# HOW BAD CAN SHORT TERMISM BE? – A STUDY OF THE CONSEQUENCES OF HIGH HURDLE DISCOUNT RATES AND LOW PAYBACK THRESHOLDS

by

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## ABSTRACT

Survey evidence suggests that the hurdle rates imposed for *DCF* analysis are often considerably in excess of any plausible estimate of firms' cost of capital, and that top level decision makers often explicitly or implicitly impose additional short payback thresholds. This paper focuses on the value loss that can arise under such 'short termist' decision criteria. It is shown that using such decision rules can help to protect the firm against the total value loss that can arise from the application of the naïve *NPV* decision rule, and that, for projects with growth prospects *and/or* moderate or greater volatility in future operating cash flows, the value loss (relative to 'optimal decision making') which arises when firms impose fixed 'short termist' thresholds can be quite small.

#### 1. INTRODUCTION

Surveys in the academic and professional literature suggest that, particularly amongst medium to large scale enterprises, discounted cash flow (*DCF*) techniques are now widely practiced in capital budgeting (Klammer and Walker [1984], Sangster [1993], Trahan and Gitman [1995], Drury and Tayles [1997], Abdel-Kader and Dugdale [1998], Godden [2001], Alkaraan and Northcott [2006]). However, it appears that the discount rate used in such *DCF* appraisal, the so called hurdle rate, is typically set significantly higher than the firm's cost of capital, with rates 5% or more above conventional estimates of the cost of capital (Hayes and Garvin [1982], Allen [1996], Schall et al [1978], Barker [1999], Adler [2000], Godden [2001], Anderson and Newell [2003], Meier and Tarhan [2006]). At the same time, the survey evidence also suggests that non-discounting measures remain popular, with additional short payback thresholds of 2-4 years often explicitly or implicitly applied by top management when considering capital budgeting proposals (Weingartner [1969], Kee and Bublitz [1988], Allen [1996], Busby and Pitts [1997], Kaplan and Atkinson [1998], Abdel-Kader and Dugdale [1998], Adler [2000], Arnold and Hatzopoulos [2000]).

The concern over apparent '*short termism*' is that it seems likely to lead to significant value loss in the context of the appraisal of long lived projects. That is, short termism seems particularly likely to reject projects with long horizon returns. However, the short termism manifest in high hurdle rates and low payback thresholds may be to an extent rationalised. It can be viewed as a crude way of adjusting for excessive optimism concerning project managers' cash flow forecasts, or because of scarcity of managerial talent, or because of financing/liquidity constraints, whether market or self imposed (Statman and Sepe [1984], Narayanam [1985], Pike [1985, 1996], Kaplan

[1986], Kaplan and Atkinson [1998], Jagannathan and Meir [2002]). Essentially, investment budgets or limited managerial capacity imply that if one accepts a project that is only marginally positive *NPV*, this may foreclose the option to undertake a better project that may come along later. In such a case, given this opportunity cost, a project needs to have sufficient positive *NPV* to be worth undertaking. Setting a higher hurdle rate, or shorter payback threshold can be viewed as a way of ensuring this. The same argument applies to the option to undertake it at a later date (when undertaking an investment forecloses the option to undertake it at a later date (when interest rates might be lower, or cash flow forecasts higher etc.). Again, it makes sense to require that projects should be activated only when they earn an *IRR* significantly above the cost of finance, or have positive *NPV* when this is measured at a higher hurdle rate, or more crudely, have a shorter payback period.

The real options explanation for apparent short termist decision criteria is now fairly well understood and the dependence of the optimal hurdle rate on key parameters (including environmental variables such as interest rates, but also project specific parameters such as expected cash flow growth rate and volatility) has been examined (Ingersoll and Ross [1992], Dixit and Pindyck [1994], Ross [1995]) as has the dependence of an optimal payback threshold on such parameters (Wambach [2000], Boyle and Guthrie [2006]). This literature has shown that optimal payback thresholds ought to be shorter, and optimal hurdle rates ought to be higher when real option effects are present. It also shows that 'by how much' depends on project specific characteristics.

However, survey evidence concerning actual practice also suggests that hurdle rate and payback thresholds are adjusted only to a very limited degree, if at all, across projects with differing characteristics (Wardlaw [1994], Busby and Pitts [1997]). This suggests a degree of 'sub-optimality' in decision making; that is, relative to optimal decision making, a degree of value loss will arise if such thresholds are applied uniformly across a range of projects with varying individual characteristics. The natural question to ask then is whether the *value loss that is likely to arise from imposing fixed hurdle rates and payback thresholds is likely to be significant or not*. If, as argued in this paper, the value loss is not significant, this may help explain the persistence of such decision criteria over time.

The above cited literature on optimal thresholds (hurdles and paybacks) has not directly addressed this value loss question, although two papers have some indirect bearing on it. Wambach [2000] uses the same 'standard investment model' adopted in this paper, focuses on the *discounted* payback threshold, and shows that this threshold does not vary much when project characteristics are varied. Whilst this observation is of interest, it should be noted that it does not answer the above value loss question; the point is that insensitivity of the optimal payback threshold to variations in parameters does not in itself indicate anything about the extent of value loss that can arise from setting a *fixed* and *non-optimal* hurdle rate or payback threshold. In particular, it gives no indication of the extent of value loss that might arise from the observed practice of setting too high a hurdle rate and too short a payback threshold. Jagannathan and Meir [2002] also use the standard investment model, but have a rather different perspective; their focus is on imprecision in *CAPM* based estimates of the cost of capital. They argue that imprecision in estimation of

the cost of capital is not such a problem as at first sight – because projects typically have option characteristics. They show that the *optimal* hurdle rate implied by the standard investment model is relatively insensitive to the number used for the cost of capital. That is, value loss arising from errors in the cost of capital, when computing and implementing an optimal hurdle rate, are likely to be small. Again, this leaves open the question addressed in the present paper, concerning value loss from the use of fixed and non-optimal decision criteria.

A further contribution of the present work lies in the fact that it also considers the use of *multiple* thresholds; the survey literature cited above shows that it is common practice for firms to set more than one threshold, and in particular, firms that routinely use a hurdle rate of discount often also impose an additional payback threshold. That is, top level decision makers often explicitly or implicitly impose additional short payback thresholds (Weingartner [1969], Wardlaw [1994]). In principle, projects may fail on either test; indeed, one would expect that which threshold proves to be the tighter may well depend on project characteristics.<sup>1</sup>

The paper also examines the value loss that rises from application of the "naïve" net present value (*NPV*) rule. Rene Stultz [1999] has argued that, whilst most business education will cover the *NPV* rule, there may be doubts as to the extent to which students are exposed to, or understand, the weaknesses of using this rule. He suggests the average MBA student understanding is that projects with positive *NPV* at an appropriate *CAPM* based risk adjusted discount rate should be accepted. Stultz focuses on the idea that not only systematic risk matters - total risk also often matters.

