Going Round the Bend

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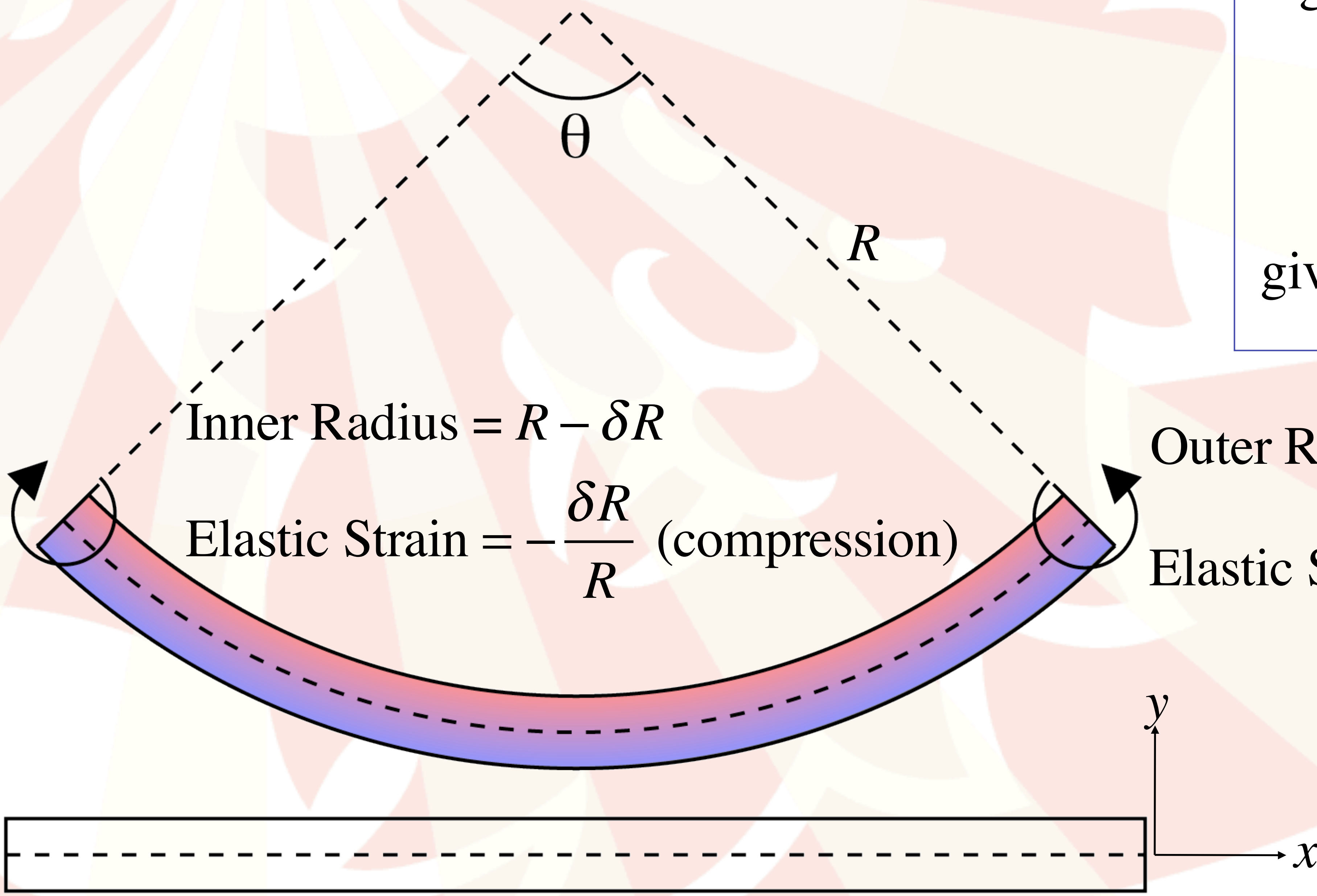
Elastic Bar in Pure Bending



Inner Radius $= R - \delta R$

Elastic Strain $= -\dfrac{\delta R}{R}$ (compression)

Outer Radius $= R + \delta R$

$$\text{Elastic Strain} = \frac{(R+\delta R)\theta - R\theta}{R\theta} = \frac{\delta R}{R} \text{ (tension)}$$

