

**ACCESS PRICING: MONOPOLY, COMPETITION AND PRICE CAP  
REGULATION IN THE PRESENCE OF UNCERTAIN DEMAND AND  
TECHNOLOGY**

**by**

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**ABSTRACT**

Opening access to 'bottleneck facilities' and allowing the firm to set the price of access confers on it significant monopoly power, hence the need for regulation. Access is typically offered on a relatively short term basis, whilst the facilities themselves are long-lived irreversible investments. In such circumstances, uncertainty creates option value through the firm 'waiting to invest'. It has recently been suggested, although without formal analysis, that the regulator should take account of this when setting the maximum price the firm is allowed to charge. This paper undertakes, in a more formal setting, a comparison of competitive, monopoly and price capped monopoly solutions to the access pricing problem under uncertainty.

## 1. INTRODUCTION

'Bottleneck facilities' arise in many industries, particularly network industries, such as telecoms, railtrack, water, electricity and gas. Firms that control such facilities have typically been required by regulators to provide open access to these facilities. Of course, access is offered at a price – and so, given the inherent monopoly power associated with access provision, the question arises as to what constitutes a fair or efficient price. The access price is the price that is paid for short term access, whilst the facilities themselves are typically long lived and largely irreversible investments. It is well known that uncertainty over key variables such as future demand, technology, interest rates etc. can give rise to option value which impacts on the firm's incentive to invest in such facilities - and that the impact can be substantial. Clearly, regulators need to take some account of this type of impact when considering the level at which to set the access price. The suggestion in recent academic work (and often repeated in confidential reports commissioned by both regulators and regulatees) is that the access price should be capped at the level that would arise if access provision was provided competitively. Whilst this is a plausible argument, to date there has been no formal analysis of the consequences of imposing such an intertemporal price cap. This motivates the present work, which compares monopoly and competitive market solutions to the hire/lease/access pricing problem under uncertainty and then analyses the impact of intertemporal price caps. The key findings are that (i) in a competitive market, uncertainty does not seem to have such a significant impact on access price as previously argued, but that (ii) monopoly power generally leads to substantial under-investment, and (iii) intertemporal price caps are helpful in controlling this, but, unlike in simple certainty settings, under uncertainty, and even if optimally chosen, intertemporal price caps are inefficient. Under uncertainty, price capped monopolists will tend to wait too long to invest, will under-invest and will tend to impose quantity rationing on their customers.

Since there has been a fair amount of work on access pricing and on the impact of option value on the incentive to invest, the rest of this introduction gives a brief review to better delineate the contribution of the present work.

The problem of access provision, and at what price, was originally conceived primarily as an atemporal problem (see for example Armstrong, Doyle and Vickers

[1996], Armstrong [1998], Baumol and Sidak [1994], Laffont, Rey and Tirole [1998*a,b*]). The focus in this literature has tended to be on the efficient recovery of access deficits arising from fixed costs/economies of scale and scope, along with the promotion of efficient downstream entry or bypass. More recently, it has also come to be recognised that there are important intertemporal issues associated with how the cost of long lived capacity should be ‘unpacked’ into period lease/hire access prices (Salinger [1998], Sidak and Spulber [1997]). Finally, the importance of uncertainty to this calculation, and the extent to which it gives rise to ‘option value’ has been raised (Hausman [1996, 1997, 1999]).

This paper focuses on a simple setting for the intertemporal access pricing problem in which any amount of long lived capacity can be purchased at a constant unit price at any given point in time. The capacity unit price may, however, trend and fluctuate over time, and the demand for access itself may also trend and fluctuate. Under certainty, in a competitive market, the hire price is given as the unit capacity cost multiplied by the sum of the interest rate, the depreciation rate, and the rate at which of technical progress reduces the price of new capacity, at least when these rates are assumed to be constant over time (see Salinger [1998], Sidak and Spulber [1997], and Laffont and Tirole [2000, p.151]).

Uncertainty changes things of course. There is now a fair body of work on the option value that arises out of the firm being able to defer the date at which irreversible capacity investment is made, along with that which arises out of being able to abandon or to temporarily mothball production (Dixit and Pindyck [1994]). Much of this work focuses on the case of the monopoly firm (McDonald and Siegel [1986], Pindyck [1988]) although the competitive case has also been addressed (Lucas and Prescott [1971], Dixit [1989]) and the similarity between the monopolist’s problem and the competitive solution has also been noted (Dixit and Pindyck [1994, p. 256]).

Hausman [1996, 1997, 1999] has utilised this work on option value to argue that, in the presence of uncertainty, the price that should be set for access should be increased above the competitive certainty price by a factor to reflect option value associated with irreversible investment, and the general thrust of his argument has been widely accepted (see Laffont and Tirole [2000]). Clearly, if regulated firms are not granted a

premium in the access price to account for option value, the incentive to invest will be adversely affected. The option value ‘multiplier’, at a generic level, is often substantial and a factor of around two is fairly typical (MacDonald and Siegel [1986], Dixit and Pindyck [1994, p. 153]). For Telecoms, Hausman [1997] provides some evidence that the multiplier may well be above three.

The first contribution of the present paper is to suggest that, whilst the above analysis properly calculates the uncertainty entry trigger price, a correct comparison with the certainty price reveals that the overall impact of uncertainty is likely raise the entry trigger price by a significantly smaller factor (by perhaps 10-50%). A new form of option value multiplier is also derived, appropriate for multiplying the certainty price estimate.

The similarity of the entry trigger price for the monopolist and the competitive industry, noted above, arises *only* in the case where the monopoly firm has a single ‘all-or-nothing’ or ‘fixed size’ project and where the choice of investment timing is the only variable under the firm’s control. When the monopoly firm can choose both the level *and* timing for its investment, its choices will generally significantly diverge from those manifest in a competitive industry. For network industries such as telecoms, electricity, gas etc., the assumption that the firm can control the level of investment as well as its timing is fairly realistic; capacity can be incrementally expanded subsequent to the initial investment being made.<sup>1</sup> This paper continues by examining this case and comparing the timing and levels of investment under competition with those under monopoly. The suggestion that a firm in control of a bottleneck facility should be allowed to set the same price as would occur if the industry was competitive seems to be fairly well accepted (see Hausman [1997, 1999], Laffont and Tirole [2000]).<sup>2</sup> However, the response of a monopolist to the imposition of such a price cap has not been formally addressed. The final objective of the present work is to analyse this problem in some detail.

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<sup>1</sup> Obviously there is some level of indivisibility regarding the consequences of such incremental investment in capacity, and investments may also tend to lead to increases in capacity in specific local areas. However, treating capacity as a continuous variable is probably a better assumption than assuming it is ‘all-or-nothing’.

<sup>2</sup> The argument can also be found in a myriad of confidential consultancy reports commissioned by regulators and those they regulate.

## 2. THE COMPETITIVE MODEL

Basic assumptions are that capacity is long lived but subject to physical depreciation, technical progress reduces the unit cost of capacity provision, and both technological progress and industry demand are uncertain. Operating (variable) costs associated with the use of capacity are taken to be zero.<sup>3</sup> In this section, the industry is assumed to be perfectly competitive. Firms, in contemplating investing in the industry need to make forecasts of the future prices they are likely to get for hiring out installed capacity. Naturally, the prices they expect depend in part on how demand evolves over time, and in part upon their own capacity investment decisions. It is possible to close the model in different ways. In common with much recent literature, this paper adopts the rational expectations assumption - that, given stable distributions for both demand and technology, the distribution for actual prices will be the same as that for each firm's anticipated prices.<sup>4</sup>

The demand function is taken to be constant elasticity, with the 'strength' of demand uncertain. The assumption of constant elasticity of demand is useful in two ways. Firstly, as a convenient parameterisation facilitating the exploration of alternative assumptions regarding this elasticity. Secondly, in that it permits the derivation of closed form solutions which are easy to interpret and debate - this is regarded as an important 'policy relevance' consideration.<sup>5</sup> This competitive model is outlined in some detail despite close similarities with earlier work primarily because it forms the foundation analysis for subsequent developments to monopoly and price capped monopoly, but also because it provides some new results and perspectives on earlier work.

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<sup>3</sup> This is a good assumption in many applications. If there are variable costs, one can think of prices here as simply the mark-up on variable cost (see Hausman [1997, p. 32]).

<sup>4</sup> See Lucas and Prescott [1971] for a lucid defence of the rational expectations assumption. Essentially, the argument is that agents will not make persistent and easy to correct forecast errors - as is reasonable, given the assumption that the underlying distributions for demand and technology are in fact stable over time (as is assumed here).