<sup>&</sup>lt;sup>1</sup> That this is in fact the case is demonstrated in section 3 below.

However, the naïve *NPV* rule can also perform poorly in the presence of real option effects. Evaluated at time zero, the naïve *NPV*-rule generates two kinds of errors. Firstly, it tends to indicate a project should be implemented when it is in fact value adding to delay implementation. Secondly, if it fails the test at time zero, the project is either rejected or at best is deferred to a time when it is evaluated as 'just viable', at which point it is activated but will again add negligible value. Given the above observations, it is thus of some interest to compare the value loss that arises from the use of payback thresholds and high hurdle rates with that which arises under the naïve *NPV* rule.

Clearly it is not possible to conduct an assessment of value loss with respect to the set of *all possible* investments; the present analysis deals with an analytically tractable but important subset with the following characteristics:

- (a) The investment cost or scale for an individual 'project' is fixed
- (b) Operating cash flows are uncertain; they evolve over time according to a geometric Brownian motion, with a fixed average growth rate and volatility
- (c) The project can either be implemented immediately or it can be deferred.
- (d) The investment once made is a sunk cost.

This is a '*standard investment model*' whose options characteristics are well understood and which have been discussed extensively in the literature (for an extended discussion of this model, see Dixit and Pindyck [1994]). This facilitates exposition, allowing the primary focus in what follows to be on the extension of the model to the study of how short termist decision criteria impact on value. Although somewhat restrictive, the above assumptions are not 'unreasonable'; projects often have a natural 'scale', and often involve repetitive production or service output which generates operating cash flows of this nature; project implementation is rarely a 'now or never' decision and investment costs are always, to lesser or greater extent sunk costs. Given this framework, value loss can be studied as a function of the key characteristics that vary across projects (initial cash flow, capital outlay, cash flow growth rate and volatility).

Conceptually, if a decision-maker knows the framework is that of the above standard investment model, and knows the parameter values for a given project, it is possible to then compute the optimal decision rule for this project. However, the perspective taken here is that, based on survey evidence, decision makers do not make 'optimal decisions'; they tend to impose fixed and relatively invariant short termist decision rules, independently of variations in project characteristics. The focus is thus on examining the value consequences of this behaviour.

It is shown that the imposition of a naïve *NPV* decision rule (i.e. accept projects if they have positive *NPV*, reject or defer them if they have negative *NPV*) can lead to total value loss. By contrast, short termist decision criteria tend to limit the extent of value loss – although there is often still *some* value loss. The key finding in this paper is that, for projects with growth prospects *and/or* moderate volatility in future cash flows, relative to optimal decision making, the percent value loss arising from the use of short termist decision rules is fairly low. As a consequence, whatever, the rationale decision makers have for the use of such decision rules of thumb, it would appear that if value loss from their use is relatively small, such rules can be evolutionarily stable.<sup>2</sup>

<sup>&</sup>lt;sup>2</sup> That is, the focus of the present paper is on the consequences that arise when decision-makers do what they say they do, rather than an enquiry into *why* they do what they say they do. If a decision rule appeared to be significantly inefficient, one might not expect it to survive in the long term. A similar argument is often made concerning cost plus pricing. That is, a cost plus pricing rule in which the plus was set independently of demand side considerations could be expected to perform poorly in many circumstances, and not to survive the rigours of a competitive market place. In this later case, survey evidence reveals that demand side factors do inform pricing decisions; the term 'cost plus pricing' thus mis-describes the price formation process (see e.g. Dorward [1987], Lucas [2003]).

The paper is structured as follows. Section 2 sets out the basic framework, outlines the structure of the optimal solution and examines the performance of the naïve NPV decision rule. Section 3 follows this by analysing decision making and valuation when projects are evaluated using a 'hurdle' discount rate significantly above the firm's cost of capital *and* are required to satisfy an additional payback threshold constraint. Section 4 then conducts a study of the value loss associated with this 'short termist' approach to project appraisal and section 5 concludes.

# 2. VALUE UNDER OPTIMAL DECISION MAKING AND THE NAÏVE DCF RULE

The basic assumptions are as follows. A project involves an initial capital outlay, denoted *K*, following which there ensues a stream of operating cash flows. The instantaneous cash flow at time *t* is denoted  $\pi_t$  and this is assumed to evolve according to a geometric Brownian motion (GBM), such that

$$d\pi_t / \pi_t = \alpha dt + \sigma d\varpi_t . \tag{1}$$

Here  $\alpha$  is the trend rate of growth in cash flow ( $\alpha$  can be positive or negative) and  $\sigma$  denotes its associated volatility. The initial cash flow is denoted  $\pi_0$  and the expected cash flow for time *t* grows at the rate  $\alpha$ :

$$E_0(\pi_t) = \pi_0 e^{\alpha t} \tag{2}$$

Finally, it is assumed that the firm has the option to defer investment (without risk that others might steal its investment opportunity). Thus the firm's decision for this project is simply one of investing at time zero or of waiting (for sufficiently favourable market developments) before investing. The value of the given project will clearly depend on project characteristics  $\pi_0, K, \alpha, \sigma$ , and on the decision rule used to determine whether to activate the project or not.

A useful way of presenting valuation results lies in first setting out value under uncertainty under the assumption that the project must be implemented, if at all, at time zero. Denote the value of such a project as  $V_i$  ('*i*' for 'immediate' investment). Then

$$V_{i} = -K + E_{0} \left( \int_{0}^{\infty} \pi_{t} e^{-rt} dt \right) = -K + \int_{0}^{\infty} \pi_{0} e^{(\alpha - r)t} dt$$
  
=  $-K + \frac{\pi_{0}}{r - \alpha} = K \left( \frac{\pi_{0}}{\pi_{i}} - 1 \right)$  (3)

where

$$\pi_i = (r - \alpha)K, \tag{4}$$

Here *r* denotes an appropriate discount rate and  $E_{\tau}$  denotes the expectations operator, with expectations formed at time  $\tau$ . In this model, notice that it must be assumed that  $r > \alpha$ , or the project will have infinite value. Notice also, from (3), that implementing the project at time zero has positive expected net present value only if  $\pi_0 > \pi_i$ . If the project has to be undertaken at time 0 or not at all, then  $\pi_i$  can be viewed as an investment 'trigger', since if the initial cash flow  $\pi_0$  is above this level, the project has positive *NPV* project and is worth implementing.