Original length (and length of neutral axis), $L = R\theta$

In general, $\varepsilon_x = \dfrac{-y}{R}$

$$\sigma_x = E\varepsilon_x = \frac{-yE}{R}$$

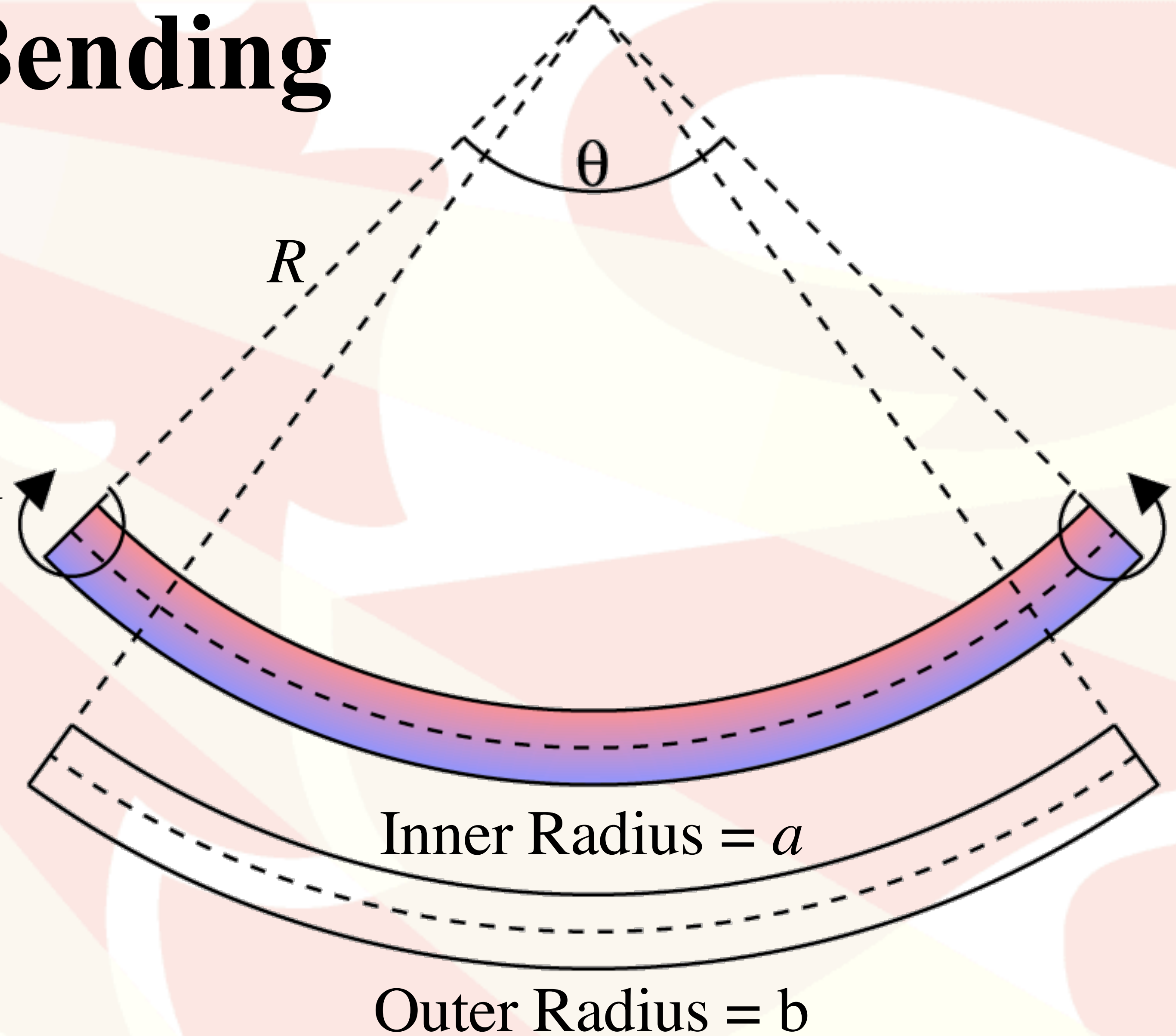
giving $-\dfrac{\sigma_x}{y} = \dfrac{E}{R}$

Curved Bar in Pure Bending

A bar with inner radius *a* and outer radius *b* has constant bending moment *M*, and constant rectangular cross-section.

The stresses in the bar are axially symmetric (or can be - it depends on how the moment is applied at the end), i.e., functions of *r* only.