<sup>5</sup> There is considerable demand from policy makers for models which are relatively easy to interpret and use.

The demand for capacity at time  $t$  is given as

$$Q_t^d = A_t p_t^\gamma, \quad (1)$$

where  $p_t$  is the price per unit of capacity and  $\gamma < 0$  is the elasticity of demand. The demand function can be inverted to write

$$p_t = (Q_t^d)^\eta A_t^{-\eta} \quad \text{where } \eta = 1/\gamma < 0 \quad (2)$$

Uncertainty enters through the level of demand variable,  $A_t$ , and also through technological progress, which is assumed to affect the unit cost of capacity, denoted  $K_t$ . In both cases, these are assumed to be geometric Brownian motions (*GBM*), as follows:<sup>6</sup>

$$\begin{pmatrix} dA_t / A_t \\ dK_t / K_t \end{pmatrix} = \begin{pmatrix} \alpha \\ -\delta \end{pmatrix} dt + \begin{pmatrix} \sigma_{AA} & \sigma_{AK} \\ \sigma_{KA} & \sigma_{KK} \end{pmatrix} \begin{pmatrix} d\bar{\omega}_t^A \\ d\bar{\omega}_t^K \end{pmatrix}. \quad (3)$$

Here  $\alpha$  is the trend rate of growth in demand (which could be positive or negative).

Individual firms purchase units of capacity at a unit cost at time  $t$  of amount  $K_t$  and technological progress reduces the price of capacity at a trend rate  $\delta > 0$ . Thus  $d\bar{\omega}_t^A$  and  $d\bar{\omega}_t^K$  denote two independent Wiener processes and the volatilities for  $A_t$ ,  $K_t$ ,

are captured in the matrix  $\begin{pmatrix} \sigma_{AA} & \sigma_{AK} \\ \sigma_{KA} & \sigma_{KK} \end{pmatrix}$ . This formulation allows that cost reducing

technological progress and demand may be correlated. Intuitively, one might expect that a higher than expected growth in demand might induce more effort and expenditure on R&D etc. and hence an increase in the rate at which the unit cost of capacity falls. In this case, one might expect that, whilst perhaps  $\sigma_{AK} \approx 0$ , it may be that  $\sigma_{KA} < 0$ . This point is returned to in section 6, where the impact of parameters on the entry hire price is examined through sensitivity analysis.

As there are no variable costs associated with the use of capacity, it follows that capacity, once installed, will be utilised so long as it can be leased or hired at a non-negative price. It is, in effect, an irreversible investment; so long as hire/lease prices

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<sup>6</sup> GBM is a common assumption in the literature. For a discussion of its pros and cons, see McDonald and Siegel [1986] and Dixit and Pindyck [1994].

are non-negative, there is no incentive to retire capacity.<sup>7</sup> An individual unit of capacity once installed is assumed to depreciate at a fixed rate  $\theta$ .<sup>8</sup> Equivalently, in terms of the ensuing mathematical analysis, one could assume that each individual unit of capacity was subject to a stochastic ‘death process’ in which the probability of the plant ceasing to be operational is a constant per unit time (i.e. is  $\theta\Delta t$  on a small time interval  $\Delta t$ ). As a consequence, at the aggregate level, the stock of capacity  $Q_t$  also depreciates at a fixed rate  $\theta$  at points in time where there is no purchase of new capacity. That is, on time intervals where there are no additions to capacity,

$$\dot{Q}_t = -\theta Q_t \quad (4)$$

Capacity leasing in this section is a competitive activity. Individuals can lease/hire capacity at time  $t$  for a short duration  $\Delta t$  at a price  $p_t\Delta t$ . The instantaneous price at each point in time  $t$  is assumed to be a market clearing price. Thus, for a given level of aggregate capacity,  $Q_t$  available at time  $t$ , the price  $p_t$  adjusts to bring this into line with demand. Hence (2) defines the relationship between the price at time  $t$  and the amount of capacity available at that time. This competitive market will clearly experience at most two types of behaviour;

**Regime 1:** Intervals on which there is no investment at all

**Regime 2:** Intervals on which there is positive entry/investment in capacity

Consider the investment problem faced by any firm thinking about installing capacity at some arbitrary time  $\tau$ . Given technology and demand are stochastic, so too is the equilibrium price - and it is assumed that the individual firm has rational expectations regarding the distribution of price at any given time  $t$  in the future. The value  $V_\tau$  of the profit flow derived from ownership of one unit at time  $\tau$  as thus viewed as being

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<sup>7</sup> Most capital investments involve sunk costs to some degree and are at least partially irreversible. This is particularly so in the case of firm specific investments (advertising etc.). However, in a competitive industry, at first glance it might appear that investment must be reversible. The key point to note however, is that there is never any incentive for an individual firm to sell capacity since the market price for it always reflects the discounted value of expected future profits to be got from it – and with zero variable costs, so long as price remains positive, there is no incentive to scrap or shut down.

<sup>8</sup> Whilst it is conventional to assume that  $\theta > 0$ , so that the quantity of capacity ‘erodes’ over time, it is also worth noting that in some situations, it is possible for  $\theta < 0$ ; this would be the case, for example where technical progress meant that capacity, having been installed, can be utilised more fully as time passes. In telecoms, network capacity has occasionally increased through changes in the way messages are sent - without any physical change in large parts of the installed system.

the expected present value of future revenues accruing from hiring out capacity over its future life;

$$v_\tau = E_\tau \left( \int_\tau^\infty p_t e^{-(\theta+r)(t-\tau)} dt \right). \quad (5)$$

Here  $r$  denotes an appropriate (risky) discount rate<sup>9</sup> and  $E_\tau$  denotes the expectations operator, where expectations are formed at time  $\tau$ . Note that the present value at time  $\tau$  of the instantaneous profit flow, at some time  $t > \tau$ , from hiring out one unit of capacity is  $p_t e^{-r(t-\tau)}$ ; however, if a unit of capacity is installed at time  $\tau$ , depreciation then affects its availability - only an amount  $e^{-\theta(t-\tau)}$  is left at time  $t$ . Hence the integrand in (5).<sup>10</sup> If it can be shown that  $p_t$  is a *GBM* process, then the value function in (5) can be written as  $v_\tau \equiv v(p_\tau, K_\tau)$ .<sup>11</sup> This is established below.

The net present value, at time  $\tau$  of investing in a unit of capacity at this time is given as

$$PV_\tau \equiv v_\tau - K_\tau. \quad (6)$$

In a competitive market, there will be no investment (regime 1) whenever firms' anticipation of future prices is such that  $PV_\tau < 0$ , whilst there is entry/investment whenever  $PV_\tau \geq 0$  (regime 2).<sup>12</sup> Entry is instantaneous and this prevents the expected present value from rising above zero. Suppose there is continuous entry on some time interval  $(\tau_1, \tau_2)$ . Then it follows that

$$v_\tau - K_\tau = 0, \quad \text{for } \tau \in (\tau_1, \tau_2), \quad (7)$$

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<sup>9</sup> Empirically, solutions are not especially sensitive to the choice of discount rate. It is also possible to take  $r$  as the riskless rate of interest, so long as expectations are calculated in a suitably 'weighted' form. See Campbell, Lo and MacKinlay [1997, ch.9] for a general discussion.

<sup>10</sup> Merton [1976] notes the result that, if physical depreciation is modelled as a Poisson death process, this feeds through into an effective increase in the discount rate (here, by the amount  $\theta$ ).

<sup>11</sup> and fixed parameters such as  $\theta, r$ , of course. These are suppressed.

<sup>12</sup> In contrast to the case of the monopoly firm, where it is possible to defer investment. Deferring investment to take advantage of possible favourable movements in underlying economic conditions (interest rates, prices etc.) makes sense only if there are no others who can jump in and steal the project. In a competitive market, firms will enter/invest as soon as present value becomes non-negative.

and hence, since  $K_\tau$  is *GBM*, so is  $V_\tau$  on this interval. On intervals where  $v_\tau - K_\tau < 0$ , there is no investment and price is given by (2). Applying Itô's lemma, and defining

$$\mu_p \equiv -\eta \left( \alpha + \theta - \frac{1}{2}(\eta + 1) (\sigma_{AA}^2 + \sigma_{AK}^2) \right), \quad (8)$$

the price process can be written as

$$dp_t = \mu_p p_t dt - \eta \sigma_{AA} p_t d\bar{\omega}_t^A - \eta \sigma_{AK} p_t d\bar{\omega}_t^K \quad (9)$$

(see appendix, A1). Thus price is also a *GBM* on the intervals where there is no investment.