However, with the option of deferral, immediate implementation may not maximise value; it may be that 'more value' can be extracted by 'waiting to invest'. The optimal solution to the value maximisation problem under uncertainty, where deferral is an option, is documented in a wide range of literature so a formal derivation is omitted (for a full exposition, see e.g. Dixit and Pindyck [1994]). In order to present

the solution, it is useful to define the intermediate variables  $R_1 \equiv \left(\alpha - \frac{1}{2}\sigma^2\right)$ ,

 $R_2 \equiv (R_1^2 + 2\sigma^2 r)^{1/2}$ ,  $\lambda_1 = (-R_1 + R_2)/\sigma^2$  and use these to define what is termed an 'option multiplier' as

$$M = \lambda_1 / (\lambda_1 - 1).$$
<sup>(5)</sup>

The optimal decision rule, and the value that accrues to the firm as a result, turns on whether initial cash flow  $\pi_0$  is above or below a 'trigger level'  $\pi_u$  ('u' for 'under uncertainty') which is defined in relation to the certainty trigger level  $\pi_i$  as

$$\pi_{\mu} = M \pi_{i} \tag{6}$$

(this explains why *M* is often referred to as an 'option multiplier'). Notice that *M* is ultimately a simple function of the parameters  $r, \alpha, \sigma$ . It can be shown that  $\lambda_1 > 1$  and so M > 1 when  $\sigma^2 > 0$ , so the trigger level under uncertainty,  $\pi_u$  is greater than the trigger level under certainty,  $\pi_i$ . Recall that  $\pi_0 > \pi_i$  implies positive *NPV*, from (3). That is, if initial cash flow is above  $\pi_i$ , the project is value adding. However, equation (6) indicates that, under uncertainty, the initial cash flow has to be higher than this if value is to be maximised; only if  $\pi_0 > \pi_u (= M \pi_i)$  should the project be implemented immediately. If  $\pi_0 < \pi_u$ , optimal decision making requires that the project be deferred until some future time *t* occurs at which the cash flow  $\pi_i$  reaches the level  $\pi_u$ . Value under optimal decision making is summarised in the following result. **Result 1.** Optimal Decision and Value:

(i) If  $\pi_0 \ge \pi_u = M \pi_i$ , the firm invests at time zero, and value V\* is given as

$$V^* = K\left(\frac{\pi_0}{\pi_i} - 1\right) = \frac{\pi_0}{r - \alpha} - K$$

(ii) If  $\pi_0 < \pi_u$ , the firm waits until a time  $\tau$  is reached when  $\pi_\tau = \pi_u$ .

Value is given as

$$V^* = K \left(\frac{\pi_0}{\pi_u}\right)^{\lambda_1} \left(\frac{\pi_u}{\pi_i} - 1\right)$$

**Proof:** (i) follows from equation (3). Part (ii) is a standard result - see e.g. Dixit and Pindyck [1994], pp. 140-145. The proof is repeated in the appendix for convenience.

Optimal decision making thus turns on whether initial cash flow  $\pi_0$  is above or below the level  $\pi_u$  in (6), where this level is higher than the level  $\pi_i$ , defined in (4), that which would arise under the naïve *NPV* rule, by the option multiplier factor *M*. The value *V*\* defined by result 1 is the benchmark against which the value that results from alternative decision criteria are compared in what follows. It is worth noting that optimal value is a function of the cost of capital *r*, and individual project characteristics; the growth rate  $\alpha$  and volatility  $\sigma$ , the initial cash flow  $\pi_0$  and the scale of the project *K*.

The first deviation from optimal decision-making to be considered is the consequence of adopting a naïve *NPV* decision rule in which projects are accepted if they have positive *NPV* but are either rejected completely or deferred if they have *NPV*<0.

**Result 2**. Decision and Value, denoted  $V_{naiveNPV}$ , under the naïve *NPV* rule. (i) If  $\pi_0 \ge \pi_i$ , the firm invests at time zero, and value is given as

$$V_{naiveNPV} = K \left( \frac{\pi_0}{\pi_i} - 1 \right)$$

(ii) If  $\pi_0 < \pi_i$ , the firm waits, and value is zero:  $V_{naiveNPV} = 0$ .

**Proof:** Part (i) as per Result 1(i). For part (ii), if  $\pi_0 \ge \pi_i$ , but if  $\pi_0 < \pi_i$ , the firm either rejects the project, so earning zero *NPV*, or defers it until it just becomes viable, and then implements, so again earning zero *NPV*.

The percent value loss that arises under such a naïve *NPV* decision rule (in which *NPV*>0 projects are accepted and *NPV*<0 projects are rejected or are deferred) is defined as

$$%VL_{naiveNPV} = 100 \times \left(V^* - V_{naiveNPV}\right) / V^* .$$
<sup>(7)</sup>

That is, percent value loss from using the naïve *NPV* rule is measured against the benchmark value that arises from optimal decision making. The key feature of the naïve *NPV* rule is that it only generates positive value if there is positive value at time zero; that is, if it is initially negative *NPV*, it is either rejected, or it is deferred until a time when it just turns positive *NPV*; in both cases, a zero value outcome. Further, as Result 1 indicates, higher value can often be had by deferring projects even when they have positive *NPV* projects at time zero. How the value loss arising from using the naïve *NPV* rule varies with the key project parameters  $\pi_0 / K$ ,  $\alpha$ ,  $\sigma$  is studied numerically in section 4 below.

# **3.** VALUE UNDER HURDLE RATES AND PAYBACK THRESHOLDS

This section extends the standard model by considering how value is affected by the widespread use of 'short termist' decision rules. Specifically, the focus is on the use of an arbitrarily high hurdle discount rate (h) and the added restriction that projects must also beat a payback threshold (PB). The use of a hurdle rate in DCF analysis can also assumed to apply over a fixed time horizon H. This could be infinite, although it seems that firms often examine cash flows only over a more limited or 'truncated' time horizon. A time horizon of 10 year time horizon is used in the analysis below, although empirically the effect of varying the threshold to a longer or even infinite horizon makes little difference, given the high hurdle rate of discount (for survey evidence on firms' time horizons, see Hayes and Garvin [1982], Ross [1986], Poterba and Summers [1995], Segelod [2000]). The survey evidence discussed in section 1 on hurdle rates suggests these are often set at least 5% above

the cost of capital, whilst for the additional payback threshold, this was commonly set, implicitly or explicitly, as short as 2-4 years.

Consider then the impact of the firm using a fixed and non-optimal hurdle rate *h* along with a fixed and non-optimal payback threshold *PB*. The decision rule is that if the project has positive *NPV* at the hurdle rate and has a lower payback period than the payback threshold, the project is activated immediately. If not, it is deferred until the cash flow improves to a point where the test is finally passed. In general, an individual project may be accepted (having positive expected *NPV* at the hurdle rate and passing the payback threshold test), or be deferred (because it has negative expected *NPV* and/or fails to satisfy the payback requirement). In terms of optimal decision making, clearly if a project has negative expected value, it *should* be deferred. Further, according to result 1, if a project has positive expected value, it may still be optimal to deferr. The question is thus whether the short termist decision criteria imply 'too much deferral' – or 'not enough'. Intuitively, over a set of projects with varying individual characteristics (initial cash flow/capital outlay, growth prospects, volatility), some will be deferred too much and some too little', in both cases with an attendant value loss relative to optimal decision making.

