-1}^* + g_t^* l_{t-1}^h \quad \text{for } t > 1,$$

$$\text{and } m_1^h = p^{+*} \sum_k \omega_1^{hk+*},$$

$$\sum_k \alpha_{k,t}^h = 1, \quad \alpha_t^h \geq 0, \quad s_t^h \geq 0, \quad l_0^h = 0,$$

$$\text{and} \quad \liminf_t \frac{l_t^h}{\prod_{\tau=1}^t g_\tau^*} \geq 0.$$

- For all firms k and for all t ,

$$(w_t^{k-*}, w_{t+1}^{k+*}) \in d^k \left(p_t^*, p_{t+1}^*, \sum_h \alpha_{kt}^h s_t^{h*} \right).$$

- At every date t ,

$$\sum_i x_t^{i*} + \sum_h x_t^{h*} - \sum_h w_t^{h*+} = 0,$$

$$\sum_h w_t^{h*-} - \sum_i e^i = 0,$$

$$\sum_i l_t^{i*} + \sum_h l_t^{h*} = 0,$$

where $\{e^i\}_{i=1}^I$ is given, $\{w_1^{hk+*}\}_{h=1, k=1}^{H, K}$ is given, and $\{w_1^{k+*}\}_{k=1}^K$ is given such that $\sum_{h=1}^H w_1^{hk+*} = w_1^{k+*}$ holds.

Aside from the details of the loan markets, this definition differs from the previous constrained equilibrium definition in that here a firm is not owned by a single capitalist and in that here the economy must be initialized by distributing ownership shares of pre-existing production among capitalists.

In an RECFE only those firms that offer the maximal rate of return on investment will receive any funds. So no inefficient firms will ever operate if for each technology at least one firm with access to the technology maximizes constrained profit.

The following theorem shows that this system of markets—spot markets for consumption loans and financial capital—is dynamically complete if all consumers have rational expectations. So if consumers have rational expectations, then all RECFE allocations are competitive equilibrium allocations.

THEOREM 6.1. *Suppose Assumptions I and D hold. Any RECFE allocation is a competitive equilibrium allocation.*

6.1. Evolution and Optimality with Dynamically Incomplete Markets

In an RECFE no selection over firms occurs (other than the trivial and immediate selection at the beginning of time) so this is not an appropriate structure in which to ask about selection for profit maximizing firms. But with the financial markets described above, inefficient firms may attract investment if some investors do not have rational expectations. In this case, the selection question shifts from direct selection over firms to the effect on firms of selection over investors with differing expectations. The interesting questions include: Will investors with rational expectations be selected for? Will this cause inefficient firms to eventually disappear? Will the equilibrium allocation converge to an RECFE allocation?

The definition of an RECFE has rational expectations built into it, but an extension to allow for differing expectations is straightforward. When a consumer makes his or her consumption, savings, and investment plans he or she does so at each date using whatever expectations he or she has at that date about future prices and rates of return. These expectations may be conditioned on any information that the consumer has. This information is all publicly available information, current and past prices, and rates of return, as well as the consumers own past choices and current wealth. A worker's decision problem yields consumption and savings decisions at each date. A capitalist's decision problem yields consumption, savings, and investment choices, as well as a choice among the set of available production plans for his or her firm at each date. The market clearing conditions for inputs and outputs at each date are unchanged. We will refer to an equilibrium with financial markets when some consumers may not have rational expectations as a *constrained financial equilibrium (CFE)*.

The success of market selection for rational expectations depends upon what is meant by the phrase "rational expectations." There are (at least) two possible definitions. Rational expectations is a constraint on investors' beliefs. The first candidate definition constrains beliefs in (rational expectations) equilibrium, but not outside equilibrium. These expectations can be viewed as either forecasting a particular price and rate of return sequence or as using a forecasting rule mapping observable information into predicted prices and rates of return. But in either case, there are no constraints on forecasts from data that are not generated in equilibrium. We call these expectations *narrow-sense* rational. We say that capitalists in a CFE have narrow-sense rational expectations if the CFE is an RECFE. The second candidate definition requires a forecasting rule that generates expectations which are correct both in and out of equilibrium, that is, expectations that always forecast correctly regardless of the behavior of other traders. Equivalently, given the expectations or forecasting rules of other traders, the rational trader's forecasting rule must generate correct expectations from any set of initial conditions for the economy. We call such expectations *wide-sense* rational. Wide-sense rational expectations might arise in a CFE which is not an RECFE, if along the equilibrium path some, but not all, traders have correct beliefs. Those with correct beliefs have wide-sense rational expectations.

Individuals with narrow-sense rational expectations need not forecast correctly in an economy in which some individuals have incorrect expectations. Thus, they may make inferior investments and their share of wealth need not converge to one. As a result, the economy need not become even asymptotically efficient. We addressed a closely related question in [2], where we showed that rational expectations equilibria need not be locally stable under a simple learning dynamic. The following example shows how market selection for narrow-sense rational expectations can fail.

EXAMPLE 6.1. We add financial markets and a new firm to the economy of Example 5.1. In that example, there are two technologies each producing a mix of the two output goods from a single input. Any allocation of input to these two firms results in a point on the production possibility frontier. So to make production inefficiency possible we also add a third dominated technology. Technology 3 produces 0.8 times as much as does technology 2 from a unit of input. This economy has a unique RECFE with constant input and output prices, $p_z = 1$, $p_x = 0.90009$, and $p_y = 0.9991$, and a constant gross rate of return on loans, $R_t = 1/\beta$. All consumers discount at rate β , so with constant goods prices and a gross rate of return on loans of $1/\beta$ the loan market does not operate. The constant outputs and the share of financial capital that is invested in firms 1 and 2 are functions of the utility parameter ρ . Firm 3 offers a lower rate of return than does firm 2 and so it never operates. We assume that $\rho = -3.0$. The RECFE share of financial capital that is invested in technology 1 is 0.533305, and the resulting outputs are $(x, y) = (0.5338, 0.5200)$.