The above analysis establishes that  $p_\tau$  is *GBM* on  $[0, \infty)$ , so the value function in (5) can be written in the form  $v_\tau \equiv v(p_\tau, K_\tau)$ . As a corollary, it also follows that present value can also be written as a function  $PV_\tau \equiv PV(p_\tau, K_\tau)$ . The functions  $PV(p_\tau, K_\tau), v(p_\tau, K_\tau)$  are specified in terms of absolute prices,  $p_\tau, K_\tau$ , and are homogenous of degree 1 in these prices; that is

$$v(\alpha p_\tau, \alpha K_\tau) = \alpha v(p_\tau, K_\tau) \quad (10)$$

Thus, writing  $\alpha = 1/K_\tau$ , and defining the function  $\psi(p_\tau/K_\tau) \equiv v(p_\tau/K_\tau, 1)$ , (10) can be written as

$$v(p_\tau, K_\tau) = v(p_\tau/K_\tau, 1) K_\tau = \psi(x_\tau) K_\tau, \quad (11)$$

where the relative price  $x_\tau$  is defined by  $x_\tau \equiv p_\tau/K_\tau$ .

The analysis of the competitive solution involves identifying the solution for  $\psi(x_\tau)$  on intervals on which there is no investment, followed by an analysis of smooth pasting conditions at the boundary at which investment commences. These are taken in turn.

### **Value on intervals of zero Investment.**

Consider the process at some time  $\tau$  where  $PV_\tau < 0$ . In what follows, for notational convenience, function arguments and time subscripts are omitted; thus  $v$  denotes the

value function  $v(p_\tau, K_\tau)$  and  $\psi$  stands for  $\psi(x_\tau)$  etc. The arbitrage equation (see Dixit [1993, p. 15]) which determines the evolution of  $v$  is given as

$$(\theta + r)vdt = pdt + E(dv). \quad (12)$$

Applying Itô's lemma and simplifying, this equation gives the fundamental differential equation for this problem as (see appendix A2)

$$\frac{1}{2}\sigma^2 x^2 \psi'' + (\mu_p + \delta)x\psi' - (\theta + r + \delta)\psi + x = 0 \quad (13)$$

where the variable  $\sigma^2$  is defined as

$$\sigma^2 \equiv \left[ \eta\sigma_{AA} + \sigma_{AK} \right]^2 + \left[ \eta\sigma_{KA} + \sigma_{KK} \right]^2. \quad (14)$$

The general solution to (13) involves the standard procedure of finding a particular solution and also finding the general solution to the homogenous part. It can be shown that the solution takes the form (see appendix A3)

$$\psi_0(x) = A_0x + A_1x^{\lambda_1} + A_2x^{\lambda_2}, \quad (15)$$

where

$$A_0 = \frac{1}{\theta + r - \mu_p} = \frac{1}{(\theta + r) + \eta \left( \alpha + \theta - \frac{1}{2}(\eta + 1)(\sigma_{AA}^2 + \sigma_{AK}^2) \right)}, \quad (16)$$

and the roots are defined as

$$\lambda_1 = (-R_1 + R_2) / \sigma^2 \quad (17)$$

$$\lambda_2 = (-R_1 - R_2) / \sigma^2, \quad (18)$$

where

$$R_1 \equiv \left( \mu_p + \delta - \frac{1}{2}\sigma^2 \right). \quad (19)$$

$$R_2 \equiv \left( \left( \mu_p + \delta - \frac{1}{2}\sigma^2 \right)^2 + 2\sigma^2(\theta + r + \delta) \right)^{1/2} \quad (20)$$

Notice that  $2\sigma^2(\theta + r + \delta) > 0$  if  $\sigma^2 > 0$ , so the roots are real and of opposite sign when uncertainty is present. The constant  $A_0$  has already been determined. The other two arbitrary constants are determined by boundary conditions. As  $x \rightarrow 0$ , if value is to be finite, it must be that  $A_2 = 0$  or the solution explodes (Dixit [1993] discusses this sort of boundary condition in more detail). The other constant is determined by smooth pasting conditions at the positive investment boundary (see below).

### Positive Investment Boundary.

At any hitting time  $\tilde{t}$  at which there is a transition between no-investment and positive investment, since smooth pasting involves value matching and first order conditions (Dixit [1993], Dumas[1991]). If the price rises sufficiently, investment is triggered. At the transition boundary, smooth pasting conditions apply. The ‘value matching’ condition requires  $PV_{\tilde{t}} = 0$ . Hence

$$PV(p_{\tilde{t}}, K_{\tilde{t}}) = 0 \Rightarrow \psi(x_{\tilde{t}}) = 1 \quad (21)$$

The ‘first order’ smooth pasting condition requires that

$$d(PV_{\tilde{t}})/dx = \psi'(x_{\tilde{t}})K_{\tilde{t}} = 0 \Rightarrow \psi'(x_{\tilde{t}}) = 0. \quad (22)$$

Thus, from (15),

$$\psi(x_{\tilde{t}}) = A_0 x_{\tilde{t}} + A_1 x_{\tilde{t}}^{\lambda_1} = 1, \quad (23)$$

and

$$\psi'(x_{\tilde{t}}) = A_0 + \lambda_1 A_1 x_{\tilde{t}}^{\lambda_1 - 1} = 0 \Rightarrow A_1 x_{\tilde{t}}^{\lambda_1} = -A_0 x_{\tilde{t}} / \lambda_1. \quad (24)$$

Thus, from (23) and (24),

$$\psi(x_{\tilde{t}}) = A_0 x_{\tilde{t}} + A_1 x_{\tilde{t}}^{\lambda_1} = A_0 x_{\tilde{t}} (1 - 1/\lambda_1) = 1, \quad (25)$$

so  $x_{\tilde{t}}$  is a constant. Denote this as  $\xi_c$ , the relative price (relative, that is, to the unit price of capacity) which triggers investment in the competitive case. Then, rearranging (25), and using (16),

$$\xi_c = \frac{\lambda_1}{(\lambda_1 - 1)} \left[ \theta + r - \mu_p \right] = \frac{\lambda_1}{(\lambda_1 - 1)} \left[ \theta + r + \eta \left( \alpha + \theta - \frac{1}{2} (\eta + 1) (\sigma_{AA}^2 + \sigma_{AK}^2) \right) \right]. \quad (26)$$

This is the relative entry trigger price (entry price relative to unit capacity cost).

Denoting the entry price that would induce entry at some arbitrary time  $t$  as  $p_e^c(t)$ ,

this is given as

$$p_e^c(t) = \xi_c K_t. \quad (27)$$

That is, entry occurs whenever price reaches the level  $\xi_c K_t$ . Under uncertainty,

depending on the evolution of capacity unit cost and demand, the solution is

characterised by an initial pulse of investment, and thereafter, intervals on which

$p_t = p_e^c(t) = \xi_c K_t$  (on which there is positive investment) interspersed with intervals

on which  $p_t < p_e^c(t)$  (on which there is zero investment).

### 3. CHARACTERISTICS OF THE COMPETITIVE SOLUTION

The first point to note about the competitive industry solution (27) is its similarity with that for the ‘single project’ monopoly case (Dixit and Pindyck [1994]). The principal difference between the model in section 2 and that described in Dixit and Pindyck [1994] (and used in Hausman [1997, 1999]) lies in the fact that the model here involves explicit modelling of the underlying processes which generate the trend and uncertainty in price,  $\mu_p$ . This is useful because it makes clear that the trend in price,  $\mu_p$ , is itself affected by uncertainty and hence that uncertainty affects the trigger price not simply through the standard option multiplier  $\lambda_1/(\lambda_1 - 1)$ ; that is, this multiplier does not in itself measure the extent to which uncertainty pushes up the entry trigger price.

It is shown below, although only under some assumptions regarding parameter values, that the competitive entry trigger price under certainty, denoted  $p_e^{cert}(t)$  can be written as

$$p_e^{cert}(t) = \xi_{cert} K_t \quad (28)$$

where

$$\xi_{cert} = [\theta + r + \delta] \quad (29)$$

Here,  $\xi_{cert}$  denotes the relative entry trigger price under certainty. This sort of certainty price has been noted in earlier work (see Salinger [1998], Sidak and Spulber [1997], Laffont and Tirole [2000, p.151]).