The payback threshold as a decision criterion requires that the project be accepted at time  $\tau$  only if the sum of expected cash flows within the interval [0, *PB*] outweighs the initial capital investment. That is, if

$$-K + \int_{\tau}^{\tau+PB} E_{\tau}(\pi_{t}) dt = -K + \pi_{\tau} \int_{\tau}^{\tau+PB} e^{\alpha(t-\tau)} dt \ge 0.$$
(8)

Clearly, this condition is satisfied if the initial cash flow is sufficiently large. A project thus passes the *PB* decision rule at time  $\tau$  if its starting cash flow at that time,

 $\pi_{\tau}$ , lies above a threshold value, denoted  $\pi_{PB}$  defined by (8): Integrating and rearranging this gives,

$$\pi_{\tau} \ge \pi_{PB} \equiv \alpha K / (e^{\alpha PB} - 1) \text{ for } \alpha \neq 0 \text{, and}$$
$$\pi_{\tau} \ge \pi_{PB} \equiv K / PB \quad \text{if } \alpha = 0 \tag{9}$$

If the cash flow is below  $\pi_{PB}$ , the firm will wait until the cash flow reaches this threshold level.

Turning now to DCF appraisal using the hurdle rate h over a time period H, this requires that

$$-K + \int_{\tau}^{\tau+H} E_{\tau}(\pi_{t}) e^{-h(t-\tau)} dt = -K + \pi_{\tau} \int_{\tau}^{\tau+H} e^{(\alpha-h)(t-\tau)} dt \ge 0 \qquad .$$
(10)

Again, this condition will be satisfied at time  $\tau$  if the initial cash flow is sufficiently large. That is, a project will have a positive *NPV* using the hurdle discount rate at time  $\tau$  if its cash flow,  $\pi_{\tau}$  at that time lies above a threshold value, denoted  $\pi_h$ . This threshold is obtained by integrating and rearranging (10) to get<sup>3</sup>

$$\pi_{\tau} \ge \pi_h = \left(\alpha - h\right) K / \left(e^{(\alpha - h)H} - 1\right) . \tag{11}$$

Again, if not, the project will be deferred until a time when it is satisfied.

As previously remarked, the aim is now to consider the extent of value loss that may arise if projects with differing characteristics are only implemented if they have positive *NPV* when evaluated under a fixed (non-optimal) hurdle rate *and* if they additionally pass the payback requirement. That is, if the cash flow  $\pi_{\tau}$  at time  $\tau$ satisfies

<sup>&</sup>lt;sup>3</sup> Note that  $\alpha - h < 0$  since  $\alpha - r < 0$  and h > r.

$$\pi_{\tau} \ge \overline{\pi} = Max[\pi_{PB}, \pi_h]. \tag{12}$$

then the project is acceptable in that it satisfies both the 'short termist' decision rules.

#### Figure 1 about here

Which of the requirements, the hurdle rate or payback thresholds, proves to be the more stringent requirement in (12) depends on the growth rate for the project.<sup>4</sup> This is illustrated in figure 1 for the case where the hurdle rate is set at 25%, the time horizon H=10 years, and the *PB* threshold is 3.5 years; the curves represent plots for the equations in (9) and (11). The interpretation of figure 1 is that a project has positive *NPV* when evaluated at the hurdle rate if, given its expected cash flow growth rate  $\alpha$ , its initial cash flow ratio to capital outlay lies above the hurdle rate curve, and it satisfies the payback criterion if it lies above the payback threshold curve. The exact positions for the two curves depends upon the levels set for the thresholds (for *h*,*H*,*PB*). In figure 1 for example, the payback requirement is more stringent for projects with growth rates above around -5% ( $\alpha = -0.05$ ), whilst the hurdle rate requirement is more stringent for projects with lower growth rates. It follows that one project may be deferred because it fails the payback threshold, whilst another project might be deferred because it fails the hurdle rate test – given this dependence on the value of  $\alpha$ , the project growth rate.

The optimal decision rule in Result 1 required immediate investment if  $\pi_0 > \pi_u$ , and deferral of investment otherwise. Under the short termist decision criteria, immediate investment occurs if  $\pi_0 > \overline{\pi}$ , with deferral otherwise. Intuition suggests that value in

<sup>&</sup>lt;sup>4</sup> That is, whether  $\pi_h > \pi_{PB}$  or not depends, for given *r*,*h*,*H*,*PB*, purely on the value of  $\alpha$ ; from (9) and (11),  $\pi_h > \pi_{PB}$  if  $\alpha / (e^{\alpha PB} - 1) > (\alpha - h) / (e^{(\alpha - h)H} - 1)$ .

this case will be the same as in Result 1, but with the trigger value  $\overline{\pi}$  replacing  $\pi_u$  in the valuation formula - and this is in fact the case:

**Result 3**. Decision and Value,  $V_{ST}$ , under short termist hurdle rate and payback threshold decision rules:

(i) If  $\pi_0 \ge \overline{\pi}$ , the firm invests at time zero, and value is given as

$$V_{ST} = K \left( \frac{\pi_0}{\pi_i} - 1 \right)$$

(ii) If  $\pi_0 < \overline{\pi}$ , the firm waits until a time  $\tau$  is reached when  $\pi_\tau = \overline{\pi}$ . Value is then given as

$$V_{ST} = K \left(\frac{\pi_0}{\overline{\pi}}\right)^{\lambda_1} \left(\frac{\overline{\pi}}{\pi_i} - 1\right).$$

**Proof:** Part (i) is as per Result 1(i). For (ii), see appendix.

There are two ways that *DCF* analysis using a hurdle rate, in conjunction with the payback threshold, can lead to sub-optimal decisions and value loss. Firstly, if for a project,  $\pi_u > \overline{\pi}$ , the tendency under the short termist decision rules is to implement the project too quickly. In this case there is no value loss if  $\pi_0 > \pi_u$  as the project is implemented immediately when this is in fact a correct decision; however, if  $\overline{\pi} < \pi_0 < \pi_u$  the project is implemented when it should be deferred, and in the case where  $\pi_0 < \overline{\pi} < \pi_u$ , the project is deferred, but will then be implemented 'too soon'. By contrast, if  $\pi_u < \overline{\pi}$ , the project will tend to be deferred too long. In this case if  $\pi_u < \overline{\pi} < \pi_0$  the project is deferred when it should be implemented immediately, and if  $\pi_0 < \pi_u < \overline{\pi}$ , it is deferred when it should be implemented immediately, and if  $\pi_0 < \pi_u < \overline{\pi}$  it is deferred when it should be deferred, but it will be deferred 'too long'. Given that  $\pi_u$  is affected by project characteristics  $\alpha, \sigma$  and  $\overline{\pi}$  is affected by  $\alpha$ , which of these outcomes is likely to occur is project specific.