Suppose that all workers, and capitalist 1, always forecast the RECFE prices. Thus, they have narrow-sense rational expectations. Capitalist 2 is irrational. He or she believes that prices and rates of return will be constant over time, but does not forecast the rational expectations prices and rates of return. Exactly what prices and returns capitalist 2 forecasts do not matter (because of the form of his or her utility function); all that matters is how he or she chooses to allocate his or her savings between the firms. We assume that capitalist 2's forecasts are such that he or she always invests share 0.875 of his or her wealth in technology 1 and the remainder in technology 3. Because all consumers forecast constant prices, and discount at rate β , the loan market clears with no trade at a constant gross rate of return of $1/\beta$. Goods prices will vary with the wealth of the capitalists because of capitalist 2's irrationality. The economy will be in an RECFE only when capitalist 1 has all of the wealth. When the wealth of capitalist 1 is $1/\beta$, he or she must invest share 0.533305 of his or her wealth in technology 1, and the remainder in technology two in order to support an RECFE. At any other wealth level, his or her expectations and allocation of wealth between the two efficient firms is not tied down by the narrow-sense rational expectations hypothesis.

Since capitalist 1 always forecasts the RECFE prices and rates of return, he or she believes that the rate of return on investment in either efficient firm is $1/\beta$ and that the rate of return on investment in the inefficient firm is less than $1/\beta$ (as it is in an RECFE). He or she is thus indifferent over investment shares between firms 1 and 2.⁷ We assume that when he

⁷ The evolution of the economy will depend only upon his or her wealth. So alternatively we can assume he or she has a forecasting rule which depends upon current market data and is such that, rationally optimizing, he or she will make the investment allocation we are about to describe.

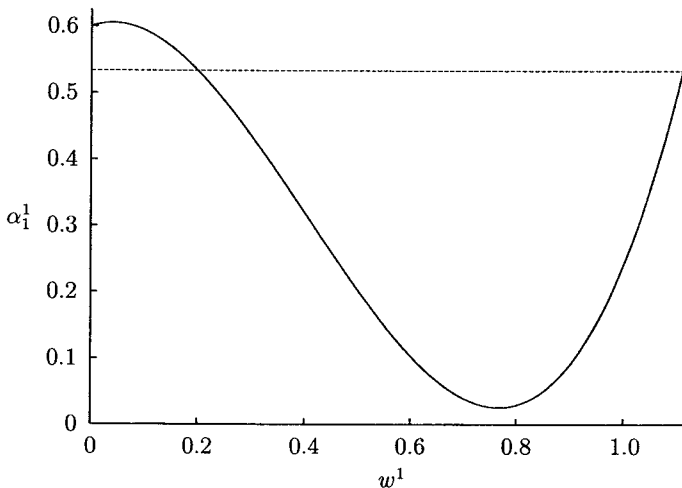


FIG. 4. Capitalist 1's allocation rule.

or she has wealth w , he or she invests fraction $0.533305 + 3(w - 0.2)(w + 0.1)(w - 1/\beta)$ in technology 1 and the remainder in technology 2. This rule has the property that when capitalist 1 has all of the wealth in the economy, $w = 1/\beta$, the share invested in technology 1, 0.533305, supports the RECFE. This allocation rule is illustrated in Fig. 4. Other than the forecast and allocation at wealth share one, the structure of this rule is not tied down by the narrow-sense rationality hypothesis.⁸

If capitalist 2 has all of the wealth in the economy, then the allocation of financial capital is incorrect, and the equilibrium allocation is not an RECFE allocation. If capitalist 1 has all of the wealth, then he or she invests correctly, the rational expectations prices are realized and the allocation is the RECFE allocation. What happens if initially both capitalists have some wealth? Figure 5 illustrates the map from the wealth of capitalist 1 at time t , to his or her wealth at time $t + 1$ for an economy with $\beta = 0.9$. This equation of evolution has four steady states. In the only locally stable steady state, capitalist 1 has all of the wealth. This is the RECFE equilibrium. But its basin of attraction is tiny. Only if the initial wealth of capitalist 1 exceeds approximately 1.1105 (a wealth share of 0.9994) will his or her wealth share converge to one. To see why this is the case, note that as

⁸ Only two properties of this rule matter for our results. First, at some wealth share less than one for the rational capitalist, the two capitalists invest so as to have equal rates of return. At this wealth the slope of capitalist 1's allocation rule is positive. Second, at wealth share one the rational capitalist invests so as to support the RECFE, and at this point the slope of the rule is positive.

capitalist 1's wealth falls from $1/\beta$ he or she invests less than the RECFE fraction in technology 1 and correspondingly more than the RECFE fraction in technology 2. Capitalist 2 invests fraction 0.875 (more than the RECFE fraction) of his or her small wealth in technology 1 and the rest in the dominated technology. Less than the RECFE fraction of total wealth is invested in technology 1, and thus the rate of return on technology 1 is greater than on technology 2. For wealth of capitalist 1 below 1.109, capitalist 2 has a greater rate of return on investments than does capitalist 1. So capitalist 2's wealth share grows. Finally, as Fig. 5 shows, no other wealth levels are mapped into a wealth for capitalist one of 1.109 or more.