Exact solution (see Timoshenko & Goodier, 'Theory of Elasticity'), where *M* is ***moment per unit thickness*** of the bar:



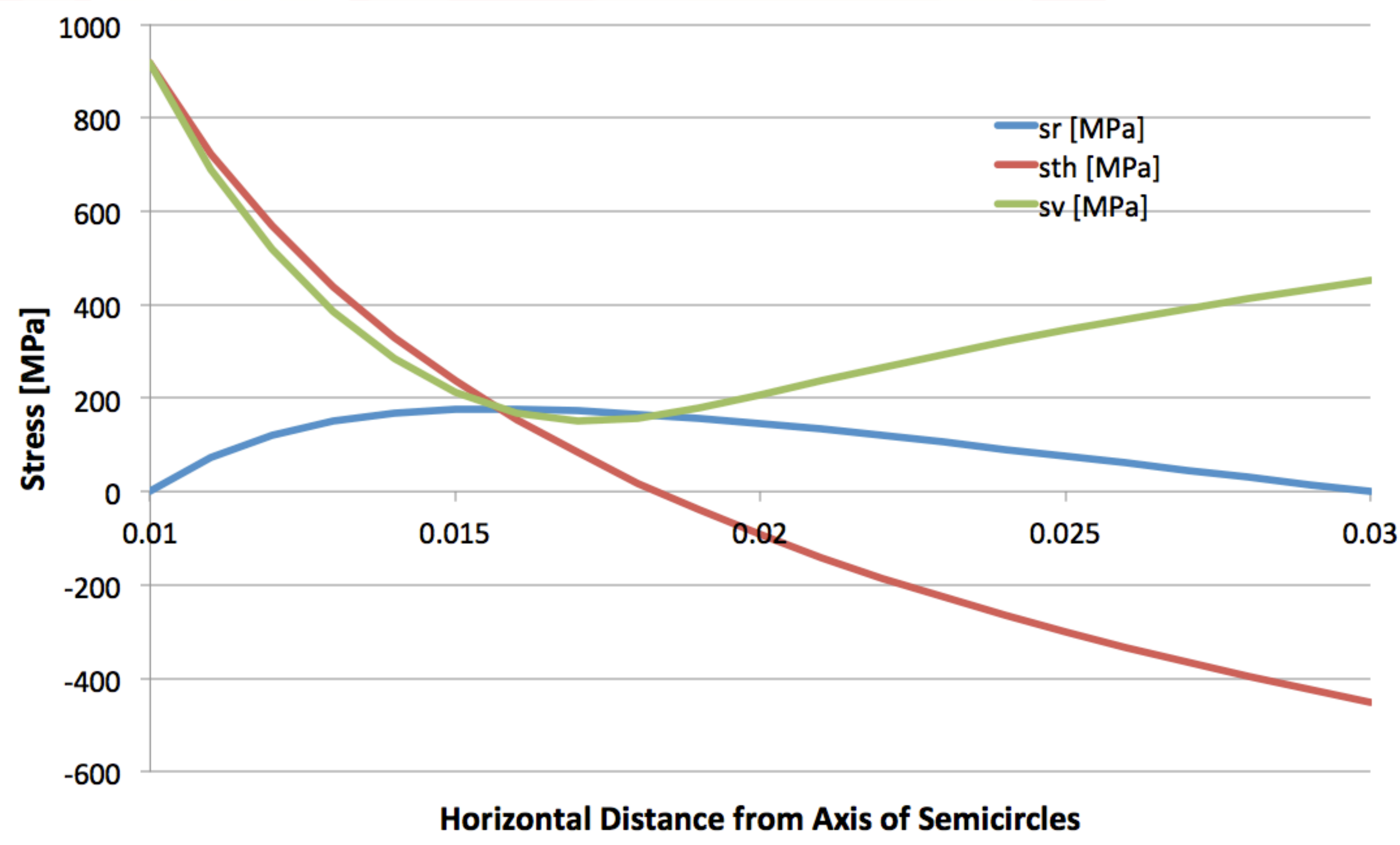
$$\sigma_r = -\frac{4M}{N}\left(\frac{a^2b^2}{r^2}\log\frac{b}{a} + b^2\log\frac{r}{b} + a^2\log\frac{a}{r}\right), \text{ where } N = \left(b^2 - a^2\right)^2 - 4a^2b^2\left(\log\frac{b}{a}\right)^2$$

$$\sigma_\theta = -\frac{4M}{N}\left(-\frac{a^2b^2}{r^2}\log\frac{b}{a} + b^2\log\frac{r}{b} + a^2\log\frac{a}{r} + b^2 - a^2\right)$$