Result 3 provides the basis for assessing the extent of value loss that can arise out of short termism and how this value loss varies with key parameters. Percentage value loss arising from the short termism decision criteria is measured as

$$%VL_{ST} = 100 \times (V * -V_{ST}) / V *$$
(13)

where  $V^*$  denotes value under optimal decision making and  $V_{sT}$  denotes value under the short termist decision rules. How this value loss varies with project characteristics  $\alpha, \sigma, \pi_0 / K$  is studied numerically in section 4 below.

# **4.** VALUE CONSEQUENCES

For projects that are sufficiently strong in terms of initial cash flow relative to capital outlay, there is of course no value loss associated with imposing short termist decision criteria, or indeed, in applying the naïve *NPV* rule; such projects are implemented immediately under all of the above decision rules. Value loss thus only arises for projects which have less favourable starting conditions. This section presents a study of how value loss depends on a project's volatility, its growth prospects and its initial cash flow/capital outlay. In the figures presented below, the discount rate is set at 10%, the payback threshold at 3 years, the hurdle rate at 25%, and the time horizon for the latter at 10 years. Clearly value loss will tend to be less if these decision criteria are relaxed – but the aim of this section is to show that even relatively demanding decision thresholds do not induce particularly high value loss, at least for projects with some growth prospects and/or volatility in future cash flows. However, before undertaking this, it is instructive, in the light of Stultz's [1999] comments discussed above, to first compare value loss under short termist decision rules with that which arises if a naïve *NPV* rule is adopted.

#### Figure 2 about here

Figure 2 illustrates the relative value loss for the case where r = 10%,  $\alpha = 5\%$ ,  $\sigma = 20\%^{5}$ , and illustrates several key points:

(a) that there is no value loss at all for projects with sufficiently high initial cash flow/capital outlay ratio (since these are implemented immediately whatever the decision rule).

(b) that the naïve *NPV* rule loses 100% of potential value on projects with sufficiently low initial cash flow/capital outlay ratio (since these are rejected outright, or deferred only until the point where they earn zero *NPV*).
(c) that short termist decision rules limit the extent of value loss (to a maximum of 20% in the above scenario) whatever the value of the initial cash flow/capital outlay ratio.

The naïve *NPV* rule loses 100% of value on projects which are initially rejected because it also indicates that they should be activated just as soon as they become marginal. That is, in being activated when they become just marginal, they never get to actually add any value. By contrast, the short termist constraints guarantee that projects are deferred to a point where they do add value. Thus figure 2 illustrates why adherence to a naïve *NPV* decision rules is so problematic. It is possible to study more extensively the performance of the *NPV* rule by examining variation in the key parameters  $\alpha, \sigma$ ; however, the essential structure of value loss under the naïve *NPV* rule remains very similar to that described in figure 2 as these parameters are varied. That is, there is always 100% value loss on a set of projects that have sufficiently low initial cash flow/capital outlay.

<sup>&</sup>lt;sup>5</sup>The table function in EXCEL enables tabulation of value under optimal decision-making (result 1), value under short termist constraints (result 3) and under the naïve NPV rule (result 2), in this case as a function of the initial cash flow/capital outlay ratio ( $\pi_0 / K$ ).

This poor performance characteristic would indeed be worrying if the average decision-maker was characterised as a 'Stultzian MBA student' (i.e. one who uses the naïve NPV rule). However, in practice, the discount rate used tends to be higher than the cost of finance - and payback thresholds are also often imposed. This makes sense given the poor performance profile for the naïve NPV rule. High hurdles and short payback thresholds significantly reduce the maximum level of value loss that can arise. The rest of this section accordingly focuses on value loss under these short termist decision rules.

Value loss arising from the short termist decision rule is given in (13) is a function of the three project characteristics, namely growth  $\alpha$ , volatility  $\sigma$ , and the initial cash flow/capital outlay ratio,  $\pi_0/K$ . Figure 3 tabulates value loss as  $\alpha$  and  $\pi_0/K$  are varied for 5% volatility in cash flow; figures 4 and 5 then repeat this for the case of 20% and 40% volatility. In all these figures, it should be noted that value loss falls to zero for projects with sufficiently high  $\pi_0/K$ . This is because the optimal decision is to implement such project immediately, and the short termist decision rules also recommend immediate implementation. It is only at lower levels of  $\pi_0/K$  where, under short termist decision rules, there is a possible divergence in investment timing, and hence the possibility of value loss.

#### **Figures 3-5 about here**

Clearly, value loss is at its most significant at relatively low volatilities. In figure 3, value loss is close to 100% when there are negative growth rates, over the full range of initial cash flow to capital outlay. However, even in this case, at higher growth rates, value loss is reduced. For example, at a 6% growth rate the value loss is limited

to a maximum of 30%, and at a 9% growth rate, it is always less than 5% whatever the initial cash flow/capital outlay ratio,  $\pi_0/K$ . Figure 4 focuses on the case where volatility is 20% (a figure used in Dixit and Pindyck [1994] to illustrate the potential magnitude of real option effects, given it is broadly consistent with volatility observed in the larger stock markets around the world). Figure 4 shows that value loss is low so long as there is some prospect for growth in cash flow over time. At a 3% growth rate, the value loss is ~30% or less, this falling to ~15% or less for when growth is 6% and to less than 4% value loss when there is a 9% growth rate. It can be argued that 20% may be a rather low volatility level for *real* projects as opposed to financial portfolios (the volatility of a stock market index, is the volatility of a significantly diversified portfolio). Moving on to figure 5, with volatility set at 40%, value loss is low *whatever* the initial cash flow/capital outlay of the project and *whatever* the growth rate (so long as the latter is not massively negative). It can be argued that volatilities of this level or higher may be quite common; in such cases, the value loss from imposing short termist decision criteria is really very small.<sup>6</sup>

Whilst the reduction in value loss arising from increased volatility is as one would expect, the value loss impact associated with growth prospects is perhaps less obvious. The general effect of the payback threshold is to make it harder for projects which have growth prospects to be activated. However, this only means such projects tend to be deferred. With strong expected growth, cash flow will tend to rise over time to a point where the project does finally pass the test. Thus the value loss from imposing a more stringent payback requirement does not necessarily translate into high value loss; indeed, quite the opposite, value loss is low for such projects.