The unstable steady state at approximately 1.1105 is the separatrix between the basin of attraction for the RECFE and an attractor contained in the interval 0.41717 and 0.79298. The dynamics in this region must be complicated—numerical investigation of the map describing the evolution of wealth shows that it contains a three-cycle for some initial condition in this region.

With cubic allocation rules it is possible to generate other phenomena, including a globally stable RECFE. We conjecture that the local stability of the RECFE is generic, but we will not pursue this here.

Alternatively, we could require a rational capitalist to always invest optimally. To do so he or she would have to be able to predict rates of return when the economy is not in an RECFE. Thus he or she would have wide-sense rational expectations. If we assume that rational capitalists have

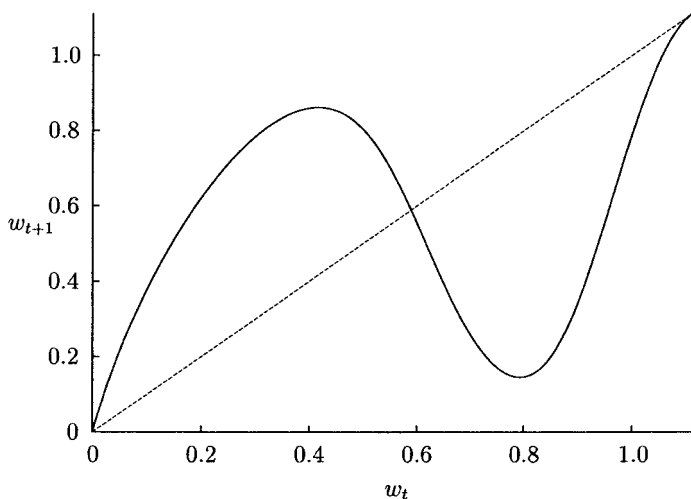


FIG. 5. Capitalist 1's wealth dynamics.

wide-sense rational expectations, then in our examples with logarithmic utility and a common discount factor, convergence to an RECFE is ensured. This occurs for two reasons. First, with log utility and a common discount factor, all consumers save at the same rate regardless of their expectations. Second, with log utility, each consumer invests so as to maximize his or her expected growth rate of wealth. The investors with correct expectations correctly maximize the growth rate of their wealth. We have shown in [3], that this behavior is selected for in the market. The following example shows how this occurs and why it is of limited interest as a selection mechanism for profit maximization.⁹

EXAMPLE 6.2. Suppose there are four firms using two technologies. Each technology is employed by one profit maximizing firm, firms 1 and 3, and one non-maximizing firm, firms 2 and 4. Each technology produces one good. Suppose there are two capitalists with logarithmic utility and equal discount factors, one of whom has wide-sense rational expectations. The other capitalist who has incorrect beliefs invests only in the non-maximizing firms. The rational investor will always invest in the profit maximizing technologies; consequently his or her share of total investment will grow relative to the investor investing in non-maximizing firms. It can be shown using Euler equation arguments that the share of investment belonging to the investor who invests in non-maximizing firms converges to 0. Thus, in this example, investors with “bad beliefs” are driven out.

But investors with incorrect beliefs who nonetheless always invest in profit maximizing firms need not be driven out. Suppose that capitalist 1 has wide-sense rational expectations and that capitalist 2 knows the true rate of return to investment in firm 1, but underestimates all the others. Only firms 1 and 3 receive any investment funds. After some finite number of dates the rate of return on investment in firms 1 and 2 must be identical. This happens as soon as investor one, the rational investor, is wealthy enough so that investing all of his or her savings in firm 3 makes its rate of return less than that of firm 1.

But how is this maintained? Capitalist 2 invests all his or her money in firm 1, and capitalist 1, the rational investor, allocates his or her money between firms 1 and 3 so as to guarantee equal rates of return. Suppose now far off in time, after this steady state is reached, capitalist 2’s expectations are such that his or her investment rule changes so that in every 13th period he or she places all of his or her investment in firm 3. If capitalist 1

⁹ More generally (with non-log utility), the completeness of markets is critical. We show in [4] that investors with wide-sense rational expectations are selected for in economies with dynamically complete markets. With incomplete markets, selection for rational expectations need not occur.

leaves his or her investment alone, firm 3 will earn less than firm 1, so investing in firm 3 would contradict the wide-sense rational expectations hypothesis. If capitalist 1 invests everything in firm 1, firm 1 will earn a lower rate of return than firm 3, which is also inconsistent with wide-sense rational expectations. Consequently, capitalist 1 must adjust his or her investment every 13th period so as to just offset capitalist 2's behavior.

The wide-sense rational expectations hypothesis requires that rational investment be responsive to the investment of irrational actors. We believe that this kind of information requirement is inconsistent with the spirit of competitive analysis. To assume that in any economy there is at least one capitalist who always correctly forecasts endogenous prices begs the question of how a capitalist whose behavior is so carefully tuned to the structure of the economy, including the behavior of any irrational capitalists, could arise.

