ERRATUM TO 'SMOOTHNESS OF NORMALISERS [...]'

There is an error in the statement of [HS16, Thm. 8.9(c)]. One must also include the case $\lambda = \mu = p-1$ when G contains a factor of type A_1.

The reason this case was missed is due to an error in the proof: the theorem [BNP04, Thm. 3.1(A,B)] does not apply directly to the module M since the high weights in M are not necessarily restricted (M is a tensor product of simple modules in $\bar{C}_{\mathbb{Z}}$).

In order to correct the lemma using [BNP04, Cor. 3.2(a)(b)] (which does apply) one should also exclude the prime $p = 3$; then the statement of [HS16, Thm. 8.9(c)] should read:

(c) If $p > 3$ is very good for G and $\lambda, \mu \in \bar{C}_{\mathbb{Z}}$ we have $\mathrm{Ext}^1_{\mathfrak{g}}(L(\lambda), L(\mu)) = 0$, or G contains a factor of type A_1, $L(\lambda)$ and $L(\mu)$ are simple modules for that factor and either (i) $\lambda = s < p-1$, $\mu = p-2-s$ and we have $\mathrm{Ext}^1_{\mathfrak{g}}(L(\lambda), L(\mu))^{[-1]} = L(1)$; or (ii) $\lambda = \mu = p-1$ and we have $\mathrm{Ext}^1_{\mathfrak{g}}(L(\lambda), L(\mu)) \cong (\mathfrak{g}^*)^{[1]}$.

(This change does not affect any of the rest of the paper.) The proof of [HS16, Thm. 8.9(c)] is correct up to the sentence which cites this theorem. That sentence and its sequel to the end of the proof should be replaced by:

Proof. In order to have $(\mathfrak{g}^*)^{[1]}$ a composition factor of $\mathrm{H}^1(G_1, H^0(\nu))$, we would need $\mathfrak{g} \cong \mathfrak{g}^* \cong H^0(\omega)$ where $\nu = p\omega - \alpha$ for $\omega \in X(T)$ and α a simple root by [BNP04, Cor. 3.2(a)(b)].

If $\mathfrak{g}$ is of type A_1, this implies that $\nu = 2p-2$ and so $\lambda = \mu = p-1$. In this case the terms in the exact sequence

$$\mathrm{H}^1(G_1, M) \to \mathrm{H}^1(\mathfrak{g}, M) \to \mathrm{Hom}^s(\mathfrak{g}, M^{\mathfrak{g}}) \to H^2(G_1, M)$$

applied to $M = L(p-1) \otimes L(p-1)$ are as follows:

$$\mathrm{H}^1(G_1, M) \cong \mathrm{Ext}^1_{G_1}(L(p-1), L(p-1)) = 0 = \mathrm{Ext}^2_{G_1}(L(p-1), L(p-1)) \cong \mathrm{H}^2(G_1, M)$$

as $L(p-1)$ is projective as a G_1-module, and $\mathrm{Hom}^s(\mathfrak{g}, M^{\mathfrak{g}}) \cong (\mathfrak{g}^*)^{[1]}$ since $M^{\mathfrak{g}} = \mathrm{Hom}_{\mathfrak{g}}(L(p-1), L(p-1)) \cong k$. Hence there is an isomorphism

$$\mathrm{Ext}^1_{\mathfrak{g}}(L(\lambda), L(\mu)) \cong (\mathfrak{g}^*)^{[1]}.$$

For the remaining types, we have

Type	A_n	B_2	B_n,C_n	D_n
$\mathfrak{g} \cong L(\omega)$ for $\omega =$	$\omega_1 + \omega_n$	$2\omega_2$	ω_2	ω_2
$\langle p\omega_\alpha - \alpha, \alpha_0^\vee \rangle$	$2p$ or $2p-1$	$4p$ or $4p-2$	$2p$, $2p-1$ or $2p-2$	$2p$ or $2p-1$

Type	E_6	E_7	E_8	F_4	G_2
$\mathfrak{g} \cong L(\omega)$ for $\omega =$	ω_2	ω_1	ω_8	ω_1	ω_2
$\langle p\omega_\alpha - \alpha, \alpha_0^\vee \rangle$	$2p$ or $2p-1$	$2p$ or $2p-1$	$2p$ or $2p-1$	$2p$ or $2p-1$	$3p$ or $3p-1$

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On the other hand, since $\lambda \in \bar{C}_{\mathbb{Z}}$ it satisfies $\langle \lambda + \rho, \alpha_0^\vee \rangle \leq p$, i.e. $\langle \lambda, \alpha_0^\vee \rangle \leq p - h + 1$. Hence any high weight μ of $M = L \otimes L^*$ satisfies $\langle \mu, \alpha_0^\vee \rangle \leq 2p - 2h + 2$. Looking at the above table, it is easily seen that this is a contradiction. Thus $(\mathfrak{g}^*)^{[1]}$ is not a composition factor of $\mathrm{H}^1(G_1, M/k)$ and the result follows. ☐

References

[BNP04] Christopher P. Bendel, Daniel K. Nakano, and Cornelius Pillen, *Extensions for finite Chevalley groups. I*, Adv. Math. **183** (2004), no. 2, 380–408. MR 2041903 (2004m:20095)

[HS16] Sebastian Herpel and David I. Stewart, *On the smoothness of normalisers, the subalgebra structure of modular Lie algebras, and the cohomology of small representations*, Doc. Math. **21** (2016), 1–37. MR 3465106