<sup>&</sup>lt;sup>6</sup> Given this observation, there is no need to report results at higher volatilities; value loss is already

It is straightforward to extend the numerical study reported above in different directions. For example, it is possible to analyse the performance of the hurdle rate criterion and the payback threshold separately. This is omitted, given space considerations, since the results are remarkable similar in structure to those reported above. It is also be possible to explore how the results are affected by changes in the discount rate (here set at r=10%) in all scenarios. The use of 10% as a nominal discount rate is probably a reasonable benchmark figure for use for the UK and US economies. However, having examined variation in r around the 10% value used in the above numerical work, it appears that the above observations concerning value loss are fairly robust to variation in the underlying interest rate r. This finding is also in line with the finding by Jagannathan and Meier [2002], that changes in r did not significantly affect the optimal level that ought to be set for the hurdle rate.<sup>7</sup>

Given the above observations, the key points are well illustrated by the scenarios considered in figures 3-5. For projects with growth prospects and/or a reasonable level of uncertainty concerning future cash flows, the value loss from exerting unduly short termist constraints (short payback thresholds, high hurdle rates), appears to be relatively small.

# **5.** DISCUSSION OF ASSUMPTIONS AND MODEL LIMITATIONS

The results in section 4 cover a wide range of scenarios and parameter values. However, the assumption that the cash flow process is geometric Brownian motion is

small at 40% volatility, and becomes yet smaller as volatility is increased.

<sup>&</sup>lt;sup>7</sup> For example, flexing r over the range 5-15%, the message is essentially the same as that reported. Note that it does not make too much sense to flex r too close to the hurdle rate – given the empirical evidence that hurdle rates tend to be set significantly above the cost of finance.

to an extent restrictive. Firstly, it should be noted that this implies that the cash flow can never go negative, and it also assumes that there is a fixed constant expected growth rate for the cash flow (although this can be varied and set as a positive or negative number). It also assumes that, if a project is not activated, the cash flow for the project still 'evolves'. This means that the value of the project evolves over time, and thus if not activated today, there may come a day when it is worth activating. This is particularly true for projects with an average positive growth rate in cash flow. Thus the model used in this paper is more appropriate for projects with longer duration, for projects in which the view of the future cash flow can change over time, and for projects that can be deferred without fear of pre-emption; it is less appropriate for short term projects, for projects for which cash flow forecasts are temporally invariant, or for projects which have to be implemented 'now or never'.

It also may *appear* that the cash flow process does not model well projects that have a cash flow life cycle (of initial growth, maturity and then decline), given the assumption of constant growth rate in expected cash flow. However, if the life cycle is sufficiently long, modelling it using a GBM process may be an acceptable approximation (since, given the discounting effect on the more distant cash flows, their impact on value is relatively small). It is also possible to model a crude form of life cycle within this standard investment model by adding a Poisson 'death process' as *per* Merton [1971] and McDonald and Siegel [1986]. This assumes that in each unit of time there is a constant probability that the cash flow process simply terminates. It is possible to pursue this quantitatively, although given space considerations it is omitted, given the central insight from so doing is reasonably straightforward. Essentially, with such a death process, if the probability of death is

significant, there is no longer such an incentive to wait to invest. In effect the hurdle rate that is optimal will be lower, the higher the death probability. It follows that, for projects which have shorter expected life because of higher 'death probabilities', the short termist decision rules may give rise to higher levels of value loss than manifest in section 4 above. By contrast, when projects have reasonably long expected lives (because of a relatively small 'death' probability), the results can be expected to be similar to those found in section 4. Finally, it is also possible to extend the model by considering scrap or resale values since these can also affect project life (given the project abandonment option). However, there is a fair amount of evidence that this has generally a relatively small impact on the optimal threshold for project acceptance (see for example Abel and Eberly [1995]).

To sum up, clearly it cannot be claimed that, for all investments, value loss is small. However, the focus in this paper concerns the impact of short termist criteria on the assessment of *long lived* projects and it can be argued that such projects often involve repetitive processes that generate cash flow streams which can be quite well characterised by the standard investment model.

## **6.** CONCLUDING COMMENTS

In the context of capital budgeting, it has often been suggested that managers appear to be 'short-termist', to over-emphasise the importance of the returns that are earned early in a project's life. In particular, surveys reveal that firms often apply a relatively high 'hurdle' discount rate (often 20%+) when evaluating projects, a rate much higher than the firm's actual cost of capital. Further, surveys also show that senior management often additionally impose payback thresholds, in that projects are also

expected to return the initial outlay in a relatively short period of time (2-4 years being common). Finally, this survey evidence also suggests the choice of hurdle rate and/or short payback threshold is not something that is varied according to project characteristics.

There is now a small but significant academic literature which discusses the potential rationality of the above forms of short-termist behaviour. The present paper focuses on option effects that are pervasive in practice – budget constraints and scarce managerial capacity to handle projects may mean that undertaking a marginal project forecloses the option to find and undertake a better one later, or foreclose the option to defer the project until a later date (when it may have better cash flow prospects etc.). In such circumstances, it is not optimal to accept a project simply because it has positive *NPV* at the firm's cost of capital. Projects should only be accepted if they are sufficiently positive *NPV*. Using a higher hurdle rate and/or shorter payback threshold is a crude way of ensuring this.

In theory, the use of a fixed and uniform 'higher hurdle rate' in *DCF* analysis, or indeed the use of a fixed payback threshold, is non-optimal, and is at best only an approximate way of taking account of real option effects. Using such fixed rules of thumb might give good decisions for some types of project but rather less good decisions for others. The overall 'cost' of such short termism thus must depend on the nature of the projects under investigation. This is essentially a quantitative rather than qualitative question, and is really one that can only be addressed by examining a class of investments for which decisions and value can be quantified. The present study did this by focusing on a 'standard investment model' featuring fixed size investments

which earn returns over time which are uncertain but characterised by constant growth rates – and investments for which the firm has control over the time at which the investment is undertaken. Although a sub-set of the class of 'all' investments, it can be argued that this is an important sub-set. The central finding is that for projects with some growth prospects and/or some volatility with respect to future cash flows, even quite restrictive short termist decision criteria (hurdle rate 25%, payback threshold 3 years), have a relatively benign impact. That is, they are not particularly costly in terms of the percent value loss they entail. For those impressed by the argument that, if there is enough competition, only the 'fittest will survive', this is perhaps what one might expect; that is, it would be surprising if investment rules used in practice entailed significant value loss and yet persisted over time.

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# **APPENDIX** [For completeness – to be included in the working paper but perhaps omitted from a published version – Opinions?]

It is useful to define the variable  $\Pi_t \equiv \pi_t / K$  (cash flow to capital outlay ratio). Value if implemented immediately is

$$V_0 = \frac{\pi_0}{r - \alpha} - K = K \left( \frac{\Pi_0}{r - \alpha} - 1 \right) \tag{A.1}$$

and so value if implemented at time  $\tau$  is  $V_{\tau} = K \left( \frac{\Pi_{\tau}}{r - \alpha} - 1 \right)$ .