7. CONCLUSION

The *market selection hypothesis* claims that market forces weed out less profitable firms in favor of more profitable firms and that the long-run behavior of markets can be described by competitive equilibria of an economy with only profit-maximizing firms. We have investigated both parts of this hypothesis, the assertion about weeding out non-maximizers and the assertion about long-run market behavior. The validity of the first claim depends upon investors expectations and the market structure. In a model without capital markets, firms grow only out of retained earnings. Market selection favors more profitable firms. Controlling for discount factors, among those firms using a given technology only the most profitable will survive in the long-run.¹⁰ In a model with capital markets, selection works on investors rather than on firms. We distinguish between *wide-sense* rational expectations and *narrow-sense* rational expectations. We argue that it is unreasonable to populate a market with investors who have wide-sense rational expectations. When the more reasonable narrow-sense concept is used, we find that the market need not select for investors with rational expectations and that unprofitable firms can stay afloat through a continual injection of outside funds.

We also investigate the second claim. When firms grow only through retained earnings, it is true that the limit firm population contains only profit maximizers (under the stated assumptions), but it does not follow that the limit behavior of the market can be described as a competitive

¹⁰ This result assumes that investors correctly forecast the marginal rate of return on investment in their decision rule.

equilibrium of an economy with profit-maximizing firms. The market allocation may not converge to an optimal allocation. We conclude that the market selection hypothesis does not justify neoclassical equilibrium analysis with profit-maximizing firms.

In retrospect, the negative answers we obtain are not surprising. To sensibly ask questions about evolution, the market structure must be incomplete. Thus what we are really asking is whether natural selection can compensate for the lack of complete markets. Of course, the incomplete markets equilibrium will not be a complete markets equilibrium from the start. But the natural selection conjecture is that from some interesting set of initial conditions (describing firms' capital or heterogeneous investors' wealths), the incomplete markets equilibrium converges to a complete markets equilibrium. Given how little structure incomplete markets equilibria have, the conjecture seems incredible and we show that it is false.

This study of selection has proceeded in an economy without stochastic shocks. Dutta and Radner [5] demonstrate that in an uncertain world, firm decision rules which maximize long-run survival probabilities are not those which maximize expected profits. Studying market selection with uncertainty is important because when profits are random, and the firm cannot be valued through arbitrage, it is unclear what objective to attribute to a firm. It is not obvious that capitalists would agree on expected profit maximization or on any other objective for the firm. In this case it is particularly interesting to see what behavioral rules the market selects for. We conjecture that, just as the investment market of [3] favors those rules with higher expected log returns, constrained equilibrium paths for an economy with uncertainty will favor those firm decision rules with higher expected log revenues.

Our study of the market selection defense of the profit maximization hypothesis is related to the literature on the market selection hypothesis in financial asset markets [3, 4, 10]. These papers address the hypothesis that, in pure exchange economies, asset markets select for rational investors. Our analysis of the retained earnings dynamic uses methods similar to those employed in these papers, but here we address different questions. Nonetheless this paper and the financial markets selection literature touch at one point. Our analysis of the capital markets justification for profit maximization reduces to the question of selection for rational expectations in repeated asset markets. The results we offer here on this issue are different from some of those in that literature for two reasons. First, in contrast to Blume and Easley [3], here we have a complete model of investor behavior. In our earlier paper, we fixed savings rates exogenously and investigated selection for "as if" rational portfolio rules. Here we endogenize all decisions. Sandroni [10] also endogenizes all decisions, but his analysis is not concerned with firms. Second, in contrast to [10], we

consider incomplete markets. In Blume and Easley [4] we examine selection for rational expectations, without firms, and show that whether markets are complete or incomplete is crucial. Our results here are consistent with those findings.

A third argument for the long-run dominance of profit-maximizing firms is adaptation. Firms adjust. By experimentation, imitation, and the like, they grope for more profitable decision rules. Firm adaptation is an important part of Winter's [12] and Nelson and Winter's [9] market selection story. In this paper we separate adaptation from selection in order to get a better understanding of how selection works. Making this distinction in biological models is certainly one big accomplishment of the neo-Darwinian synthesis. But for two reasons we believe that putting adaptation back into the market selection mechanism will not change our answers. First, suppose adaptation really drives firms to profit-maximizing behavior. The coordination problem illustrated in Example 5.2 still remains. Selection cannot always allocate capital optimally in incomplete markets. Second, we have studied the interaction of adaptation and equilibrium in other, related contexts [2, 3] and have found mixed results. An adaptive process that can converge to optimal behavior in a stationary environment may be dysfunctional when the environment and the behavior coevolve.

The dynamics of adaption and selection in the models discussed here are quite different from biological dynamics. The models developed here are certainly motivated by an elementary understanding of the Darwinian view of natural selection in biological processes. But nonetheless the analogy between biological and economic process is surprisingly rough given how influential biological thought has been in the lore of economics if not in serious economic analysis. What is the economic analog of a species? What is the biological analog of a firm? The difference between adaptation and selection in social and economic settings and in biological contexts suggests that the population dynamics of the econosphere look very different than that of the biosphere. At a crude level we have found the biological analogy to be helpful in motivating our thinking, but we believe that successful models of the population ecology of social and economic organizations will look very different than their biological analogs.

APPENDIX: PROOFS

Proof (Theorem 3.1). Under our assumptions, each capitalist's optimal path solves the optimization problem

$$\begin{aligned}
& \max_{\{z_t^j, R_t^j\}} && \sum_t \beta_j^{t-1} v^j(p_t, z_t^j) \\
& \text{s.t.} && z_t^j + y_t^j = m_t^j \\
& && m_{t+1}^j = R^j(p_t, p_{t+1}, y_t^j) \\
& && 0 \leq y_t^j \leq m_t^j
\end{aligned}$$

for $j = h, k$, where m_t^j is revenue at the beginning of period t , z_t^j is consumption expenditure, y_t^j is expenditure on inputs, and $v(p, z^j)$ is the capitalist's one-period indirect utility function evaluated at prices p and expenditure on consumption z^j .