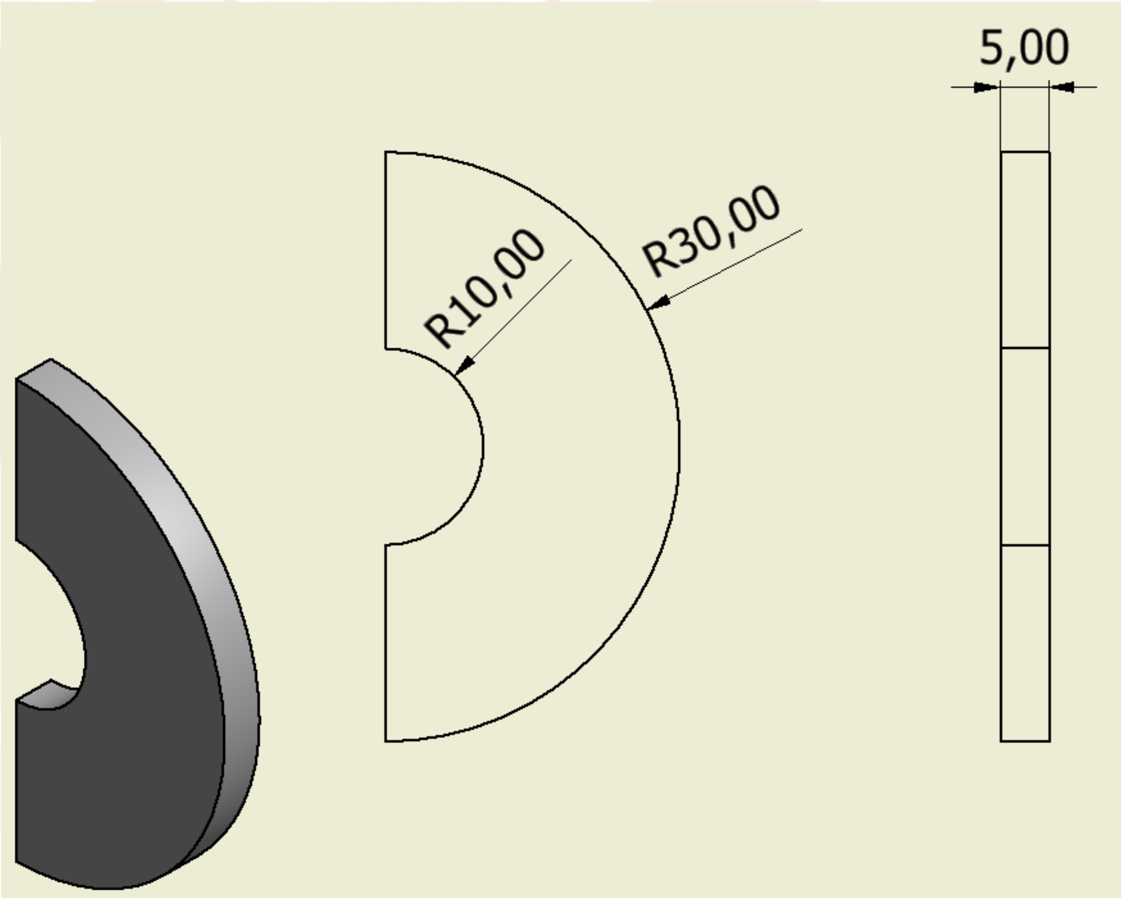
$$\tau_{r\theta} = 0$$

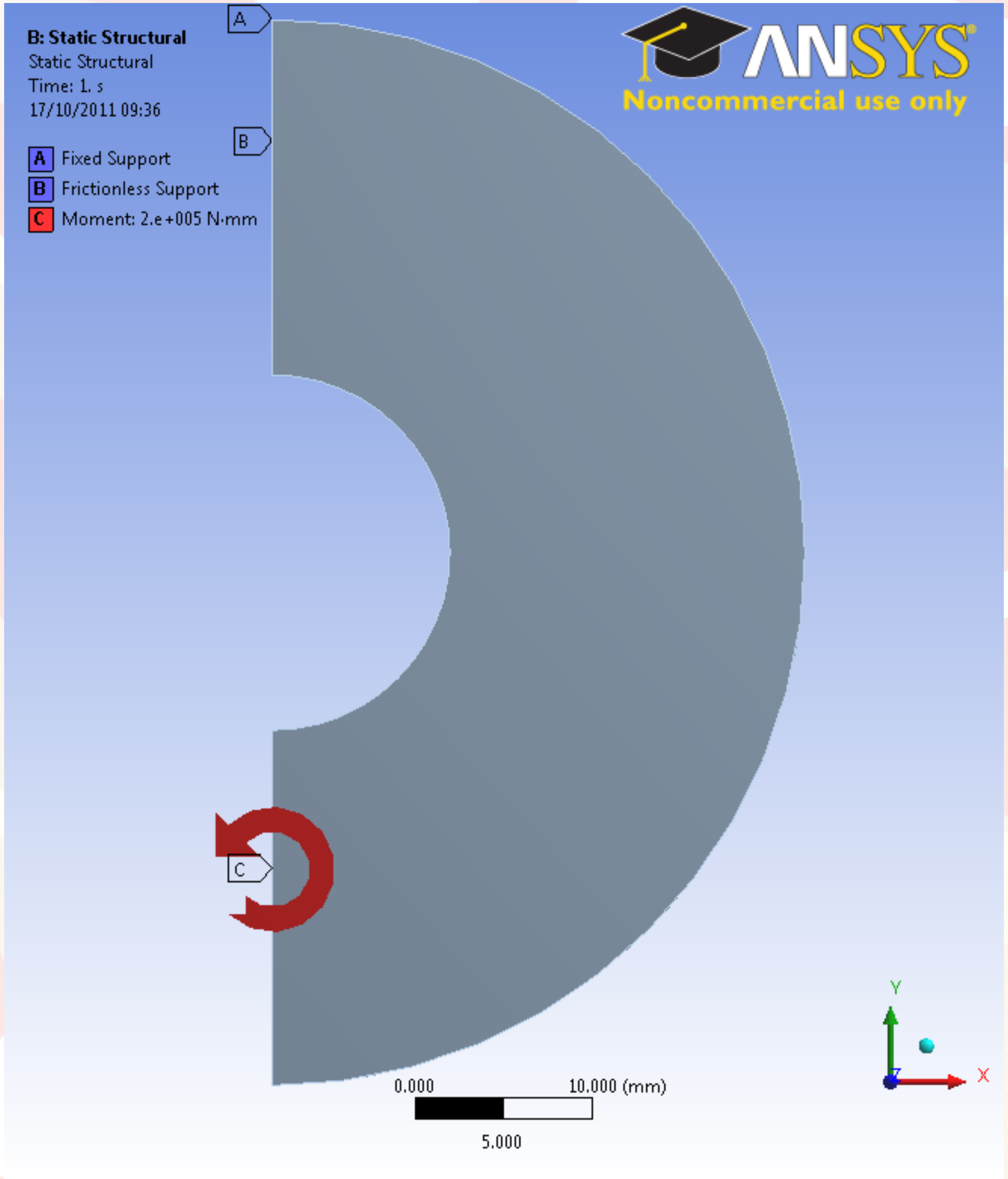
Curved Bar in Pure Bending

For the example on the next slide, these lead to the following stresses:



