Combined heat and power economics

Ian M. Dobbs

Combined heat and power is a joint product system generating electricity and heat, both relatively 'non-storable' commodities with temporally fluctuating demands. A 'peak-load pricing' model of the CHP system is developed to investigate the pricing and capacity decisions involved in this two market system. Various market structures are considered and the pricing implications investigated. The solutions have several interesting features, including possible peak-load switching. Where a decentralized CHP system exports electricity to the central system and operates in a local heat market, then, ceteris paribus, *higher central electricity system prices imply lower optimal local heat market prices. In this latter case, the tariff offered by the electricity supply industry for CHP generated electricity has implications for investment and for pricing in the heat market – this tariff is therefore examined further. The case for marginal cost pricing is shown to have several attractive features.*

Keywords: Economics; Electricity; CHP

Combined heat and power (CHP) is the principle of recovering heat normally rejected in the production of electricity and power, the recovered heat being of a 'grade' useful for various purposes of which perhaps the potentially most important in the future could be district heating.* This paper, however, is only concerned with analysing the economic and technical structure of the CHP joint production generating system *per se* and not with its many possible applications.

The attraction of CHP is that by producing heat, power and electricity jointly they can be, overall, more thermally efficient than when provided separately. No doubt stimulated by recent surges in the real price of energy (also expected future prices) several studies of CHP have recently been undertaken,[2–9] and this long established† technique has become the focus of much attention.

CHP is basically an investment in which a future stream of energy cost savings can be achieved by additional initial capital outlay. There are however many aspects to the problem since it also links with questions of resource conservation and the economic decentralization of electricity supply.‡ As it is not possible in this paper to discuss all of the issues pertinent to the economic analysis of CHP,§ the focus will be on pricing and investment decisions.

CHP plant outputs may form inputs for an industrial concern, a local community, for national electricity and heat grids and so on. The way such plant is operated depends upon the 'price schedules' it faces. In the UK it is often the case that the local CHP plant exports and/or imports electricity from the national grid whilst generating locally used heat. Terms offered to such local electricity producers by the electricity supply industry (ESI) may thus be important in determining a CHP scheme's economic viability. Certainly in the UK there have been criticisms of (ESI) policy in this respect; sufficient at any rate to stimulate the ESI to publish information[12] in defence of their policy.♦

* For most important and also the most recent study of the potential for district heating see Marshall.[1]

† The Singer factory on Clydebank for example has had CHP since 1898.[10]

‡ These issues are discussed in some detail by Dobbs.[11]

§ Other aspects (reliability, cost benefit analysis, decentralization) will be the subject of further papers.

♦ 'The Electricity Boards are well aware of the criticism levelled at them, that schemes for the combined production of heat and electricity would go ahead more frequently were it not for Electricity Boards' terms. . .'[13]

The author is with the Department of Economics, University of Newcastle upon Tyne, Newcastle upon Tyne, NE1 7RU, UK.

Final manuscript received 30 September 1981.

 0140-9883/82/040276-10 $03.00 © 1982 Butterworth & Co (Publishers) Ltd

The pricing problem

CHP produces primarily two products – electricity and heat. In the most general case, the demand for these products is temporarily fluctuating; an analysis of the pricing problem will therefore be initiated by developing a 'Williamson-type'[14] deterministic peak load pricing model based upon 'Surplus maximization'.¶ Such a model is useful for characterizing the nature of the CHP technology as it lends itself to the investigation of changes in the objective function and also in the market structures.

The welfare maximizing monopoly case

The first case considered is that of the isolated CHP plant providing for local electricity and heat markets. In practice this might represent a CHP scheme supplying the needs of a small isolated community (Finland provides some such cases). It is assumed that neither electricity nor heat can be stored economically** so that peak-load pricing is in principle applicable in both cases.

CHP technology can be divided into two categories; (a) where electricity and heat production are positively correlated and (b) where they are negatively correlated. Most prime movers belong to category (a) (diesels, dual-fuel engines, petrol engines, gas turbines) whilst steam turbines belong in (b). In (a) greater electricity output implies greater waste heat output. In (b) heat is 'generated' by bleeding steam from intermediate points on the steam turbine, thus generating heat at the expense of electricity power output. The following analysis is based on a type (a) CHP system (type (a) and (b) systems are considered by Dobbs[16]).