Aggregate endowments are fixed and used at each date to produce the next date's output, so aggregate consumption is finite. Individuals have perfect foresight, so the value of each individual's decision problem is finite. These observations along with Assumptions *I* and *R* imply that the Euler equations are necessary. Therefore along any equilibrium path,

$$v_z^j(p_t, z_t) = \beta_j R_{y_t}^j v_z^j(p_{t+1}, z_{t+1}),$$

where $R_{y_t}^j = R_y^j(p_t, p_{t+1}, y_t^j)$. Consequently,

$$\begin{aligned}
\frac{v_z^h(p_{t+1}, z_{t+1}^h)}{v_z^k(p_{t+1}, z_{t+1}^k)} &= \frac{\beta_k R_{y_t}^k v_z^h(p_t, z_t^h)}{\beta_h R_{y_t}^h v_z^k(p_t, z_t^k)} \\
&= \frac{v_z^h(p_1, z_1^h)}{v_z^k(p_1, z_1^k)} \prod_{\tau=1}^{t-1} \frac{\beta_k R_{y_\tau}^k}{\beta_h R_{y_\tau}^h}.
\end{aligned}$$

From the definition of the marginal utility of income it follows that for any consumption good j ,

$$\frac{D_j u^h(c_{t+1}^h)}{D_j u^k(c_{t+1}^k)} = \frac{v_z^h(p_1, z_1^h)}{v_z^k(p_1, z_1^k)} \prod_{\tau=1}^{t-1} \frac{\beta_k R_{y_\tau}^k}{\beta_h R_{y_\tau}^h} \quad (\text{A.1})$$

Suppose that $\prod_{\tau=1}^{t-1} (\beta_k R_{y_\tau}^k / \beta_h R_{y_\tau}^h)$ converges to 0. Then the right hand side of (A.1) converges to 0. Since consumption is bounded from above along any equilibrium path, the numerator of the left hand side is bounded away from 0. Consequently the denominator of the left hand side must be converging to $+\infty$, and so c_t^k converges to 0.

Finally, we need to show that capitalist k 's share of retained earnings converges to 0. Suppose not. Then there is an $\epsilon > 0$ such that infinitely often he or she can purchase at least fraction ϵ of the aggregate endowment. Since preferences are strictly monotone, in any such period he or she could use it to produce a consumption bundle that would give utility

$u^k(0) + \delta$ were he or she to consume it. If capitalist k carries out this plan at some date far in the future, its utility exceeds the continuation utility of the optimal plan with $c_t^k \rightarrow 0$. This is a contradiction. ■

Proof. Consider Eq. (A.1) in the proof of Theorem 3.1. For the profit-maximizing firm h , $R_{yt}^h = r_t^h$, while for the other firm concavity of R^k in y implies that $R_{yt}^k \leq r_t^k$. Consequently

$$\frac{v_z^h(p_{t+1}, z_{t+1}^h)}{v_z^k(p_{t+1}, z_{t+1}^k)} \leq \frac{v_z^h(p_1, z_1^h)}{v_z^k(p_1, z_1^k)} \prod_{\tau=1}^{t-1} \frac{\beta_k r_\tau^k}{\beta_h r_\tau^h}$$

and the rest of the argument follows as in the proof of the theorem. ■

Proof (Corollary 3.2). From Corollary 3.1, and noting that $\beta_h = \beta_k$, it is sufficient to show that

$$\prod_{\tau=1}^{t-1} \frac{r_\tau^k}{r_\tau^h} \rightarrow 0.$$

Note that as capitalist h maximizes constrained profits r_τ^h is independent of y_τ^h . So for all dates τ we have from assumption (1) of the corollary that

$$r_\tau^h = \frac{R^h(p_\tau, p_{\tau+1}, y_\tau^k)}{y_\tau^k} \geq \frac{R^k(p_\tau, p_{\tau+1}, y_\tau^k)}{y_\tau^k} = r_\tau^k.$$

From assumption (2) of the corollary we have that infinitely often

$$r_\tau^h = \frac{R^h(p_\tau, p_{\tau+1}, y_\tau^k)}{y_\tau^k} \geq \alpha \frac{R^k(p_\tau, p_{\tau+1}, y_\tau^k)}{y_\tau^k} = r_\tau^k,$$

for $\alpha > 1$. So

$$\prod_{\tau=1}^{t-1} \frac{r_\tau^k}{r_\tau^h} \rightarrow 0.$$

■

Proof (Corollary 3.3). If both firms are profit maximizers, then $R_{yt}^j = r_t^j$, and the result now follows immediately from Theorem 3.1. ■

Proof (Theorem 4.1). In equilibrium, utility maximization for workers implies that $\sum_t q_t e^i < \infty$. Consequently the first welfare theorem is valid and so every competitive equilibrium is Pareto optimal. Let $((x_t^i)_{i=1}^I, (\omega_t^h, x_t^h)_{h=1}^H)$ denote the equilibrium allocation, and consider the allocation with the same first period consumptions, and such that the following properties hold: For all $t \geq 1$, $\omega_t^{h-} = (1 - \beta) \sum_{\tau \geq 1} \beta^{\tau-1} \omega_\tau^{h-}$. For

all $t \geq 2$, $\omega_t^{h+} = (1 - \beta) \sum_{\tau \geq 2} \beta^{\tau-2} \omega_\tau^{h+}$. For all $t \geq 2$ and for every consumer j (capitalist or worker), $x_t^j = (1 - \beta) \sum_{\tau \geq 2} \beta^{\tau-2} x_\tau^j$. This allocation is feasible. If the equilibrium consumption plan is not stationary, this allocation is also Pareto preferred, which is a contradiction. ■