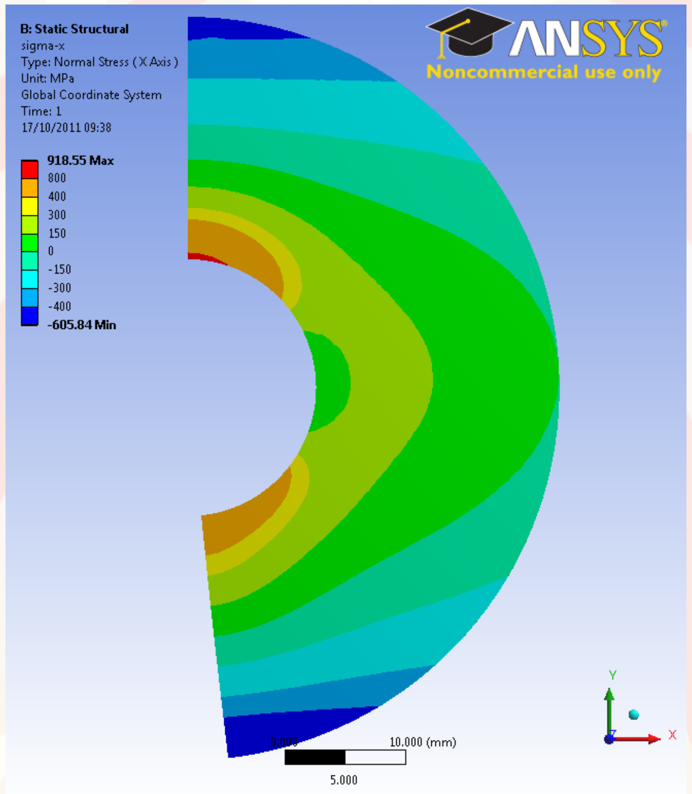
Curved Bar in Pure Bending

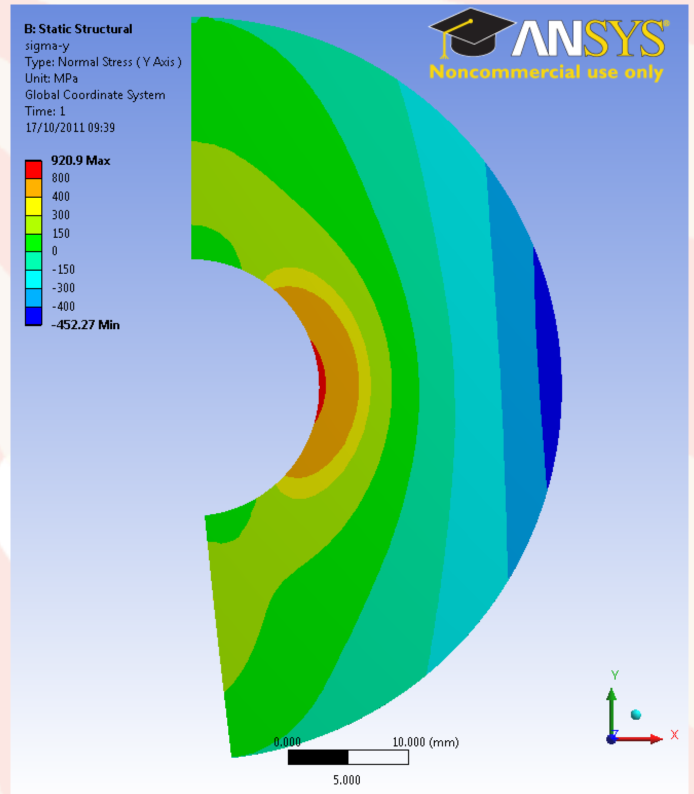


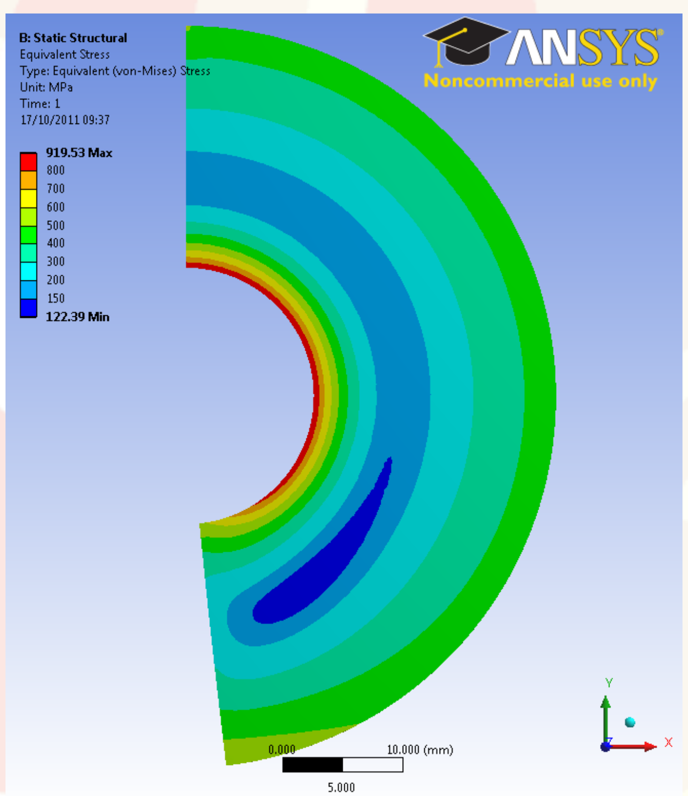