The typical CHP system is multi-set in structure – unlike the ESI system which operates several different types of plant of varying ages, whereas CHP plants are usually similar in both age, efficiency and size. It can be shown that although the part-load performance of an individual prime mover may be non-linear over certain output ranges, the behaviour of the multi-set system, if operated in cost-minimizing manner, approximates very well to the assumption that production is homogeneous of degree one, overall efficiencies remaining fairly constant throughout the output range.[17] This assumption of linearity is therefore adopted here.††

¶ The assumptions underlying the use of such a maximand are well known and need no recapitulation here. The adoption of a deterministic framework is primarily for simplicity – a justification can be constructed however upon the analysis of Crew and Kleindorfer[15] who conclude that '. . . where uncertainty is small, deterministic problems do in fact serve as reasonable approximations for corresponding problems under uncertainty'.

** Heat is in practice more readily storable than electricity and it is also the case that the larger scale heat generation systems have significant thermal inertia and effective thermal storage; Short-run supply thus does not in general have to correspond exactly to demand although it must do so over longer time periods.

†† For diesel and dual-fuel engines, heat recovery does not significantly affect electricity generation efficiency. This is also true, but to a less good approximation, for gas turbines.

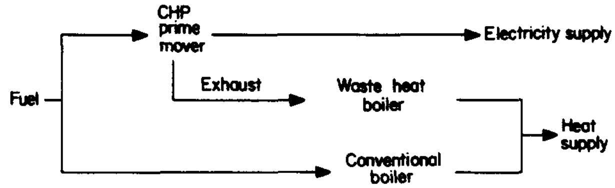

Figure 1. The CHP system.

The CHP system generates electricity and heat (here called 'waste heat'). Additional heat may be required – if so it is assumed that this is generated using conventional heat raising boilers.‡‡ It is assumed that the heat required is of constant 'grade'.§§ Time periods are defined for intervals during which both heat and electricity demand schedules are stationary. Let there be n such time periods, each of duration t_i with total time normalized to unity so that:

$$\sum_{i=1}^{n} t_i = 1 \tag{1}$$

The CHP structure is represented diagramatically in Figure 1. In each period i fuel input (F_i) is divided between the prime mover (F_{pi}) and conventional boiler (F_{bi}).

$$F_i = F_{pi} + F_{bi} \tag{2}$$

Electricity supplied (E_i) must be less than or equal to that generated:

$$E_i \leqslant e \cdot F_{pi} \tag{3}$$

where e denotes the electricity generating efficiency of the prime mover (assumed constant; see above). Heat supply (H_i) is the sum of that generated by the prime movers waste heat boilers (H_{wi}) and conventional boilers (H_{bi}):

$$H_i = H_{wi} + H_{bi} \tag{4}$$

The heat generating efficiencies are respectively w, b; heat supply in each case must be less than or equal to that generated;

$$H_{wi} \leqslant w \cdot F_{pi} \tag{5}$$

$$H_{bi} \leqslant b \cdot F_{bi} \tag{6}$$

The capacity constraints are:

$$H_{wi} \leqslant Q_w \tag{7}$$

‡‡ In the case of gas turbine systems it is possible to generate additional heat through the use of an 'after-firing' facility whereby fuel is injected into the hot exhaust gases and fired in the 'waste' heat boiler. This method of additional heat raising is particularly efficient (ie ~95%). There is a limit to the additional heat that may be raised in this fashion (the oxygen content of the exhaust limits this facility) – additional heat would then be raised in conventional boilers. This case is analysed in Dobbs.[17]

§§ By 'grade' is meant the physical conditions which define the nature of the heat (temperature; if steam, pressure etc). This has to be assumed because the efficiency with which heat is generated depends, *ceteris paribus*, upon its 'grade'; to generate heat at higher temperature generally implies a lower heat raising thermal efficiency. This is discussed in the context of CHP in Dobbs.[18]

$$H_{bi} \leqslant Q_b \tag{8}$$

$$E_i \leqslant Q_e \tag{9}$$

where Q_e, Q_b, Q_w are installed capacities for which the unit capacity costs are $\beta_e, \beta_b, \beta_w$. P_f denotes the input fuel price; f_i, E_i the electricity demand schedule and electricity output in period i; g_i, H_i the heat demands schedule and heat output in period i.