Take the ratio of the uncertainty and certainty trigger prices,

$$p_e^c(t)/p_e^{cert}(t) = \left( \frac{\lambda_1}{\lambda_1 - 1} \right) \left( \frac{\theta + r - \mu_p}{\theta + r + \delta} \right) \quad (30)$$

where the trend  $\mu_p$  in the hire/lease price  $p_t$  is given by (8). With no uncertainty, from (26), the term  $-\mu_p = -\eta(\alpha + \theta)$ , and this term can be larger or smaller than  $\delta$ , depending on parameter values. For example, it is likely to be significantly smaller if there is a downward trend in prices; that is if  $\alpha < 0$ . The overall effect of uncertainty on the relative price in (30) is thus rather less than might be expected if the standard

option multiplier is applied to the certainty price (see section 6 for numerical estimates).

This implies no criticism of earlier work on the entry trigger price under uncertainty (e.g. Dixit and Pindyck [1994], Hausman [1997, 1999]) *per se* since an equation of type (26) is used in this literature. However, it should be clear from the above analysis that it is inappropriate to use  $[\theta + r - \mu_p] K_t$  as the benchmark for the certainty price in the competitive industry - and so when it is suggested that uncertainty increases the entry price by a factor of 2, or 3, or more, this is not really a ‘fair’ comparison. In fact the actual overall impact of uncertainty is likely to be much less than this (see below and also section 6 for numerical estimates and comparisons).

It is worth considering in more detail whether the entry price (26) converges on the certainty price (28), as it is not immediately obvious that this will in fact be the case.

Thus, from (26), note that

$$\begin{aligned} & \lim_{\sigma_{AA} \rightarrow 0} \lim_{\sigma_{AK} \rightarrow 0} [\theta + r - \mu_p] \\ &= \lim_{\sigma_{AA} \rightarrow 0} \lim_{\sigma_{AK} \rightarrow 0} \left[ \theta + r + \eta \left( \alpha + \theta - \frac{1}{2} (\eta + 1) (\sigma_{AA}^2 + \sigma_{AK}^2) \right) \right] \\ &= [\theta + r + \eta (\alpha + \theta)] \end{aligned} \quad (31)$$

so this begs the question “what happened to the term  $\theta + r + \delta$  ?” For brevity, define

$$Lim \equiv \lim_{\sigma_{KK} \rightarrow 0} \lim_{\sigma_{AA} \rightarrow 0} \lim_{\sigma_{AK} \rightarrow 0} \lim_{\sigma_{KA} \rightarrow 0} . \quad (32)$$

Then it is straightforward, though tedious, to establish that, taking limits,

$$\begin{aligned} & Lim \frac{\lambda_1}{(\lambda_1 - 1)} \left[ \theta + r + \eta \left( \alpha + \theta - \frac{1}{2} (\eta + 1) (\sigma_{AA}^2 + \sigma_{AK}^2) \right) \right] \\ &= Max [\theta + r + \delta, \theta + r + \eta (\alpha + \theta)] \end{aligned} \quad (33)$$

(see appendix A4). Thus the formally correct entry trigger price under certainty is given by (28) but where

$$\xi_{cert} = Max [\theta + r + \delta, \theta + r + \eta (\alpha + \theta)] . \quad (34)$$

The rationale for this turns out to be reasonably straightforward; the solution converges on one which is essentially driven by the trend in demand or, one which is driven by the rate of technical progress. At time 0, clearly there has to be an instantaneous influx of investment in capacity. Thereafter, under certainty, it can be

proved<sup>13</sup> that there is either continuous investment in capacity and the hire price is always that given in (28) – or there is no further investment in capacity. Essentially, there is no further investment in capacity if erosion in the level of demand is sufficiently fast whilst physical depreciation is sufficiently slow. This occurs when

$$\theta + r + \delta < \theta + r + \eta(\alpha + \theta), \quad (35)$$

which requires that

$$(\alpha + \theta)/\gamma > \delta \quad \text{or equivalently, as } \alpha < \gamma\delta - \theta \quad (36)$$

Since the elasticity of demand  $\gamma < 0$  and depreciation  $\theta \geq 0$  (usually), this can only occur if demand is falling sufficiently fast ( $\alpha$  sufficiently negative). When this occurs, the demand effect depresses prices at a faster rate than is indicated in (28), and hence the initial hire price has to be higher to compensate for the ensuing faster decline in the price profile (to motivate the initial investment, firms must expect future hire prices to be such that the investment is at least a zero *NPV* transaction). If the inequalities in (35), (36) are reversed, then there is an initial pulse of investment, followed by positive levels of investment thereafter. Under certainty, price in this case tracks the trigger price function; that is, (28) holds everywhere on  $[0, \infty)$ . This is really the ‘normal’ limiting case – demand has to fall at a fairly high trend rate to overturn it. Thus, in the rest of the paper, it is convenient to assume that  $\alpha > \gamma\delta - \theta$  so that (29) does indeed describe the entry trigger price under certainty.

Whilst the root  $\lambda_2$  plays no role in the solution in equation (15) (because  $A_2 = 0$ ), an interesting and somewhat remarkable result is that

$$\left( \frac{\lambda_1}{\lambda_1 - 1} \right) [\theta + r - \mu_p] = \left( \frac{\lambda_2 - 1}{\lambda_2} \right) [\theta + r + \delta]. \quad (37)$$

(Proof: appendix A5). Thus it turns out that it is after all possible to write the competitive entry price as an ‘option value’ mark-up on the certainty price; that is, (26) can be rewritten as

$$\xi_c = \left( \frac{\lambda_2 - 1}{\lambda_2} \right) [\theta + r + \delta], \quad (38)$$

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<sup>13</sup> The proof that there are only two possible solutions in the certainty case is rather tedious and is omitted.

although the ‘option multiplier’  $(\lambda_2 - 1)/\lambda_2$  is not one which seems to have been noted in previous work. For plausible parameter values, this multiplier takes a smaller value than  $\lambda_1/(\lambda_1 - 1)$ . Section 6 establishes that the overall impact of uncertainty will raise the trigger entry price by a factor of around 10-60% of the certainty price.<sup>14</sup>

#### 4. THE MONOPOLY FIRM

This section analyses the case where a single firm controls the level of capacity in the industry. The broad characteristics of the solution parallel that described in the competitive case, so the analysis is abbreviated where possible. Demand is assumed elastic ( $\gamma < -1$ ); with a constant elasticity demand curve, there is clearly no solution if demand is inelastic.<sup>15</sup> This implies revenue increases with quantity sold so, for any given level of installed capacity, the firm will choose to fully utilise this capacity (by setting price appropriately).

As before, there may be intervals on which the firm does not invest (regime 1), and intervals on which it chooses a positive level of investment (regime 2). The strategy parallels that for the competitive case; first the evolution of value in the non-investment regime is established, followed by an analysis of smooth pasting conditions at the boundary at which investment commences.

The price process, on time intervals on which there is no entry/investment is given by (9). The firm is assumed to maximise expected present value; at some time  $\tau$  during an interval of non-investment, this is

$$V_\tau(p_\tau, K_\tau, Q_\tau) = E_\tau \left\{ \int_\tau^{\tilde{t}} p_t Q_t e^{-r(t-\tau)} dt + V(p_{\tilde{t}}, K_{\tilde{t}}, Q_{\tilde{t}}) e^{-r(\tilde{t}-\tau)} \right\}. \quad (39)$$

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<sup>14</sup> Notice that it would be a mistake to use the trend rate in the cost of capacity in place of the trend rate in hire prices, and it would also be a mistake to multiply a previously estimated certainty access price (that takes account of interest rates, depreciation and technological progress) by a factor equal to the ‘option multiplier’  $\lambda_1/(\lambda_1 - 1)$  since this would generally give a significant overestimate of the access price.

<sup>15</sup> For the usual reason that revenue goes to  $+\infty$  as quantity is cut to zero. Thus the firm, whatever its installed capacity would increase its profit by making less of that capacity available to the market; clearly this is not a plausible description of the limiting behaviour of demand price.

Here  $r$  denotes an appropriate discount rate<sup>16</sup> and  $E_\tau$  denotes the expectations operator, where expectations are formed at time  $\tau$ . The time  $\tilde{t}$  denotes the end of the period of non-investment, a point in time at which new investment adds to capacity.