Proof (Lemma 4.1). Multiplying price vectors by positive scalars leaves the workers' budget sets unchanged, so their demand is invariant to the change in prices. The same is true for capitalists. To see this, consider a plan $(x_t, \omega_t)_{t \geq 1}$ in capitalist h 's budget set. First observe that, for fixed ω_t^+ , the set of affordable (x_t, ω_t^-) pairs is invariant to the proposed change in scale of prices. Finally observe that, due to homogeneity, $(\omega_t^-, \omega_{t+1}^+) \in d^h(p_t, p_{t+1}, p_t \cdot \omega_t^-)$ if and only if $(\omega_t^-, \omega_{t+1}^+) \in d^h(\lambda_t p_t, \lambda_{t+1} p_{t+1}, \lambda_t p_t \cdot \omega_t^-)$. ■

Proof (Theorem 4.2). Because the consumption path is stationary, all the output prices are collinear. Rescale prices (according to Theorem 4.1) so that $p_t^* = \beta^{t-1} p_1^*$. This price sequence supports the stationary consumption path of all consumers in the competitive equilibrium consumer choice problem.

It follows from Corollary 3.2 that all active firms maximize constrained profits, but it remains to show that all active firms maximize (unconstrained) profits. To do this we show that for each capitalist the gross rate of return on investment is constant over time and that its maximal value is one.

Output price ratios are constant and the level is falling at rate β . Consequently the value of each capitalist's output falls at rate β . But, due to piecewise linearity, the capitalist's problem does not restrict input prices. However, since the output prices are falling at rate β , it follows that the value of each consumer's consumption falls at the same rate. Since each worker's budget constraint is satisfied, the value of each worker's endowment falls at rate β . Consequently the value of aggregate expenditures falls at the same rate. Since each capitalist's share of input expenditure is constant, each capitalist's input expenditures falls at rate β . So the gross rate of return on investment is constant over time.

We turn now to the decision problem of a typical capitalist (the superscript h is dropped for clarity). This capitalist is spending amount z_t on consumption and y_t on inputs in period t . Let $r = p_{t+1}^+ \cdot \omega_{t+1}^+ / p_t^- \cdot \omega_t^-$ denote the gross rate of return on a dollar invested in the firm at time t . (We have already seen that this number is constant through time). Constrained profit maximization implies that the firm is run so as to maximize r . So we only need show that $r = 1$. The capitalist solves the following decision problem

$$\begin{aligned} \max_{\{y_t, z_t\}} \quad & \sum_{t=1}^{\infty} \beta^{t-1} v(\beta^{t-1} p_1, z_t) \\ \text{s.t.} \quad & y_t + z_t = m_t, \\ & m_{t+1} = r y_t, \\ & m_1 > 0 \quad \text{given,} \\ & 0 \leq y_t \leq m_t, \end{aligned}$$

where $v(p, z)$ is the capitalist's indirect utility function for the one-period problem. The Euler equation is necessary (see the proof of Theorem 3.1) and is given by

$$v_z(p_t, z_t) = \beta r v_z(p_{t+1}, z_{t+1}).$$

From the 0-degree homogeneity of indirect utility and stationarity,

$$v_z(p_t, z_t) = \beta r v_z(p_{t+1}, z_{t+1}) = r v_z(\beta^{-1} p_{t+1}, \beta^{-1} z_{t+1}) = r v_z(p_t, z_t).$$

Differential strict monotonicity of utility functions implies that $v_z > 0$, so $r = 1$. ■

The proof of Theorem 4.3 requires a result about the continuity properties of constrained equilibrium.

LEMMA A.1. *Suppose that Assumptions H and ND hold, and that $(p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H)_{t=1}^{\infty}$ is a constrained equilibrium. If the equilibrium allocations converge to $((x^{i*})_{i=1}^I, (x^{h*}, \omega^{h*})_{h=1}^H)$ as t grows large, then there are positive scalars λ_t and a price vector p^* such that:*

- $(\beta^{-(t-1)} p^*, (x^{i*})_{i=1}^I, (x^{h*}, \omega^{h*})_{h=1}^H)_{t=1}^{\infty}$ is a constrained equilibrium, and
- $\beta^{-(t-1)} \lambda_t p_t$ converges to p^* .

Proof. Consider the price sequence $\{\|p_t^+\|^{-1} p_t^+\}_{t=1}^{\infty}$ for consumption goods. These prices all lie in a compact set. Any sub-sequential limit supports each x^{i*} and x^{h*} . From our assumptions on preferences there is a unique such consumption goods price vector of length 1. Call it p^{+*} , and observe that the sequence of consumption goods prices p_t^+ converges to the ray defined by p^{+*} . Assumption ND implies there is a unique p^{-*} which solves the workers' budget constraints when consumption goods prices are p^{+*} and consumptions are x^{i*} . Upper hemi-continuity of the solution correspondence for linear equations implies that p^{-*} is the limit of the

sequence $\{\|p_t^+ \|^{-1} p_t^-\}_{t=1}^\infty$. Taking $\lambda_t = \beta^{t-1} \|p_t^+ \|$ satisfies (2). From Lemma 4.1, $(\beta^{-(t-1)} \lambda_t p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H)_{t=1}^\infty$ is a constrained equilibrium such that, in addition to the convergence of the allocation, prices converge to p^* . The properties of equilibrium are all closed, and so $(p^*, (x^{i*})_{i=1}^I, (x^{h*}, \omega^{h*})_{h=1}^H)_{t=1}^\infty$ is a constrained equilibrium. Finally, renormalizing prices as per Lemma 4.1 gives point 1. ■