Defining welfare as the sum of producers' and consumers' surplus, the maximand is:

$$W = \sum_{i=1}^{n} t_i \int^{E_i} f_i(E)\mathrm{d}E + \sum_{i=1}^{n} t_i \int^{H_i} g_i(H)\mathrm{d}H - p_f \Sigma t_i F_i - Q_e\beta_e - Q_w\beta_w - Q_b\beta_b \tag{10}$$

The first term represents the total surplus derived from electricity consumption, the second from heat consumption, the third the cost of total fuel consumed.

Some common sense simplifications are possible; given typical values of the parameters, it is never sensible to generate more electricity than is required.♦♦ The implication is that Equation (3) holds with equality. A similar argument applies for Equation (6). Equations (3), (4) and (6) then imply that:

$$F_i = \frac{E_i}{e} + \frac{H_{bi}}{b} = \frac{E_i}{e} + \frac{(H_i - H_{wi})}{b} \tag{11}$$

and Equations (3) and (5) that:

$$H_{wi} \leqslant \frac{w}{e} E_i \tag{12}$$

Equation (10) is therefore to be maximized subject to the constraints:

$$H_{wi} \leqslant \frac{w}{e} E_i \tag{13}$$

$$H_{wi} \leqslant H_i \tag{14}$$

$$H_{wi} \leqslant Q_w \tag{15}$$

$$H_i - H_{wi} \leqslant Q_b \tag{16}$$

$$E_i \leqslant Q_e \tag{17}$$

The maximand is concave and the constraints convex (linear) so a unique solution exists. The Lagrangian is defined as:

$$\mathcal{L} = W + \sum_{i=1}^{n} \lambda_{1i}\left(\frac{wE_i}{e} - H_{wi}\right) + \sum_{i=1}^{n} \lambda_{2i}(H_i - H_{wi}) + \sum_{i=1}^{n} \lambda_{3i}(Q_w - H_{wi}) + \sum_{i=1}^{n} \lambda_{4i}(Q_b - H_i + H_{wi}) + \sum_{i=1}^{n} \lambda_{5i}(Q_e - E_i) \tag{18}$$

which yields the following necessary first order conditions:

$$-\beta_e + \Sigma\lambda_{5i} \leqslant 0; \quad Q_e \geqslant 0 \tag{19}$$

$$-\beta_w + \Sigma\lambda_{3i} \leqslant 0; \quad Q_w \geqslant 0 \tag{20}$$

$$-\beta_b + \Sigma\lambda_{4i} \leqslant 0; \quad Q_b \geqslant 0 \tag{21}$$

$$t_i P_{ei} - \frac{P_f t_i}{e} + \lambda_{1i}\frac{w}{e} - \lambda_{5i} \leqslant 0; \quad E_i \geqslant 0 \quad i = 1, \ldots n \tag{22}$$

$$t_i P_{hi} - \frac{P_f t_i}{b} + \lambda_{2i} - \lambda_{4i} \leqslant 0; \quad H_i \geqslant 0 \quad i = 1, \ldots n \tag{23}$$

$$\frac{t_i P_f}{b} - \lambda_{1i} - \lambda_{2i} - \lambda_{3i} - \lambda_{4i} \leqslant 0; \quad H_{wi} \geqslant 0 \quad i = 1, \ldots n \tag{24}$$

$$H_{wi} \leqslant \frac{w}{e} E_i; \quad \lambda_{1i} \geqslant 0 \quad i = 1, \ldots n \tag{25}$$

$$H_{wi} \leqslant H_i; \quad \lambda_{2i} \geqslant 0 \quad i = 1, \ldots n \tag{26}$$

$$H_{wi} \leqslant Q_w; \quad \lambda_{3i} \geqslant 0 \quad i = 1, \ldots n \tag{27}$$

$$H_i - H_{wi} \leqslant Q_b; \quad \lambda_{4i} \geqslant 0 \quad i = 1, \ldots n \tag{28}$$

$$E_i \leqslant Q_e; \quad \lambda_{5i} \geqslant 0 \quad i = 1, \ldots n \tag{29}$$