The value function  $V_\tau$  is homogenous in prices and also linear in  $Q_\tau$ , and so can be written as

$$V_\tau \equiv \psi(x_\tau) K_\tau Q_\tau \quad \text{where } x_\tau = p_\tau / K_\tau \quad (40)$$

denotes the relative price. It is also useful to define the ‘per unit capacity’ value function (as in section 2) as

$$v_\tau(x_\tau, K_\tau) = V_\tau(x_\tau, K_\tau, Q_\tau) = \psi(x_\tau) K_\tau. \quad (41)$$

Thus,

$$\begin{aligned} V_\tau(p_\tau, K_\tau, Q_\tau) &= \psi(x_\tau) K_\tau Q_\tau \\ &= E_\tau \left\{ \int_\tau^{\tilde{t}} p_t Q_t e^{-r(t-\tau)} dt + \psi(x_\tau) K_\tau Q_\tau e^{-r(\tilde{t}-\tau)} \right\} \\ &= E_\tau \left\{ \int_\tau^{\tilde{t}} p_t Q_\tau e^{-(r+\theta)(t-\tau)} dt + \psi(x_\tau) K_\tau Q_\tau e^{-(r+\theta)(\tilde{t}-\tau)} \right\} \end{aligned}$$

and so

$$\psi(x_\tau) K_\tau = E_\tau \left\{ \int_\tau^{\tilde{t}} p_t e^{-(r+\theta)(t-\tau)} dt + \psi(x_\tau) K_\tau e^{-(r+\theta)(\tilde{t}-\tau)} \right\} \quad (42)$$

So, in terms of the per unit capacity value function,

$$v_\tau(x_\tau, K_\tau) = E_\tau \left\{ \int_\tau^{\tilde{t}} p_t e^{-(r+\theta)(t-\tau)} dt + v_{\tilde{t}}(x_{\tilde{t}}, K_{\tilde{t}}) e^{-(r+\theta)(\tilde{t}-\tau)} \right\}. \quad (43)$$

To reduce notational clutter, time subscripts and function arguments are dropped in what follows (where this results in no loss of intelligibility). From (43), the arbitrage equation (Dixit [1993, p. 15]) is

$$(r + \theta)v dt = p dt + E(dv). \quad (44)$$

Then applying Itô’s lemma and simplifying, this yields the following fundamental differential equation (appendix A3):

$$\frac{1}{2} \sigma^2 x^2 \psi'' + (\mu_p + \delta) x \psi' - (\theta + r + \delta) \psi + x = 0. \quad (45)$$

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<sup>16</sup> Empirically, solutions are not especially sensitive to the choice of discount rate. It is also possible to take  $r$  as the riskless rate of interest, so long as expectations are calculated in a suitably ‘weighted’ form. See Campbell, Lo and MacKinlay [1997, ch.9] for a general discussion.

This equation is identical to (13), for the competitive case. The general solution is thus given by (15) where as before,  $\lambda_1$  is given by (17) and  $A_2 = 0$  for the same reasons as in the competitive case. Thus

$$\psi_0(x) = A_0x + A_1x^{\lambda_1} \quad (46)$$

where the constant  $A_0$  is given by (16).

### Boundary Conditions:

Under monopoly, it pays to restrain investment in capacity (in order to enjoy higher subsequent hire prices). It is useful to define a ‘net value’ function for the monopoly case as

$$\pi_{\tilde{t}} = \psi(x_{\tilde{t}})K_{\tilde{t}}Q_{\tilde{t}} - K_{\tilde{t}}Q_{\tilde{t}} \quad (47)$$

where  $Q_{\tilde{t}} = A_{\tilde{t}}p_{\tilde{t}}^{\gamma}$ . Initially, there is a major investment in capacity (when  $\tilde{t} = 0$ ) and this represents the overall net value of the initial investment. The optimal initial choice of  $p_{\tilde{t}}$ , and so  $x_{\tilde{t}} (= p_{\tilde{t}} / K_{\tilde{t}})$ , at  $\tilde{t} = 0$  (equivalently, choice of capacity  $Q_0$ ) is one which maximises this function. At any subsequent time  $\tilde{t}$  at which the firm wishes to add further to capacity, (47) also measures the rate at which net value changes with the choice of trigger price  $p_{\tilde{t}}$ . At such boundaries, smooth pasting conditions apply; in this case the requirement is that<sup>17</sup>  $\partial\pi_{\tilde{t}} / \partial p_{\tilde{t}} = 0$  and

$\partial^2\pi_{\tilde{t}} / \partial p_{\tilde{t}}^2 = 0$ . Thus

$$\partial\pi_{\tilde{t}} / \partial p_{\tilde{t}} = 0 \Rightarrow \gamma[\psi(x_{\tilde{t}}) - 1] + x_{\tilde{t}}\psi'(x_{\tilde{t}}) = 0 \quad , \quad (48)$$

$$\begin{aligned} \partial^2\pi_{\tilde{t}} / \partial p_{\tilde{t}}^2 = 0 \Rightarrow \\ \gamma(\gamma - 1)[\psi(x_{\tilde{t}}) - 1] + (\gamma - 1)x_{\tilde{t}}\psi'(x_{\tilde{t}}) \\ + \{(1 + \gamma)x_{\tilde{t}}\psi'(x_{\tilde{t}}) + x_{\tilde{t}}^2\psi''(x_{\tilde{t}})\} = 0. \end{aligned} \quad (49)$$

After some rather tedious manipulations (see appendix, A8), it can be shown that (46), (48), (49) can be simplified to obtain the appealing simple condition that

$$x_{\tilde{t}} = \frac{\gamma}{(1 + \gamma)} \frac{\lambda_1}{(\lambda_1 - 1)} (\theta + r - \mu_p) \quad (50)$$

Thus the hitting value  $x_{\tilde{t}}$  is a constant as in the competitive case. Denoting this as

$\xi_M$ , this gives a nice comparison for pricing and output choices under monopoly *vis a*

<sup>17</sup> Since the problem involves a free choice for the trigger price, in this case the smooth pasting conditions require setting the first and second derivatives (with respect to this price) of the net value function to zero (Dixit [1993], Dumas [1991]).

vis those under competition. That is, in view of (16) and (26), the relative entry trigger price under monopoly is given as

$$\xi_M = \xi_c / (1 + 1/\gamma). \quad (51)$$

Dixit, Pindyck, and Sodal [1999], in dealing with an ‘all-or-nothing’ monopoly investment problem, interpret the option multiplier  $\lambda_1 / (\lambda_1 - 1)$  as an elasticity mark-up. Here, it can be seen in (50) that, when the firm controls both timing and level of investment, there is an additional (and ‘genuine’) demand elasticity mark-up. The associated monopoly entry price  $p_e^M(t)$  at which the firm is induced to add to capacity is given as

$$p_e^M(t) = \xi_M K_t = p_e^c(t) / (1 + 1/\gamma) \quad (52)$$

Thus in this model, the entry trigger price at which a monopolist starts to invest is given as the competitive entry trigger price (under uncertainty) plus the standard monopoly mark-up.<sup>18</sup> Hence, since  $\xi_M > \xi_c$ , the monopolist only adds to capacity when price reaches a higher value than would be the case under competition. Thus, prices will generally be higher under monopoly than under competition, whilst, concomitantly, installed capacity will be less. It is possible to compare the levels of investment; since the entry trigger price is higher in monopoly by the factor  $\xi_M / \xi_c = 1 / (1 + 1/\gamma)$ , then from (1), the initial level of installed capacity is lower than in the competitive case, this being captured by the ratio  $(\xi_M / \xi_c)^\gamma$ . These points are explored numerically in section 6.

## 5. MONOPOLY SUBJECT TO PRICE CAP

Given a firm which controls the market will tend to delay investment and will underinvest, a natural question to pose is whether some form of regulation might be beneficial. Hausman [1997], having established the competitive entry price, suggests this is the price the monopoly provider of capacity should be allowed to set if it is required to offer access. In practice, this would take the form of a price cap, an upper

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<sup>18</sup> In the single period Monopoly pricing problem under certainty, profit maximisation requires setting a price  $p_M = MC / (1 + 1/\gamma)$  (where  $MC$  denotes marginal cost which would also correspond to the competitive price  $p_c$  in a competitive market). That is,  $p_M = p_c / (1 + 1/\gamma)$ .

bound to the price that the monopolist is allowed to set. However, whilst such a regulatory price rule appears plausible, the question of how a monopolist will respond to such a price cap has not been formally addressed. This section shows that, if a price cap is imposed, the best choice of cap is indeed the competitive price – but that even when the price cap is optimally chosen, outcomes may diverge significantly from those that would occur in a competitive market.

The price the firm is allowed to set at time  $t$ , denoted  $p_t^s$ , is restricted by a price cap constraint of the form

$$p_t^s \leq \bar{p}_t \quad \text{where } \bar{p}_t = \bar{\xi} K_t \quad (53)$$

and  $\bar{\xi}$  is a constant chosen by the regulator. In what follows, it is shown that the general consequence of imposing a constraint, if it binds at all, is that the firm will tend to under-invest. It will also, when the price cap binds, tend to wish to impose quantity rationing. However, it is also shown that, if a price cap of this form is set, then choosing  $\bar{\xi} = \xi_c$ , the competitive industry value, realises the best, albeit imperfect, response from the firm.