Proof (Theorem 4.3). Let $(\beta^{(t-1)} p, (x^i)_{i=1}^I, (x^h, \omega^h)_{h=1}^H)_{t=1}^\infty$ be a stationary, locally stable constrained equilibrium, while $(\beta^{(t-1)} p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H)_{t=1}^\infty$ denotes a constrained equilibrium whose allocation converges to the stationary allocation. According to Lemma 1.1, there is no loss of generality in assuming that $\beta^{-(t-1)} p_t$ converges to p . We will refer to the stationary equilibrium and the converging equilibrium, respectively.

For each worker and capitalist, $\beta^{-(t-1)} p_t^+ \cdot x_t^j$ is converging to a limit z^j , and for each capitalist, $\beta^{-(t-1)} p_t^- \cdot \omega_t^{h-}$ converges. The argument of Theorem 4.2's proof shows that all active firms are profit maximizing and earning 0 profits. It remains only to show that a vanishing firm could not make positive profits if it became active in the limit, that is, that there is no stationary nonzero production plan with gross rate of return greater than 1.

The Euler equation holds along any equilibrium path. Thus

$$v_z^h(p_t, z_t^h) = v_z^h(p_1, z_1) \left(\beta^{t-1} \prod_{\tau=1}^{t-1} r_\tau^h \right)^{-1}$$

where z_t is the expenditure on date t consumption goods and r_t^h is the gross rate of return on a dollar invested in firm h for one period at date t . Since indirect utility is homogeneous of degree 0, its partial derivatives are homogeneous of degree -1 . Consequently for any capitalist h ,

$$\lim_t v_z^h(\beta^{-(t-1)} p_t, \beta^{-(t-1)} z_t^h) = \lim_t \frac{1}{\prod_{\tau=1}^{t-1} r_\tau^h} v_z^h(p_1, z_1^h).$$

The price sequence $\beta^{-(t-1)} p_t$ converges to p . If firm h vanishes, then

$$0 \leq \lim_t \beta^{-(t-1)} z_t^h \leq \lim_t \beta^{-(t-1)} p_t^+ \omega_t^{h+} = p^+ \cdot 0 = 0.$$

The Inada condition on utility functions then implies that the left hand limit of the iterated Euler equation is $+\infty$ for vanishing firms. Therefore $\lim_t \prod_{\tau=1}^t r_\tau^h = 0$, that is, the long-run gross rate of return on investment in firm h is 0.

Finally, observe that $r_t^h = p_{t+1}^+ \cdot \omega_{t+1}^{h+} / p_t^- \cdot \omega_t^{h-}$ converges to $r^h = \beta p^+ \cdot \omega^{h+} / p^- \cdot \omega^{h-}$, the rate of return for firm h in the stationary equilibrium. If firm h

is inactive in the stationary equilibrium, it is vanishing in the converging equilibrium. But if $r^h > 1$, then for t large enough, $r_t^h > 1$, which contradicts the conclusion of the previous paragraph that the long-run gross rate of return on firm h investment is 0. ■

Proof (Theorem 6.1). We show that in an RECFE markets are dynamically complete and all operating firms maximize (unconstrained) profits. Thus the equilibrium allocation is a complete markets competitive equilibrium allocation (Definition 2.2).

By Assumption D there is, for each technology, at least one firm that maximizes profit. All capitalists have rational expectations so clearly no non-maximizing firm will receive any investment. We thus ignore such firms. Clearly in any period all operating firms offer the same rate of return, $r_{kt}^* = r_t^*$.

LEMMA A.2. $r_t^* = g_t^*$.

Proof. Suppose at some date t , $r_t^* > g_t^*$. Let $(x_t^{h*}, \alpha_t^{h*}, s_t^{h*}, l_t^{h*})_{t=1}^\infty$ be the optimal plan for capitalist h . Consider the plan $(\hat{x}_t^h, \alpha_t^{h*}, \hat{s}_t^h, \hat{l}_t^h)_{t=1}^\infty$ which agrees with the supposed optimal plan at all dates other than t and $t+1$ and has: $\hat{l}_t^h = l_t^{h*} - 1$, $\hat{s}_t^h = s_t^{h*} + 1$, $\hat{x}_t^h = x_t^{h*}$, $\hat{l}_{t+1}^h = l_{t+1}^{h*}$, $\hat{s}_{t+1}^h = s_{t+1}^{h*}$, and $\hat{x}_{j,t+1}^h = x_{j,t+1}^{h*} + (r_t^* - g_t^*) / (p_{j,t+1}^* \text{Con})$ for each good j . This plan is clearly feasible and has a higher value than the supposed optimal plan. So $r_t^* \leq g_t^*$.