$\lambda_{5i}, \lambda_{4i}, \lambda_{3i}$ here represent the 'shadow' prices associated with, respectively, in the ith period, prime mover capacity, waste heat boiler capacity and conventional boiler capacity. P_{ei} and P_{hi} are the prices to be set in the electricity and heat market in period i. Complementary slackness is assumed to hold throughout (19) to (29).

Solutions to this problem, for given parameter values (e, w, b etc), depend essentially upon the schedules of electricity and heat demand. The nature of the solution is intrinsically straightforward: CHP (for the particular case under consideration – diesel, dual-fuel or gas turbine driven systems) generates electricity and waste heat in fixed proportions (viz: 1 unit fuel input generates e units of electricity and w units of heat). In some cases further heat is raised by conventional boilers (or in the gas turbine case, by 'after-firing').¶¶ The marginal cost of supplying heat and electricity then depend upon which constraints are binding.

♦♦ In principle this might occur if conventional boiler plant were relatively very expensive to install whilst at the same time there was a high willingness to pay for heat; in such a situation it might make sense to uprate prime mover output to generate more waste heat (to avoid having to install conventional boiler capacity), the surplus of electricity then being 'disposed of'.

¶¶ See footnote §§.

It is possible to derive prices for all possible demand conditions for the above model. A complete exposition, even for the 2-period model is both algebraically tedious and lengthy. The approach is therefore to outline the solution for a small number of cases, illustrating in particular the possibility of peak-period switching.

The non-trivial solution would require $Q_e, Q_w, E_i, H_{wi}, H_i > 0$ in all periods; this implies conditions (19), (20), (22), (23) and (24) hold with equality. (22), (23) and (24) imply that:

$$P_{ei} = \frac{P_f}{e} + \frac{\lambda_{5i}}{t_i} - \frac{\lambda_{1i}}{t_i}\frac{w}{e} \tag{30}$$

$$P_{hi} = \frac{P_f}{b} + \frac{\lambda_{4i}}{t_i} - \frac{\lambda_{2i}}{t_i} \tag{31}$$

or

$$P_{hi} = \frac{1}{t_i}(\lambda_{1i} + \lambda_{3i}) \tag{32}$$

These pricing equations parallel the standard peak-load results. In Equations (30) and (31) the first two terms of the right-hand sides correspond exactly to the prices for stand-alone electricity and heat systems. In Equations (30) P_f/e is the marginal generation cost and λ_{5i}/t_i the capacity related cost for electricity whilst for heat in Equation (31) these are P_f/b and λ_{4i}/t_i respectively. The other terms represent the benefits of interaction between the two markets. Conditions in one market can thus effectively reduce marginal cost of output in the other market – and vice versa. The usual peak/off-peak effects are present – in a firm peak case with period one as peak $\lambda_{51} = \beta_e$, $\lambda_{52} = 0$, or with shifting peak $\Sigma\lambda_{5i} = \beta_e$ when there is a demand on electrical capacity in both periods. Just one case will be examined in detail. Suppose $Q_b > 0$ so that (21) holds with equality. Clearly (26) and (28) are related:

$$H_{wi} < H_i \Leftrightarrow H_i - H_{wi} = Qb$$

$$\lambda_{2i} = 0 \Leftrightarrow \lambda_{4i} > 0$$

In the two period problem with $Q_w > 0$ there must be demand on capacity in one period or the other or both. Let:

$$H_{w1} = H_1 \Rightarrow \lambda_{31} > 0$$

$$H_{w2} < H_2 \Rightarrow \lambda_{32} = 0; \quad \lambda_{31} = \beta_w$$

If $H_{w1} = H_1$ then since $Q_b > 0$ it must be that the demand on conventional boiler occurs in period 2: $\lambda_{42} > 0$ and $\lambda_{41} = 0$ so $\lambda_{42} = \beta_b$. Equation (31) implies $P_{h2} = P_f/b + \beta_b/t_2$; this implies that to expand period two output requires additional conventional boiler capacity. In period 1, $H_{w1} = H_1$. If Equations (25) or (27) do not bind then from Equation (32) $P_{hi} = 0$; clearly therefore (25) or (27) must bind. If (27) binds the price is $P_{h1} = \beta_w/t_i$; this corresponds to the case where waste heat is available but waste heat boiler capacity would have to be expanded to increase heat output. If (25) binds, then to expand *WH* output requires increasing electrical output too. Let us take the case where (27) binds and examine the implications for the electricity market. In this case heat prices are:

$$P_{h1} = \beta_w/t_1$$

$$P_{h2} = \frac{P_f}{b} + \beta_b/t_2$$

and optimal capacities may be computed since:

$$P_{h1}^* = g_1(H_1^*) = \beta_w/t_1; \quad Q_w^* = H_1^* = H_{w1}^*$$

Optimal conventional boiler capacity is deduced from the period two price equation:

$$P_{h2}^* = \frac{P_f}{b} + \frac{\beta_b}{t_2} = g_2(H_2^*)$$

$$Q_b^* = H_2^* - \frac{wE_2}{e}$$

where wE_2/e is the waste heat output from the prime mover. Equation (30) may be rewritten using Equation (32) as:

$$P_{ei} = \frac{P_f}{e} + \frac{\lambda_{5i}}{t_i} - \frac{w}{e}\left(P_{hi} - \frac{\lambda_{3i}}{t_i}\right)$$

so, for the case under consideration the prices are tabulated in Table 1.

In this case therefore the cost of expanding electricity output in period 1 is unaffected by the heat market whilst in the heat market the cost is that of expanding waste heat boiler capacity. In period 2 the cost of expanding heat output is that it requires an addition to boiler capacity. The associated cost of expanding electricity is however affected by the conditions in the heat market – in effect, expanding electricity output increases waste heat output – there is spare waste heat boiler capacity so this allows a reduction in conventional boiler capacity and output in the heat market – all this is a byproduct of expanding electricity output – hence the reduction of

$$\frac{w}{e}\left(\frac{P_f}{b} + \frac{\beta_b}{t_2}\right)$$

in marginal cost.

Note that the above results are reversed if it is assumed that $H_{w1} < H_1$ and $H_{w2} = H_2$. The former case however points to the possibility of peak-load 'switching', since in Table 1, in the heat market, if period 1 is the conven-

Table 1.

Period	Heat price	Electricity price
1	β_w/t_1	$\frac{P_f}{e} + \lambda_{51}/t_i$
2	$\frac{P_f}{b} + \beta_b/t_2$	$\frac{P_f}{e} + \frac{\lambda_{52}}{t_2} - \frac{w}{e}\left(\frac{P_f}{b} + \frac{\beta_b}{t_2}\right)$

tional peak period, $P_{h2} > P_{h1}$ (since β_w is only slightly larger than β_b and t_1 and t_2 will be of comparable magnitudes). This is possible because although it may be that with period 1 as the peak period, $H_1 > H_2$, the demand on conventional boiler is in fact

$$H_i - \frac{wE_i}{e}$$

since heat output of wE_i/e can be provided by waste heat boiler ie it is possible that

$$H_1 - \frac{wE_1}{e} < H_2 - \frac{wE_2}{e}$$

If the CHP pricing Equations (30) and (31) are compared to separated systems operating peak-load pricing (if the efficiencies were the same these would be

$$P_{ei} = \frac{P_f}{e} + \lambda_{5i}/t_i; \quad P_{hi} = \frac{P_f}{b} + \lambda_{4i}/t_i\Big)$$

it would appear that they are generally lower – this is not the case however since the peak/off-peak periods for the joint-product system do not necessarily correspond to those for the separate systems – the peak-switching noted above is a case in point. It also does not necessarily follow that both heat *and* electricity consumers benefit from the change to CHP.