In contrast to the cases analysed in sections 2-4, in this section there are three possible regimes. When current installed capacity is too ‘large’, price will be below the price cap – and the firm will wait (regime 1). As demand and technology evolve over time, the price at which capacity is utilised may hit the level imposed by the intertemporal price cap. At this point, the firm is price constrained. If the only constraint on the firm is the price cap, then it may choose to defer investment yet further. That is, it may choose to impose quantity rationing on customers (regime 2). However, if demand increases sufficiently, it can be expected that the firm will eventually wish to add to capacity (regime 3).

Let  $p_t$  stand for the *demand price*, the price which would reduce the level of demand to the currently available capacity, such that (2) gives the relationship between this demand price and capacity. However, the firm is required to set a price which

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satisfies the price cap (53). Denote the price the firm sets as  $p_t^s$ ; then clearly, the set price must satisfy,

$$p_t^s = \text{Min}[p_t, \bar{p}_t] = \text{Min}[p_t, \bar{\xi} K_t]. \quad (54)$$

where  $\bar{\xi}$  is a constant chosen by the regulator. If and when the demand price  $p_t$  is allowed by the firm to rise above the price cap  $\bar{p}_t$ , this entails quantity rationing; that is, from (1) demand is  $Q_t^d = A_t \bar{p}_t^\gamma$  whilst installed capacity is related to the demand price by  $Q_t = A_t p_t^\gamma$ . Thus, given  $\gamma < 0$ , when  $p_t > \bar{p}_t$ , clearly  $Q_t^d > Q_t$ ; there is excess demand.

Let  $\psi_0$  (as before) denote the solution when there is no investment and no price constraint whilst  $\psi_c$  denotes the solution when the price constraint applies but there is no investment. As before, the solution process involves characterising the process in each of the two non-investment regimes, followed by a study boundary conditions.

### **Regime 1: Unconstrained price, no investment.**

The solution here is naturally identical to that already established for the unconstrained monopoly firm. The fundamental equation is thus given by (45) and the solution is again

$$\psi(x) = A_0 x + A_1 x^{\lambda_1} + A_2 x^{\lambda_2} \quad (55)$$

where  $A_0$  is given by (16). As before, note that, as  $x \rightarrow 0$ , if value is to be finite, it must be that  $A_2 = 0$  or the solution explodes. Hence

$$\psi(x) = A_0 x + A_1 x^{\lambda_1} \quad (56)$$

The constant  $A_1$  is determined later via an analysis of the boundary conditions to the overall problem.

### **Regime 2: Price constrained, no investment**

In this region,  $p_t = \bar{\xi} K_t$ ; the arbitrage equation becomes

$$(r + \theta)v dt = \bar{\xi} K dt + E(dV) \quad (57)$$

The analysis parallels that for (44); it yields (compare with (45)):

$$\frac{1}{2} \sigma^2 x^2 \psi'' + (\bar{\mu}_p + \delta) x \psi' - (\theta + r + \delta) \psi + \bar{\xi} = 0, \quad (58)$$

as the fundamental equation. The general solution is thus

$$\psi_c(x) = \left( \bar{\xi} / (\delta + \theta + r) \right) + C_1 x^{\lambda_1} + C_2 x^{\lambda_2} \quad (59)$$

where the arbitrary constants  $C_1, C_2$  are determined by a consideration of boundary conditions.

**Analysis of transition boundary conditions:**

Let  $\tilde{t}_1$  denote a hitting time at which there is a transition between the regimes 1 and 2 whilst  $\tilde{t}_2$  denotes a hitting time at which new investment commences.

**Regime 1/2 boundary:**

Here the price cap binds, and the level here of the price cap is exogenously fixed by the regulator. Thus,  $x_{\tilde{t}_1} = \bar{\xi}$  and smooth pasting involves matching value and first derivatives (with respect to  $x$ ) for the solutions as they meet at the boundary (Dumas [1991]). Since  $v(x_t) = \psi(x_t)K_t$ , this entails<sup>19</sup>

$$\psi(\bar{\xi}) = \psi_c(\bar{\xi}), \quad (60)$$

$$\psi'(\bar{\xi}) = \psi'_c(\bar{\xi}) \quad (61)$$

**No investment/Positive Investment Boundary - Optimal Stopping:**

It is convenient to write  $x_{\tilde{t}_2} = \xi$  (which, as will be seen, is a constant if the smooth pasting conditions are to be satisfied). That is,  $\xi$  denotes the relative trigger market clearing price at which the firm would choose to start to invest when the firm is subject to a price cap. That is, whilst being required to set the price  $p_t^s \leq \bar{\xi} K_t$ , for its given level of capacity, the market clearing price  $p_t$  is the price the firm would choose to set if it was unconstrained. Whenever this market clearing price  $p_t \rightarrow \xi K_t$ , the firm will start to add to capacity (in what follows it is shown that in general,  $\xi > \bar{\xi}$ ). Since the choice of  $\xi$  is free, the smooth pasting conditions at  $\tilde{t}_2$  require the first and second derivatives of the value function in regime 2 to satisfy equivalent conditions to those specified above in the unconstrained monopoly case. That is,

$$\partial \pi_{\tilde{t}_2} / \partial p_{\tilde{t}_2} = 0 \Rightarrow \gamma[\psi(\xi) - 1] + \xi \psi'(\xi) = 0 \quad (62)$$

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<sup>19</sup> where, from the definitions of  $\psi, \psi_c$ , these are calculated as  $\psi(\bar{\xi}) = A_0 \bar{\xi} + A_1 \bar{\xi}^{\lambda_1}$ ,  $\psi'(\bar{\xi}) = A_0 + \lambda_1 A_1 \bar{\xi}^{\lambda_1 - 1}$ ,  $\psi_c(\bar{\xi}) = \left( \bar{\xi} / (\theta + r + \delta) \right) + B_1 \bar{\xi}^{\lambda_1} + B_2 \bar{\xi}^{\lambda_2}$  and  $\psi'_c(\bar{\xi}) = \lambda_1 B_1 \bar{\xi}^{\lambda_1 - 1} + \lambda_2 B_2 \bar{\xi}^{\lambda_2 - 1}$ .

$$\begin{aligned} \partial^2 \pi_{i_2} / \partial p_{i_2}^2 = 0 \Rightarrow \\ \gamma(\gamma-1)[\psi(\xi)-1] + (\gamma-1)\xi\psi'(\xi) \\ + \{(1+\gamma)\xi\psi'(\xi) + \xi^2\psi''(\xi)\} = 0 \end{aligned} \quad (63)$$

**Analysis of smooth pasting conditions:**

It is possible to solve the equations (60)-(63) to determine the arbitrary constants  $A_1, B_1, B_2$  and the value of  $\xi$  as a function of  $\bar{\xi}$  and the other parameters in the problem.

It is possible to solve the equations (56),(59),(62),(63) (see appendix A10) to obtain the value of  $\xi$  as a function of  $\bar{\xi}$  and the other parameters in the problem. After extensive manipulation, this can be written as<sup>20</sup>

$$\xi = \left( \left( \frac{\bar{\xi} - \xi_{cert}}{\xi_c - \xi_{cert}} \right) \left( \frac{\xi_c \bar{\xi}^{\lambda_2 - 1}}{1 + \eta \lambda_2} \right) \right)^{1/\lambda_2} \quad (64)$$

where as previously established,  $\xi_{cert} = \delta + \theta + r$  is the certainty competitive entry trigger relative price, and  $\xi_c = \left( \lambda_2^{-1} / \lambda_2 \right) (\delta + \theta + r)$  is the uncertainty competitive entry trigger relative price whilst  $\xi_M = \xi_c / (1 + \eta)$  is the uncertainty unconstrained monopoly entry trigger relative price.<sup>21</sup> Equation (64) gives the relative demand price; it is also useful to define the associated *demand price* at which the price capped monopolist would choose to start investing as

$$p_e^{MPC}(t) = \xi K_t \quad (65)$$

where  $\xi$  is defined in (64). Recall that this *demand price*  $p_e^{MPC}(t)$  at which entry is triggered is not that which is observed in the market place (because the price cap is binding); the monopolist holds back investment in capacity until a time arrives when demand is such that, if the monopolist was allowed to set price freely, it could hire out all its currently installed capacity at this price. The difference between the market clearing demand price in (65) and the price cap gives an index of the extent to which the firm is under-investing. An alternative measure is to consider the level of capacity each would install. Imagine a time at which, given current installed capacity levels,