Now suppose that at some time t , $r_t^* < g_t^*$. Assumption I implies that in an RECFE all consumer goods are produced and so some firm(s) operate. Each active firm has an investor. The reverse of the argument above implies that for such investors there is an alternative feasible plan that has a higher value than their supposed optimal plan. So $r_t^* \geq g_t^*$. ■

DEFINITION A.1. The present value of profits from $(\omega_{t-1}^{k-}, \omega_t^{k+})$ is

$$\pi_t^k = \frac{p_t^{k+*} \cdot \omega_t^{k+}}{g_t^*} - p_{t-1}^{k-*} \cdot \omega_{t-1}^{k-}.$$

LEMMA A.3. $\pi_t^k = 0$ for all k, t .

Proof. For any inactive firm profits are clearly 0. Suppose that firm k is active. Then from Lemma A.2 and the observation that all active firms earn the same rate of return,

$$g_t^* = r_t^* = \frac{p_t^{k+*} \cdot \omega_t^{k+}}{p_{t-1}^{k-*} \cdot \omega_{t-1}^{k-}}.$$

Therefore,

$$\pi_t^{k*} = \frac{p_t^{k+*} \cdot \omega_t^{k*+}}{g_t^*} - p_{t-1}^{k-*} \cdot \omega_{t-1}^{k*-} = 0.$$

■

Thus in an RECFE, all constrained profit-maximizing firms are unconstrained profit maximizers with respect to present value prices,

$$q_t^* = \left(\frac{1}{\prod_{\tau=1}^t g_\tau^*} \right) p_t^*.$$

Worker i 's RECFE budget set can be written as:

$$B^i(p^*, g^*) = \left\{ x^i: x_t^i \in C \text{ for all } t \text{ and there exists } \{l_t^i\}_{t=1}^\infty \text{ such that } p_t^{+*} x_t^i + l_t^i \leq m_t^i = p_t^{-*} e^i + g_t^* l_{t-1}^i \text{ for all } t, l_0 = 0, \text{ and } \liminf_t \frac{l_t^i}{\prod_{\tau=1}^t g_\tau^*} \geq 0 \right\}.$$

DEFINITION A.2. The complete markets budget set for worker i is:

$$\hat{B}^i(p^*, g^*) = \left\{ x^i: x_t^i \in C \text{ for all } t \text{ and } \liminf_T \sum_{t=1}^T q_t^{*-} \cdot e^i - \sum_{t=1}^T q_t^{+*} \cdot x_t^i \geq 0 \right\}.$$

LEMMA A.4. $B^i(p^*, g^*) = \hat{B}^i(q^*, g^*)$ for all workers i .

Proof. $\hat{B}^i \subset B^i$:

Let $x^i \in \hat{B}^i(q^*, g^*)$. Define $z_t^i = p_t^{-*} \cdot e^i - p_t^{+*} \cdot x_t^i$. Then

$$\liminf_T \sum_{t=1}^T \frac{z_t^i}{\prod_{\tau=1}^t g_\tau^*} \geq 0.$$

Define $l_0^i = 0$ and $l_t^i = z_t^i + g_t^* l_{t-1}^i$ for all $t \geq 1$. Note that for any T ,

$$\frac{l_T^i}{\prod_{\tau=1}^T g_\tau^*} = \sum_{t=1}^T \frac{1}{\prod_{\tau=1}^t g_\tau^*} z_t^i.$$

Thus $x^i \in B^i(p^*, g^*)$ and is supported by the consumption loan sequence l^i . $B^i \subset \hat{B}^i$:

For any $x^i \in B$ there is a consumption loan sequence l^i satisfying the constraints in B . Substitution shows that for any T ,

$$\frac{l_T^i}{\prod_{\tau=1}^T g_\tau^*} = \sum_{t=1}^T \left(\frac{1}{\prod_{\tau=1}^t g_\tau^*} \right) (p_t^{-*} \cdot e^i - p_t^{+*} \cdot x_t^i).$$

Thus $x^i \in \hat{B}^i$. ■

It follows from the observation that all active firms in period t earn rate of return r_t^* and Lemma A.2 that the CFE budget set for capitalist h with rational expectations can be written as:

$$B^h(p^*, g^*) = \left\{ x^h: x_t^h \in C \text{ for all } t \text{ and there exists } \{l_t^h, s_t^h\}_{t=1}^\infty \text{ such that} \right. \\ \left. p_t^{+*} \cdot x_t^h + s_t^h + l_t^h \leq m_t^h = r_t^* s_{t-1}^h + g_t^* l_{t-1}^h \text{ with } s_t^h \geq 0 \text{ for all } t \geq 1, \right. \\ \left. m_1^h \text{ given, and } \liminf_t \frac{l_t^h}{\prod_{\tau=1}^t g_\tau^*} \geq 0 \right\}.$$

DEFINITION A.3. The complete markets budget set for capitalist h is

$$\hat{B}^h(p^*, g^*) = \left\{ x^h: x_t^h \in C \text{ for all } t \text{ and} \right. \\ \left. \limsup_T \sum_{t=1}^T q_t^{+*} \cdot x_t^h \leq q_1^{+*} \cdot \sum_k \omega_1^{hk+*} \right\}.$$

LEMMA A.5. $B^h(p^*, g^*) = \hat{B}^h(q^*, g^*)$ for all capitalists h .

Proof. See the proof of Lemma A.4. ■

As each consumer faces the complete markets budget set, and each firm maximizes unconstrained profits, any RECFE allocation is a competitive equilibrium allocation. ■

ACKNOWLEDGMENTS

The authors are grateful for the hospitality and support of the Santa Fe Institute. We thank seminar participants at Columbia, Carnegie-Mellon, Cornell, UCLA, Washington University, and Wisconsin for their comments and suggestions.

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