It is in principle possible for there to be positive optimal CHP capacity but with one set of consumers being subject to a welfare loss whilst the other group makes a welfare gain. Clearly, however, net 'welfare' as defined must increase if positive CHP capacity is to be optimal.

Alternative market structures

In the case where the CHP scheme is operated to maximize profits, ie where it operates as a local monopoly supplier of electricity and heat the first two terms of Equation (10) are replaced by the revenue functions:

$$\sum_{i=1}^{n} t_i P_{ei} E_i$$

and

$$\sum_{i=1}^{n} t_i P_{hi} H_i$$

The analysis is identical except that marginal revenue terms replace prices.

If the periods are of relatively short duration and the system is of significant size, then what is often referred to as 'thermal inertia' relaxes the constraint (14) to some extent. 'Thermal inertia' expresses the idea that any heat generation system will possess some inherent capability for heat storage (which may be extended by investment in further heat storage equipment), a capability which tends to increase with the size of the system. This allows the possibility of some degree of 'smoothing' of the heat demand/supply relation since heat can to an extent be generated in 'off-peak' periods for supply in 'peak' periods. The effect on the model specification is as follows: Equation (4) no longer needs to hold, but must be replaced by an overall heat conservation equation:

$$\sum_i t_i H_i = \sum_i t_i H_{wi} + \sum_i t_i H_{bi} \tag{33}$$

Equation (11) can now only be written as:

$$F_i = \frac{E_i}{e} + \frac{H_{bi}}{b}$$

and Equation (14) no longer applies. The extent to which storage allows transfer to heat supply between periods would then need to be specified by a series of inter-period constraints – the general formulation of these would presumably be quite complex. However, in the case of completely free thermal transfer possibilities, the problem can be easily formulated. In this case there are no thermal inter-period constraints: fuel cost may be written as:

$$P_f \sum_i t_i F_i = P_f \sum_i t_i \left(\frac{E_i}{e} + \frac{H_{bi}}{b}\right) \tag{34}$$

whence, using Equation (33), (34) can be written as:

$$P_f \Sigma t_i F_i = P_f \Sigma t_i \left(\frac{E_i}{e} + \frac{H_i - H_{wi}}{b}\right)$$

which is exactly as in the Equation (18). Thus with the free thermal transfer possibility, the only change in the model specification is to knock out the constraint (14). The solution for this case is not pursued further here however as the 'no' thermal transfer case is considered more realistic than the 'free' thermal transfer case (although of course much depends upon the length of the time periods involved).

CHP with electricity sold to the national grid

The market structure whereby the CHP supplies a local heat market and exports electricity to a national grid is considered to be a very important case – such 'decentralized' generation units could (as distinct from should) well form the basis for a substantial development of CHP in the UK.*** In effect, the CHP producer becomes a 'price-taker' in the electricity market: the modification

*** This is not to prejudge the issue of whether it would be 'better' to have a system of decentralised CHP units generating heat for local communities and feeding electricity into the ESI system – or – a system in which CHP units are integrated directly into the ESI system itself. The economic, as distinct from political, pro's and con's would presumably turn on the relative economies and diseconomies of scale and centralization. For example, the direct control approach might yield overall planning and operational benefits as compared to the 'less forceful' price signal control of the decentralized case – cost control however, as part of 'unionised-big-industry', might be expected to be less successful, not being subject to the 'rigours of competition'. The assessment of costs and benefits of such issues so far remains conjectural (and ideological).

to the model specification in this case is that the term

$$\sum_{i=1}^{n} t_i \int^{E_i} f_i(E)\, \mathrm{d}E$$

in Equation (10) is replaced by the benefit of selling electricity to the grid $\Sigma_i t_i P_{ei} E_i$. Now the P_{ei} $i = 1, \ldots n$ are given parameters in the CHP optimizing problem, parameters set by the ESI. Apart from the fact that the electricity prices are parametric, the first order condition (19) to (29) remain the same. There are now two broad categories of solution: either no CHP at all, or a positive CHP prime mover and waste heat boiler capacity.