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<sup>20</sup> Naturally, for a given specification of  $\bar{\xi}$ , it is also possible to determine the arbitrary constants  $A_1, B_1, B_2$  although this is not pursued here.

both a monopolist and a competitive industry would choose to invest. From (1) and (65), the monopolist would add capacity to the point where  $Q_e^{MPC}(t) = A_t (\xi K_t)^\gamma$ ; by contrast, the competitive industry would add capacity until  $Q_e^c(t) = A_t (\xi_c K_t)^\gamma$ . Hence the level of under-investment by the monopolist can be measured as

$$Q_e^{MPC}(t) / Q_e^c(t) = (\xi / \xi_c)^\gamma. \quad (66)$$

In section 6, in the sensitivity analysis, the behaviour of  $\xi$  as a function of  $\bar{\xi}$  is explored numerically. However, the following three general properties can be usefully established for the function  $\xi(\bar{\xi})$ . Firstly, since  $\lambda_2 < 0$ , fairly clearly, as the price cap  $\bar{\xi}$  is varied,

$$\lim_{\bar{\xi} \downarrow \xi_{cert}} \xi(\bar{\xi}) = +\infty. \quad (67)$$

This makes sense. At the certainty price, the firm gets zero net present value from investing only if it is *guaranteed* to be able to hire out its capacity at the certainty price for ever (recall, the certainty price is falling over time at the rate  $\delta$ ). Given uncertainty, there is the possibility it will not get this price if demand shifts adversely. Thus as the price cap is tightened to this level, the firm needs a higher and higher demand price to induce it to add capacity. In the limit, as  $\bar{\xi} \rightarrow \xi_{cert}$ , so  $\xi \rightarrow +\infty$ ; the firm will not install any capacity at all.

Secondly, and intuitively, is that

$$\lim_{\bar{\xi} \uparrow \xi_M} \xi(\bar{\xi}) = \xi_M. \quad (68)$$

This merely states that the price cap ceases to have an impact as it is relaxed to the level the unconstrained monopolist would choose. Thirdly, and more interestingly, it turns out that

$$\xi_c = \operatorname{argmin}_{\xi_{cert} \leq \bar{\xi} \leq \xi_M} \xi(\bar{\xi}) \quad (69)$$

(Proof: appendix A11). That is, if the object is to get as close to the competitive solution as possible, setting the price cap equal to the competitive industry entry trigger price is the best one can do. If one does set  $\bar{\xi} = \xi_c$ , then (64) simplifies to give

$$\xi(\xi_c) = (1 + \eta \lambda_2)^{-1/\lambda_2} \xi_c. \quad (70)$$

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<sup>21</sup> Relative to capacity unit price, that is.

Given uncertainty ( $\sigma_{AA}, \sigma_{KA}$  and/or  $\sigma_{AA} > 0$ ), then since  $\eta, \lambda_2 < 0$ , it follows that  $(1 + \eta\lambda_2) > 1$  and so

$$\xi(\xi_c) > \xi_c. \quad (71)$$

However, fairly clearly, from (70), using the definition (32), since  $\text{Lim } \xi_c = \xi_{cert}$  and  $\text{Lim } (1 + \eta\lambda_2)^{-1/\lambda_2} = 1$  it is also true that

$$\text{Lim } \xi(\xi_c) = \xi_{cert} \quad (72)$$

(Proof: appendix A11). Thus (72) establishes that, under certainty, setting the intertemporal price at the competitive entry level does indeed induce the monopolist to emulate the competitive market, whilst (71) establishes that, in the presence of uncertainty, the price capped monopolist will tend to under-invest and wait too long to invest. In regime 3, in the absence of investment and with the firm not adding to capacity, demand is quantity rationed. This follows from (1); when price capped, quantity demanded is  $Q_t^d = A_t \bar{p}_t^\gamma$  whilst installed capacity is related to the demand price by  $Q_t = A_t p_t^\gamma$ . Thus when demand price exceeds the price cap ( $p_t > \bar{p}_t$ ), given  $\gamma < 0$ , so demand exceeds installed capacity;  $Q_t^d > Q_t$ .

The essential reason why the firm may choose to ‘under-invest’ (relative to the competitive benchmark) is that, given demand uncertainty, it will take into account the possibility of future adverse market movements when choosing how much capacity to add. Restraining investment in periods where the price cap is binding allows it to enjoy higher prices if demand falls at a later date. The possible numerical magnitude of this type of effect is examined in some detail in section 6. As indicated there, the level of under-investment can be substantial.

## 6. SENSITIVITY ANALYSIS

This section examines the empirical significance of the above results by conducting a sensitivity analysis focused on ballpark figures commonly used in the Telecom industry. These are given in table 1.

**Insert Table 1 here**

One (rather crude) way to tackle demand uncertainty lies in focusing on the revenue process, since it is straightforward to gather sales revenue data for any given firm.

The revenue process in unconstrained regions parallels the price process and is given as

$$dR_t = \mu_R R_t dt - \eta R_t (\sigma_{AA} d\bar{w}_t^A + \sigma_{AK} d\bar{w}_t^K) \quad (73)$$

where

$$\mu_R = -\eta\alpha - (\eta+1)\theta + \frac{1}{2}\eta(\eta+1)(\sigma_{AA}^2 + \sigma_{AK}^2), \quad (74)$$

so the variance is<sup>22</sup>

$$v_R = \eta^2 (\sigma_{AA}^2 + \sigma_{AK}^2). \quad (75)$$

Notice that the variance of this process is the same as for the price process; compare (73) with (9). The variance for capacity unit cost is, from (3),

$$v_K = \sigma_{KA}^2 + \sigma_{KK}^2. \quad (76)$$

Define  $\sigma_K = (v_K)^{1/2}$  and  $\sigma_R = (v_R)^{1/2}$  as the instantaneous standard deviations and the correlation between the revenue and capacity cost processes as

$$\rho = \frac{COV(dK/K, dR/R)}{[Var(dR/R)Var(dK/K)]^{1/2}} = \frac{-\eta(\sigma_{AA}\sigma_{KA} + \sigma_{AK}\sigma_{KK})}{\sigma_K\sigma_R}. \quad (77)$$

As discussed in section 2, let  $\sigma_{AK} = 0$  whilst allowing that  $\sigma_{KA}$  may be non-zero.

That is, technological progress may be affected by demand side factors but not vice versa. Then it is straightforward to parameterise the model in terms of the parameters

$r, \alpha, \delta, \theta, \gamma, \rho, \sigma_K, \sigma_R$ , since using (75)-(77), it follows that  $\sigma_{KK} = \sigma_K \sqrt{1 - \rho^2}$ ,

$\sigma_{AA} = -\sigma_R / \eta$  and  $\sigma_{KA} = \rho \sigma_K$  (see appendix A12). Given values for these

parameters, and a value for capacity cost at the time entry takes place (standardised at  $K_0 = \text{£}100$ ), it is straightforward to first compute  $\sigma_{AA}, \sigma_{KK}, \sigma_{KA}$  and then values for

$\xi_{cert}, \xi_c, \xi_M$ , the relative entry trigger prices. These hold for all  $t \geq 0$ ; however,

multiplying these by the initial benchmark figure  $K_0 = 100$  also gives the initial entry

trigger prices as  $p_e^{cert}, p_e^c, p_e^M$ . Given any specification for  $\bar{\xi}$  the value for  $\xi(\bar{\xi})$ ,

the relative demand price at which the price constrained monopolist would choose to

enter, can be obtained from (64), and hence the value for the demand price  $p_e^{MPC}$ .

<sup>22</sup> The derivation parallels that for price. The linkage is very approximate since, within the model, volatility will differ on intervals of zero investment from intervals on which there is positive investment. It is possible to pursue this calculation, but since the general thrust is merely to obtain ballpark figures, this is not pursued. The calculation is also crude in that it is based on aggregate sales data.

The benchmark (risky) discount rate is taken as 10%, although a range up to 30% is reported; the value for  $\theta$  is 10% with a range from 0-50% is considered (infinite life down to 2 years expected life); the value for elasticity is  $-2$ , with a range from  $-1$  to  $-8$ . The trend in demand,  $\alpha$ , is set to zero, with a range from  $-30$  to  $+30\%$ . The benchmark for  $\delta$  used in Table 1 as suggested in Hausman [1997] for the Telecom industry, at 8%, with a range from 0-12%. Looking at sales data for the FTSE100 companies for example, the average across all companies for  $\sigma_R$  was about 15% (with most lying between 5% and 30%) over the last 15 years. Financial regulation has an impact of course, and many of the European Telecom companies have quite low values (4% for British Telecom, Telecom Italia, 6% for Deutsche Telecom etc.). The volatility of share prices (or price indexes) is often referred to as giving some idea for volatility. For example, Dixit and Pindyck [1994] use 20% as an estimate (based on the volatility of the S&P index). In Table 1, the benchmark value used for both revenue and capital unit cost is 20%; this is arguably a little on the high side, especially for the latter. However, since it is argued in this paper that uncertainty has a rather smaller impact than appears to be the case for the ‘all-or-nothing’ monopolist, it seems reasonable to take such a value, and to use sensitivity analysis to look at a range from 0% to 40%. Finally, an arbitrary value of £100 is used for unit capacity cost.

