ERRATUM TO 'MAXIMAL SUBALGEBRAS OF CARTAN TYPE IN THE EXCEPTIONAL LIE ALGEBRAS'

There is an error in the statement of [HS16, Lemma 2.8]. The first subalgebra $W \cong W_1 \subseteq H = H(2;\Phi(\tau))^{(1)}$, whose basis is given in the lemma is not a p-subalgebra. This is because, for the first element in the basis—the element representing ∂ in W_1—namely, $(1 - X^{(p-1)}Y^{(p-1)})\partial_X \in H$, one has the identity

$$\Big((1 - X^{(p-1)}Y^{(p-1)})\partial_X\Big)^{[p]} = -Y^{(p-1)}\partial_X,$$

which is not 0, and not an element of W. In fact it is the element 'Θ' as in [FSW14, (5.6)] and we shall refer to it as such in this erratum.

Now, one has that the p-closure, $W_p = W \oplus \langle\Theta\rangle$ and one has $\Theta^{[p]} = 0$. We check that this does not affect the remainder of the paper.

Observation 0.1. Suppose H and W are restricted Lie algebras, with W a subalgebra of H such that the p-closure $W_p \cong W \oplus \mathfrak{n}$ where $\mathfrak{n}$ is a subspace of H consisting of p-nilpotent elements which commute with H. Then for any restricted representation V of H, we have that the restriction of V to W has restricted composition factors.

Proof. Let U be a simple W-submodule of V. As V is restricted, $\mathfrak{n}$ acts nilpotently on V, hence by Schur's lemma, $\mathfrak{n}$ acts trivially on U. Thus the image of W in $\mathfrak{gl}(U)$ is restricted. This is to say that U is a restricted representation for W. The general result follows by induction after factoring out U. □

From the observation it follows that the statement of Lemma 2.9 can remain the same. Each representation under consideration is restricted for the minimal p-envelope Z of H. The calculations in the proof for the case of the adjoint representation find vectors killed by the action of ∂ and their $X\partial$ weights, where ∂ and $X\partial$ represent the usual elements in W. By Observation 0.1 any simple submodule is restricted, hence must contain such an vector and the $X\partial$ weight on it determines the isomorphism type of the simple restricted submodule, thus the conclusion in this case can remain the same.

The arguments finding the restriction to W of Verma modules $M(r)$ with $r \geq 2$ can be improved. Since we are only interested in the composition factors of $M(r)|W$ we may, by Observation 0.1 assume in the ensuing calculations that the element representing ∂, that is $x = (1 - X^{(p-1)}Y^{(p-1)})\partial$, acts such that $x^p = 0$, since it will do so on any simple submodule. In particular it does no harm to assume that x kills $x^{p-1}y^b \otimes v_i$, for all $0 \leq a \leq p-1$, $i \in \{-r, -r+2, \ldots, r\}$. Thus $M(r)$ can be graded with $x^a y^b \otimes v_i$ in grade $2a$ and one has that $x(M(r)(2a)) \subseteq M(r)(2a+2)$. As also each $x^a y^b \otimes v_i$ is a weight vector for $h = X\partial_X - Y\partial_Y$ representing $X\partial$ in W, with weight $i - a + b$, we have that $M(r)$ as a graded $\langle x, h\rangle$ module is identical to its analogue in the context of the proof of Lemma 2.6. Then Lemma 2.1 and Proposition 2.2 guarantee that the composition factors as a W-module are the same as those given in 2.6, (which concur with those in Lemma 2.9).

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Now let us recall the main strategy to prove that there are no Hamiltonian-type subalgebras of $\mathfrak{g}$, if $\mathfrak{g}$ is exceptional in good characteristic. The lists of all possible composition factors of $\mathfrak{g}|W$ are given in Tables 3 and 5. It is straightforward to compare these with those coming from Lemmas 2.6 and 2.9 and to see that this is incompatible with any W_1 subalgebra of any Hamiltonian Lie algebra.

References

[FSW14] Jörg Feldvoss, Salvatore Siciliano, and Thomas Weigel, *Restricted lie algebras with maximal 0-pim.*

[HS16] Sebastian Herpel and David I. Stewart, *Maximal subalgebras of Cartan type in the exceptional Lie algebras*, Selecta Math. (to appear) (2016).