If at time 0, investment is deferred, then value is no longer given by (A.1), although it clearly remains a function of  $\pi_t$ , K and is homogenous of degree one in these prices. Thus, value at any time prior to implementation can be written as a function

$$V_t = \psi(\pi_t / K) K = \psi(\Pi_t) K \tag{A.2}$$

where  $\pi_t$  is the cash flow that would hold at that time *if* the project was implemented then. A stochastic dynamic programming approach can be used to determine the optimal decision rule which maximises value; this determines the cash flow threshold at which point the firm should invest, and determines the value that results from this decision rule. Details of the solution to this standard problem involves an analysis of the Bellman equation of dynamic programming and can be found in Dixit and Pindyck [1994,pp. 140-145]. For convenience, the analysis is repeated below. The Bellman equation requires that (dropping time subscripts in what follows to avoid notational clutter)

$$rVdt = E(dV). \tag{A.3}$$

The next step involves evaluating E(dV) using Itô's lemma (note – working with relative price in what follows):

$$dV = V_{\Pi}d\Pi + \frac{1}{2}V_{\Pi\Pi}d\Pi^2 \tag{A.4}$$

where

$$V_{\Pi} = \psi'(\Pi) K, \qquad V_{\Pi\Pi} = \psi''(\Pi) K,$$

Also, from (1),

$$d\Pi = \alpha \Pi dt + \sigma \Pi d\varpi \Rightarrow d\Pi^2 = \Pi^2 \sigma^2 dt \tag{A.5}$$

Hence the term dV is given as

$$dV = V_{\Pi} d\Pi + \frac{1}{2} V_{\Pi\Pi} d\Pi^2 = \psi' (\Pi) K d\Pi + \frac{1}{2} \psi'' (\Pi) K d\Pi^2$$
(A.6)

Now, using (A.5),

$$dV = \psi' \left[ \alpha \Pi dt + \sigma \Pi d \, \varpi \right] + \frac{1}{2} \psi'' K \left( \sigma^2 \Pi^2 dt \right)$$

Thus taking expectations,

$$E(dV) = \left(\frac{1}{2}\sigma^2 \Pi^2 \psi'' + \alpha \Pi \psi'\right) K dt$$
(A.7)

Hence the arbitrage equation becomes

$$r\psi Kdt - \left(\frac{1}{2}\sigma^2 \Pi^2 \psi'' + \alpha \Pi \psi'\right) Kdt = 0$$
(A.8)

Dividing through by *Kdt* then gives the usual ordinary second order differential equation

$$\frac{1}{2}\sigma^{2}\Pi^{2}\psi'' + \alpha\Pi\psi' - r\psi = 0.$$
 (A.9)

A solution for the function  $\psi$  is now sought. Consider a trial solution of the form  $\psi(\Pi) = \Pi^{\lambda}$  (A 10)

$$\psi(\Pi) = \Pi^{\lambda} \tag{A.10}$$

Thus, 
$$\psi'(\Pi) = \lambda \Pi^{\lambda-1}$$
 and  $\psi''(\Pi) = \lambda(\lambda-1)\Pi^{\lambda-2}$ . Substituting into (A.10) gives  
 $\frac{1}{2}\sigma^2 \lambda(\lambda-1)\Pi^{\lambda} + \alpha^2 \Pi^{\lambda} + \pi \Pi^{\lambda} = 0$  (A.11)

$$\frac{1}{2}\sigma \lambda(\lambda-1)\Pi^{2} + \alpha \lambda\Pi^{2} - \eta \Pi^{2} = 0$$
(A.11)

which would hold if

$$\frac{1}{2}\sigma^2\lambda^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\lambda - r = 0$$
(A.12)

It is convenient to define

$$R_1 \equiv \left(\alpha - \frac{1}{2}\sigma^2\right) \tag{A.13}$$

$$R_2 \equiv \left( \left( \alpha - \frac{1}{2} \sigma^2 \right)^2 + 2\sigma^2 r \right)^{1/2}$$
(A.14)

so the roots to the quadratic equation (A.17) are

$$\lambda_1 = \left(-R_1 + R_2\right)/\sigma^2 \tag{A.15}$$

and

$$\lambda_2 = (-R_1 - R_2) / \sigma^2$$
. (A.16)

$$\frac{1}{2}\sigma^2\lambda^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\lambda - r = 0 \tag{A.17}$$

It is convenient to define

$$R_1 \equiv \left(\alpha - \frac{1}{2}\sigma^2\right) \tag{A.18}$$

$$R_2 = \left( \left( \alpha - \frac{1}{2} \sigma^2 \right)^2 + 2\sigma^2 r \right)^{1/2}$$
(A.19)

so the roots to the quadratic equation (A.17) are

$$\lambda_1 = \left(-R_1 + R_2\right)/\sigma^2 \tag{A.20}$$

and

$$\lambda_2 = \left(-R_1 - R_2\right) / \sigma^2 \,. \tag{A.21}$$

The general solution is thus

$$\psi(\Pi) = B_1 \Pi^{\lambda_1} + B_2 \Pi^{\lambda_2}$$
 (A.22)

where  $\lambda_1(>0), \lambda_2(<0)$  are defined by (A.20) and (A.21). As  $\Pi \to 0$  value must be finite so this boundary condition entails  $B_2 = 0$ , and so

$$\psi(\Pi) = B_1 \Pi^{\lambda_1} \,. \tag{A.23}$$

At time  $\tau$  where the investment is implemented, smooth pasting requires that

$$\psi(\Pi_u)K = \left(\frac{\Pi_u}{r - \alpha} - 1\right)K \tag{A.24}$$

(value matching) and that the first derivatives be equal (see Dixit [1993] for a clear exposition of these so called smooth pasting conditions); that is

$$\psi'(\Pi_u)K = \left(\frac{1}{r-\alpha}\right)K.$$
(A.25)

Equation (A.24) implies

$$B_{1}\Pi_{u}^{\lambda_{1}}K = \left(\frac{\Pi_{u}}{r-\alpha} - 1\right)K \Longrightarrow B_{1}\Pi_{u}^{\lambda_{1}} = \frac{\Pi_{u}}{r-\alpha} - 1, \qquad (A.26)$$

whilst (A.25) gives

$$B_{1}\lambda_{1}\Pi_{u}^{\lambda_{1}-1}K = \left(\frac{1}{r-\alpha}\right)K \Longrightarrow \lambda_{1}B_{1}\Pi_{u}^{\lambda_{1}} = \frac{\Pi_{u}}{r-\alpha}.$$
(A.27)