### Table 2 here

Table 2 uses the benchmark values of Table 1, and reports the impact of unilaterally varying each parameter in turn. It shows that the rate of growth in demand  $\alpha$ , the elasticity of demand  $\gamma$ , the correlation between technological progress and demand  $\rho$ , the rate of depreciation  $\theta$ , and the discount rate  $r$ , over their plausible ranges, have an effect, but sensitivity is really quite limited, and even with quite wide parameter variations, the relative trigger price  $\xi_c (= p_e^c / p_e^{cert})$  hardly gets above 50% (note the ranges reported in tables 2 and 5 feature a rather wider range for parameter values than might be construed as a 95% confidence interval). Uncertainty *per se* has an impact, but only when volatility is fairly high. When  $\sigma_K = \sigma_R = 30\%$  or less, the mark-up is less than 60%. Panel (b) of table 2 illustrates the fact that the impact on the initial level of installed capacity demand is not much affected by  $r$  or  $\rho$  but is relatively sensitive to the other parameters, especially demand elasticity (as one would

expect). Uncertainty enters through  $\sigma_R$  and  $\sigma_K$ ; whilst it is possible to vary each independently, for brevity, the right hand column in both panels merely reports the impact for the case where they are set equal and then varied. The price ratio naturally increases with volatility; for example the entry trigger price is only 1.08 times the certainty price when  $\sigma_R (= \sigma_p) = 10\%$  but is 1.28 times it when  $\sigma_R (= \sigma_p) = 20\%$ . However, to get a relative price of twice or more requires really quite high volatility (40% or so).<sup>23</sup>

As suggested in previous work, the ‘option value’ multiplier  $\lambda_1 / (\lambda_1 - 1)$  for Telecoms can easily be of the order of magnitude 2-4. However, as explained above, the option multiplier that multiplies the certainty price is  $(\lambda_2 - 1) / \lambda_2$  and this takes a generally significantly smaller value. This is illustrated in Table 3 below.

**Table 3 here**

Notice that the multiplier  $\lambda_1 / (\lambda_1 - 1)$  lies above, and converges on the value 1.866 as predicted by the limit analysis in section 3 and appendix A4 (see equation (A.60)), where it is shown that  $\text{Lim } \lambda_1 / (\lambda_1 - 1) = (\theta + r + \delta) / (\theta + r + \eta[\alpha + \theta]) (= 1.866 \text{ at Table 1 benchmark values})$ . Note that column 2 gives the implied value for the volatility measure  $\sigma$  defined in (14), for reference purposes.

**Table 4 and Figures 1, 2 here**

Table 4 gives the response of the price capped monopolist to variations in the tightness of the price cap whilst Figure 1 illustrates the relative demand price effect, and Figure 2, the impact on relative level of investment (relative to the competitive uncertainty case). Notice, in Figure 1, that choosing the competitive level for the price cap gives the best response, as in (69), whilst the limit behaviour indicated by (67) and (68) is also manifest. At the Table 1 benchmark values, the imperfect nature of the firm’s response is clear. Thus the certainty competitive entry price is £28, under uncertainty, it is £35.94 but even with the best choice for the price cap, setting  $\bar{\xi} = \xi_c = 0.3594$ , the initial demand price at which the price capped firm enters is

<sup>23</sup> Of course, the sensitivity analysis only considers moving each parameter value unilaterally. If *all* parameters are moved to levels which tend to increase the mark-up, a larger figure can be produced. For example, pushing  $\alpha \rightarrow -0.1$ ,  $\gamma \rightarrow -3$ ,  $\theta, \delta \rightarrow 0$  and  $r \rightarrow 0.05$ , then  $p_e^c / p_e^{cert} = 2.16$ . However, these are all rather extreme values.

£47.945 and, if the firm was commencing at time zero, it would install only just over 56.2% of the capacity the competitive industry would supply. This illustrates the general argument presented above that the price cap cannot be used to realise the competitive outcome in the presence of uncertainty.

#### Table 5 here

Table 5 explores the consequences of varying each parameter from the Table 1 values, whilst maintaining an optimal price cap ( $\bar{\xi} = \xi_c$ ), on the relative demand price that would induce the price capped monopolist to start to invest,  $\xi$ , where  $\xi$  is given by (64). That is, the price cap is optimally reset for each variation from the benchmark case. This gives some idea of the conditions under which the optimally set price cap is most effective. The first panel in Table 5 reports the relative entry price  $p_e^{MPC} / p_e^c = \xi / \xi_c$ . In the benchmark case this takes the value 1.334 (demand price at which entry occurs is 33.4% higher than that for the competitive case). As can be seen, for all variations, the price cap performs fairly poorly - except for the case where uncertainty is small. Of course, in the limit, under certainty, the price cap works perfectly well, in view of (72); this is illustrated in the last column in both panels of table 5.

## 7. CONCLUDING COMMENTS

This paper suggests that uncertainty does have an impact on the price at which firms choose to invest in capacity, but that, in the competitive industry case, relative to the standard certainty estimate for the access price, uncertainty probably adds less than a 60% mark-up (and perhaps significantly less). It also shows that Monopoly *per se* may be a problem; if the incumbent (e.g. network operator) is free to set prices and choose the capacity of the network, as in the certainty case, it will tend to invest too little and wait too long, and for too high an entry price, before choosing to invest. The extent to which it does so depends on estimates of the various parameters involved, but most notably, on the estimate of the demand elasticity; as in the single period monopoly problem, unless demand is really quite elastic, the mark-up is likely to be substantial. The regulated firm will also have an incentive to try to claim that the access price should in fact be higher than is indicated in the competitive case (in this model, by the elasticity mark-up  $\gamma/(1+\gamma)$ ) and so will have an incentive to try to

massage estimates of capital costs (and other parameters) to try to achieve a higher allowed level for the access price.

Under certainty, it was shown that subjecting the firm to a well chosen intertemporal price cap would resolve this problem by forcing the monopoly firm to emulate the competitive ideal (assuming the trend in demand is not too negative). However, under uncertainty, it has been shown that an intertemporal price cap may be beneficial, but cannot be used to fully realise the competitive solution. The essential problem is that when the price cap is set at a level below the monopoly entry trigger price, the firm does not start investing as soon as the price cap is hit. It delays investment, and sheds demand through quantity rationing, until a point is reached where demand is sufficiently strong to motivate it to invest. The rationale for not expanding capacity when price capped is that the firm is taking account of the future possibility that demand may fall sufficiently for the price cap no longer to bind. Having less installed capacity allows it to enjoy higher prices later on when the price cap does not bind. Thus, as in the unconstrained case, the firm subject to a price cap also realises option value through ‘waiting to invest’.

To sum up, although in this model the best choice of price cap appears to be that where the firm is allowed to charge prices not higher than would arise in a competitive industry, it has been shown that a price capped firm will have a general incentive to both under-invest and to quantity ration. This would be manifest in service industries by the firm allowing the quality of service to degrade. For example, particularly in periods where there is a significant upswing in demand such that the price cap binds, the firm has a clear incentive to drag its feet on investment, an incentive to find excuses for why it cannot keep up with such ‘unexpected’ upswings in demand etc.<sup>24</sup> Although there are important factors (such as brand loyalty and reputation), which are not explicitly modelled here, that would tend to reduce this incentive, it does suggest that some care must be given to the design of price cap regulation. One would predict that quantity rationing and under-investment are likely to be more prevalent in industries where brand loyalty or quality of service are

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<sup>24</sup> In Telecoms, quantity rationing would manifest itself through falls in the quality of service. Interestingly, this kind of problem is beginning to manifest itself in the UK - although, perhaps, for

regarded as less important factors, and also perhaps in industries where quantity rationing is harder to monitor. In such industries, if price cap regulation is introduced, there may be a need to also consider how quality/service standards are monitored and enforced.

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rather different reasons since the recent burgeoning of internet traffic also calls into question the intertemporal structure of retail tariffs (which time periods are peak and off peak etc.).

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