Assuming CHP exists, $Q_e, Q_w > 0$. Now $H_i > 0$ so (23) holds with equality. Suppose CHP does not export electricity in period i so that $E_i = 0$. Then $H_{wi} = 0$ so the heat market must be supplied entirely by conventional boiler. Clearly $H_{wi} < H_i \Rightarrow \lambda_{2i} = 0$; if $H_i < Q_b$ then $\lambda_{4i} = 0$ and $P_{hi} = P_f/b$ whilst if $H_i = Q_b$ then $\lambda_{4i} > 0$ and

$$\Sigma\, t_i P_{hi} = \Sigma \frac{t_i P_f}{b} + \beta_b$$

where the summation runs over those i for which $H_i = Q_b$ (if there is just one peak period, its price is $P_{hi} = (P_f/b) + \beta_b/t_i$). In the 2-period problem, given $Q_w > 0$ then if $H_{wi} = 0$ in one period it must be that $H_{wi} > 0$ in the other; Q_w will be chosen so that in this latter period $H_{wi} = H_i$ so it must be that $H_i = Q_b$ in the former period. If period 1 is the peak period so that typically $P_{e1} > P_{e2}$, then $H_{w1} > 0$ and $H_{w2} = 0$. Then $P_{h2} = (P_f/b) + \beta_b$. This again shows the switching possibility.

More typically, $E_i > 0$ so that $H_{wi} > 0$ too. Thus (18), (19), (21), (22) and (23) hold again with equality. Manipulation yields the result that:

$$P_{hi} = \frac{P_f}{w} + \frac{1}{t_1}\left(\frac{e}{w}\lambda_{5i} + \lambda_{3i}\right) - \frac{e}{w}P_{ei} \qquad (35)$$

It is again possible to deduce the optimal capacities in the usual way. Equation (35) is indicative of the nature of the pricing inter-relationships; the term

$$\frac{1}{t_i}\left(\frac{e}{w}\lambda_{5i} + \lambda_{3i}\right)$$

represents the marginal cost of expanding CHP capacity to supply heat in period i; clearly if $H_{wi} < Q_w$ and $E_i < Q_e$ then $\lambda_{3i} = 0$ and $\lambda_{5i} = 0$ so this term would be zero. It reaches its maximum value if there is a single period j in which demand occurs for both capacities (highly likely in the 2-period case), elsewhere being off-peak, when $\lambda_{5j} = \beta_e$ and $\lambda_{3j} = \beta_w$.

The term $(e/w)P_{ei}$ represents the impact of selling electricity to the grid upon the effective marginal cost of supplying heat – the higher P_{ei}, the lower the effective marginal heat generation cost. There are the usual firm peak and shifting peak solutions associated with Equation (35). The implications of the above analysis may be summarized in the following:

- *Ceteris paribus*, an increase in electricity price received by the CHP producer reduces the optimal price in the heat market.
- Compared to district heating provided by conventional boilers, if positive CHP capacity is economic then the use of that capacity will of course imply greater overall benefits in the heat market. However, although prices under CHP will in general be lower than those under the corresponding conventional system, this will not always be the case.

The first proposition follows from Equation (4). As for the second, if CHP operates, then $H_{wi} > 0$; then (24) and (32) imply that:

$$P_{hi} = \frac{1}{t_i}(\lambda_{1i} + \lambda_{3i}) = \frac{P_f}{b} + \frac{\lambda_{4i}}{t_i} - \frac{\lambda_{2i}}{t_i}$$

The corresponding conventional heat market price is:

$$P_{hi} = \frac{P_f}{b} + \frac{\lambda_{4i}}{t_i}$$

(where λ_{4i} is associated with the constraint $H_i - H_{wi} \leqslant Q_b$). Clearly, if λ_{4i} took on similar values in both cases then since $\lambda_{2i} \geqslant 0$ the CHP prices would be lower. However they need not be since peak-switching analogous to that in the welfare maximizing monopoly case, as discussed above, can occur.