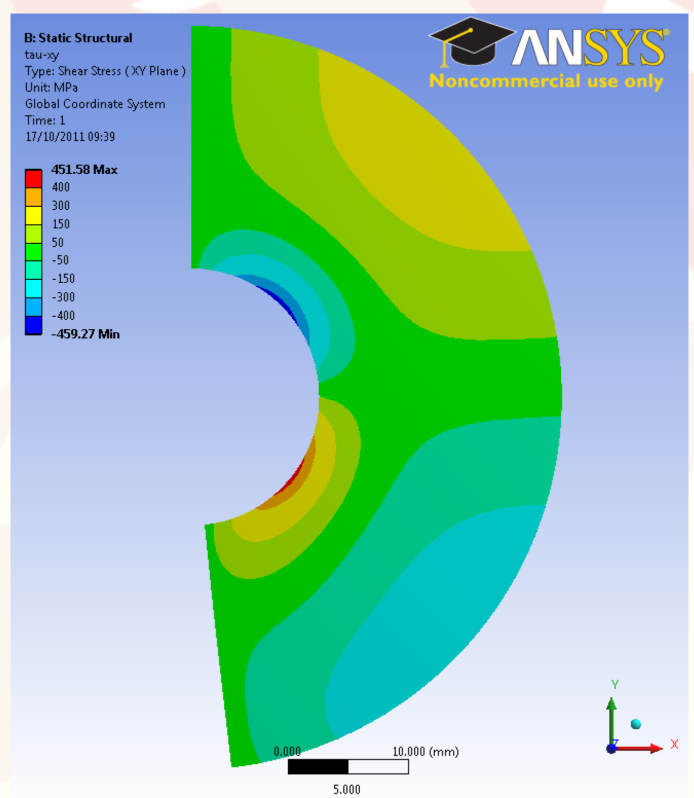
Semicircular bar, fixed at the top edge, with a frictionless support on the upper flat surface, and a moment applied at the lower flat surface.