From (A.26) and (A.27), we get

$$\left(\lambda_{1}-1\right)B_{1}\Pi_{u}^{\lambda_{1}}=1 \Longrightarrow B_{1}\Pi_{u}^{\lambda_{1}}=1/\left(\lambda_{1}-1\right),$$
(A.28)

and hence substituting back into (A.27), that

$$\lambda_1 B_1 \Pi_u^{\lambda_1} = \frac{\Pi_u}{r - \alpha} \Longrightarrow \Pi_u = \frac{\lambda_1}{\lambda_1 - 1} (r - \alpha), \qquad (A.29)$$

and so

$$\Pi_{u} = \frac{\lambda_{1}}{\lambda_{1} - 1} (r - \alpha). \tag{A.30}$$

This is equation (6) in the paper and is a standard result in the literature (see Dixit and Pindyck [1994], p. 145). The decision at time zero is to implement the project only if  $\Pi_0 \ge \Pi_u$  (A.31)

If it is implemented at time zero, then value is simply

$$V = K \left( \frac{\Pi_0}{r - \alpha} - 1 \right) \tag{A.32}$$

If, however,  $\Pi_0 < \Pi_u$  then value is given by (A.1). This requires determination of  $B_1$  using (A.28). Thus, from (A.28),

$$B_1 = 1/\prod_u^{\lambda_1} \left(\lambda_1 - 1\right) \tag{A.33}$$

so

$$V = \psi(\Pi_0) K \tag{A.34}$$

where

$$\psi(\Pi_{0}) = B_{1}\Pi_{0}^{\lambda_{1}} = (\Pi_{0} / K)^{\lambda_{1}} / \Pi_{u}^{\lambda_{1}} (\lambda_{1} - 1)$$
  
=  $(\Pi_{0} / \Pi_{u})^{\lambda_{1}} / (\lambda_{1} - 1)$  (A.35)

Since  $\Pi = \pi / K$  and  $1/(\lambda_1 - 1) = M - 1 = \left(\frac{\pi_u}{\pi_i} - 1\right)$  from (5) and (6), this establishes

result 1(ii), that  $V^* = K\left(\frac{\pi_0}{\pi_u}\right)^{\lambda_1} \left(\frac{\pi_u}{\pi_i} - 1\right).$ 

# Investment under short termist decision criteria

Here, given the inequality in (12), the firm chooses to invest at time 0 only if

$$\Pi_0 \ge \Pi \tag{A.36}$$

This constraint only plays a role if  $\overline{\Pi} > \Pi_u$  where  $\Pi_u$  is defined by (A.30). Suppose the firm waits until a time  $\tau$  at which this price can be achieved. At the time of investment, as per (A.1), value is given as

$$V_{\tau} = K \left( \frac{\overline{\Pi}}{r - \alpha} - 1 \right) \tag{A.37}$$

whilst, as before, for  $t < \tau$ ,

$$V_t = \psi(\Pi_t) K \tag{A.38}$$

where  $\psi(\Pi) = B_1 \Pi^{\lambda_1}$  from (A.23). At the point of investment, only the value matching condition applies (see Dixit [1993] for a discussion of smooth pasting conditions). That is, at  $\tau$ ,

$$V_{\tau} = \psi(\Pi_{\tau})K = K \left(\frac{\overline{\Pi}}{r - \alpha} - 1\right).$$
(A.39)

hence,

$$B_1\left(\overline{\Pi}\right)^{\lambda_1} K = K\left(\frac{\overline{\Pi}}{r-\alpha} - 1\right)$$
(A.40)

so

$$B_{1} = \left(\frac{\overline{\Pi}}{r-\alpha} - 1\right)\overline{\Pi}^{-\lambda_{1}}$$
(A.41)

and hence

$$\psi(\Pi) = B_{\Pi} \Pi^{\lambda_{1}} = \left(\frac{\overline{\Pi}}{r-\alpha} - 1\right) \overline{\Pi}^{-\lambda_{1}} \Pi^{\lambda_{1}} = \left(\frac{\overline{\Pi}}{r-\alpha} - 1\right) \left(\Pi / \overline{\Pi}\right)^{\lambda_{1}}$$
(A.42)

Value at time zero is thus

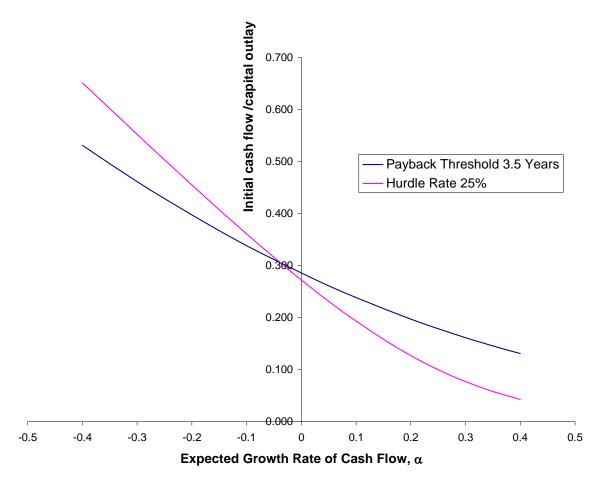
$$V_0 = \psi(\Pi_0)K = K\left(\Pi_0/\overline{\Pi}\right)^{\lambda_1} \left(\frac{\overline{\Pi}}{r-\alpha} - 1\right).$$
(A.43)

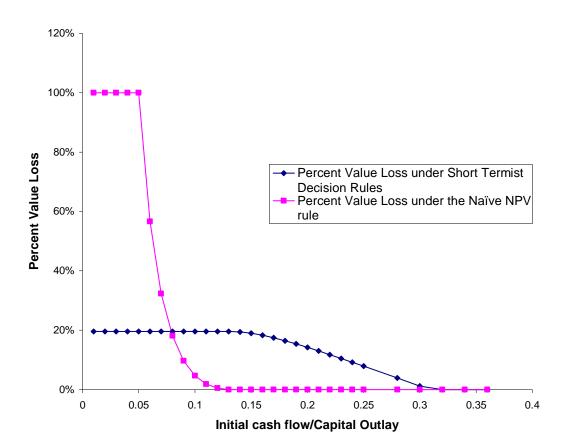
Since  $\Pi = \pi / K$ , using (4),  $\left(\frac{\overline{\Pi}}{r - \alpha} - 1\right) = \left(\frac{\overline{\pi}}{\pi_i} - 1\right)$ , result 3(ii) is established, that

value under the short termist decision rule is

$$V_{ST} = K \left(\frac{\pi_0}{\bar{\pi}}\right)^{\lambda_1} \left(\frac{\bar{\pi}}{\pi_i} - 1\right). \tag{A.44}$$

Figure 1 Comparing Hurdle Rate and Payback Thresholds





**Figure 2:** Comparing Short Termist Constraints and the Naïve NPV rule (setting  $\alpha = 5\%$ ,  $\sigma = 20\%$ , r = 10%, h = 25%, PB = 3 years)