Curved Bar in Pure Bending









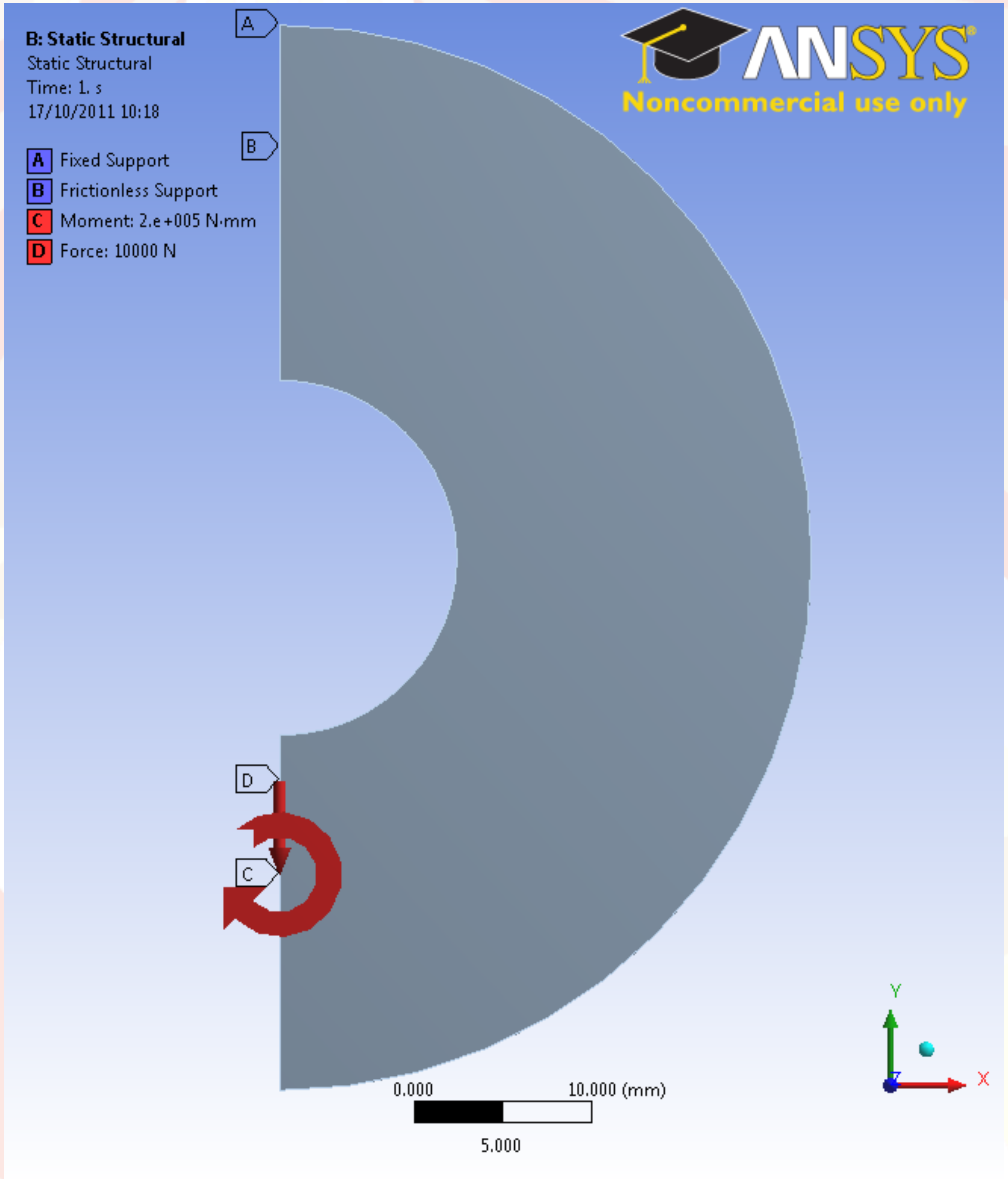
Notes

The shear stress is zero on the horizontal section at half-height.

The asymmetry (most apparent in the equivalent stress contours, bottom left) is caused by the way the moment is applied on the lower flat surface.

The vertical normal stress (upper right) is large and compressive at the outside edge, and large and tensile on the inner edge.

Curved Bar in Bending + Opposite Moment



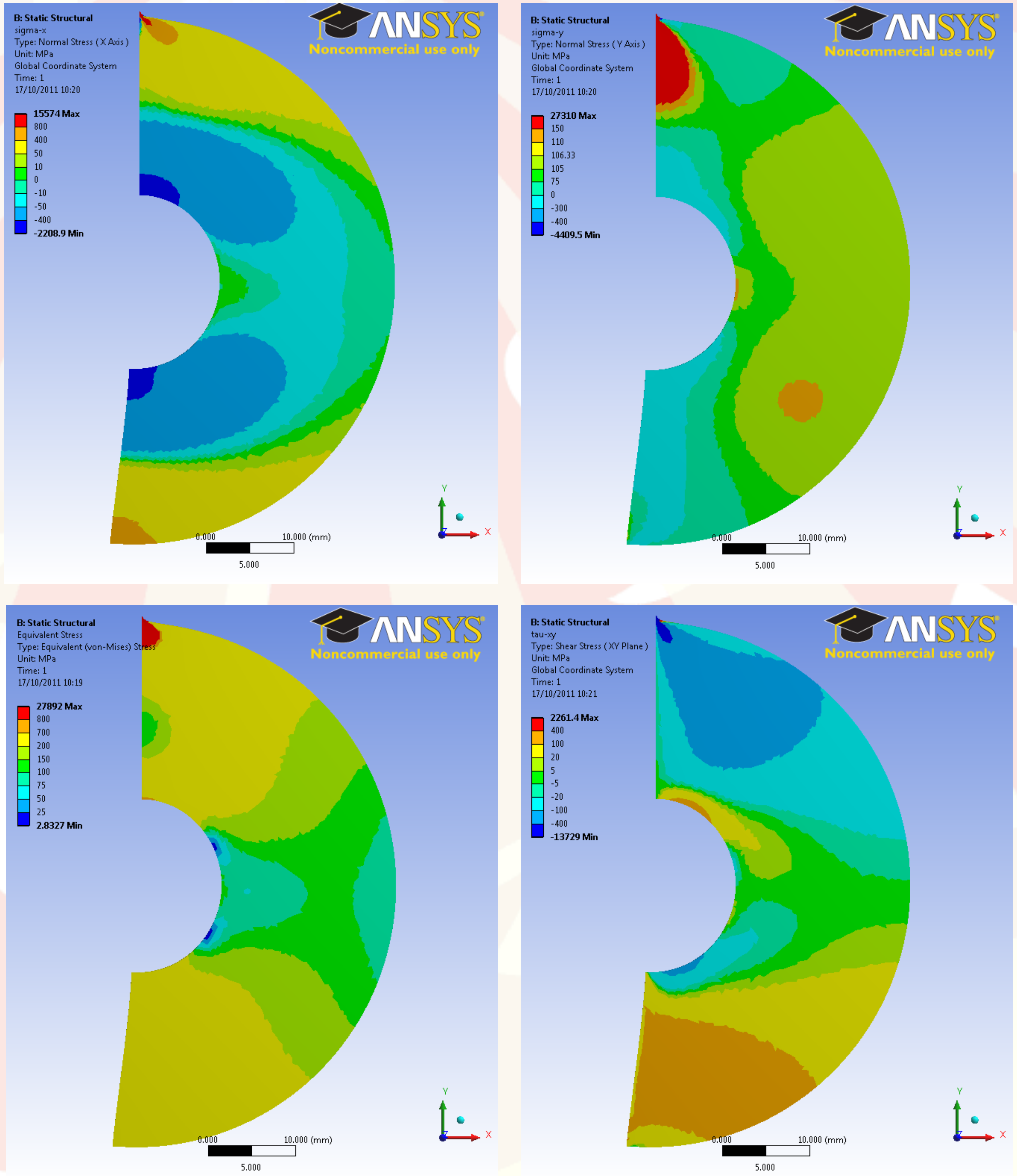
In this case, the previous moment is reversed, and a vertical load downwards is applied also.

The moment applied by the vertical force around the centre of the horizontal section at half-height, is 10kN × 20mm = 200Nm, i.e., the bending moment along the section should be zero.

Horizontal Section at Half-Height

Essentially, this combination shows the stresses induced by the applied force with the bending effects removed. The vertical stress (top right on the next slide) on the horizontal section is fairly uniform, suggesting that a reasonable approximation for the stresses on this section is to calculate stresses for pure bending, and add the mean axial stress.

Curved Bar in Bending + Opposite Moment



The high stresses in the top corner are caused by the fixed constraint on the top edge.

Horizontal Section at Half-Height

The horizontal normal stress (top left) is small and mostly negative, i.e., slightly compressive.

The vertical normal stress (top right) is approximately 106MPa across most of the section with no high stress peaks.

The applied load is 10kN and the cross-sectional area is $20mm \times 5mm = 100mm^2$, so the average vertical stress should be 100MPa.

The equivalent (von Mises) stress is in the range 75-150MPa.

The shear stress is zero, or very close.