On-site CHP

The last case worthy of brief discussion is that where the CHP plant is used to generate on-site electricity and heat, a mode of operation common in industry. It is usually the case that the time-varying loads to be satisfied, E_t, H_t, are parametrically given. Heat is generated on-site by the combination of conventional and waste-heat boiler and on-site electricity demand is satisfied by the prime mover with possibly additional import or export of electricity to the central grid. The price of fuel is constant whilst the prices for import/export electricity will generally differ (certainly in practice) and be time varying. The benefit function (10) is now linear and the problem, which is essentially one of choosing the optimal capacity levels for prime mover, waste-heat boiler and conventional boiler, can be solved using straightforward linear programming.†††

ESI – CHP pricing policy

For the decentralized operation of CHP, the price offered by the ESI for privately generated electricity is clearly a key variable in determining the private economic viability of such schemes. This is especially the case when the CHP system exports all of its generated electricity (as it would probably do if supplying district heat) but it is also, though to a lesser

††† This problem is discussed in detail in Dobbs Ref 9, Chapter 2.2.

extent, important to the on-site CHP system. In the UK there are currently few cases of the former mode of operation but the latter are present in significant numbers, with around 20% of industries' electricity requirement being generated in this manner.‡‡‡

There has been much debate in the CHP technical literature on the issue of what the price schedule should be. This policy oriented discussion has not always been well-informed and this section therefore examines the more important contributions in the light of economic theory.

The virtually unanimous view of those writing about CHP seems to be that the 'correct' or 'fair' price should correspond to central system avoided costs.§§§ In the past it has been the case that the ESI has not offered such terms and commentators have thus often claimed 'unfair' discrimination against the CHP producer. The current ESI position on this buy-back price probably accords more closely with the 'avoided cost' price although the only substantive evidence published by the ESI (Burchnall[21]) suggests a policy that moves only partway towards it. Burchnall's article is clearly not the definitive statement of ESI policy♦♦♦ but it equally clearly sets down various principles which have probably not been modified since that date – it is therefore worth considering. It is convenient to examine the pricing problem in two stages – firstly the relation between price and marginal cost and secondly what the relevant marginal costs are.

The price-marginal cost relation

The argument that price should be set at short-run marginal avoided cost is of course based upon the welfare economics analysis of a 'first best world'. In practice, in a 'second best' world there are many reasons why prices should diverge from marginal costs – for example because of divergences in other sectors, for equity reasons etc.¶¶¶

The ESI position as presented by Burchnall is that price is a matter for negotiation and that it will in general be set so that it effects a division of benefits which is in some sense even handed or 'fair' as between the general body of electricity consumers and the CHP electricity producers:

As regards the kilowatt-hour price element, this will lie between the producer's cost and as a maximum the public supply marginal cost of production at the relevant time of day/year. If this maximum is greater than the cost of private production there is a benefit that has to be shared between the general body of consumers and the private producer, which can only be a matter of negotiation; an equal sharing of the benefit seems reasonable, especially bearing in mind that ordinary consumers are paying for the grid whereby the surplus electricity is to be usefully distributed.[25]

In practice the ESI are set financial targets: maximizing welfare subject to such a constraint provides some rationale for the above policy. Thus suppose a minimum target profit level for the ESI of $\bar{\Pi}_e$ so that $\Pi_e \geqslant \bar{\Pi}_e$. The ESI demand schedule may be represented as $P_e = h(q)$ where P_e, q are the price paid and quantity consumed by the consumer. A CHP export-electricity supply function may be postulated as $P_c = g(q_c)$. It is assumed that $h' < 0, g' > 0$. Suppose the ESI cost of production of an output q_e is $c(q_e)$. Now clearly total electricity supply will equal consumer demand so $q = q_e + q_c$. ESI operating profits are:

$$\Pi_e = P_e q - P_c q_c - c(q_e)$$

$P_e \cdot q$ is revenue from sales, $P_c \cdot q_c$ is cost of CHP electricity (q_c) purchased at a price P_c and $c(q_e)$ the cost of centrally generated electricity. The local CHP producer's electricity generation profits are:

$$\Pi_c = P_c q_c - \int_0^{q_c} g(Q)\,\mathrm{d}Q$$

Final market consumer surplus is:

$$CS = \int_0^{q} h(Q)\,\mathrm{d}Q - P_e q$$

Overall welfare is:

$$w = CS + \Pi_e + \Pi_c$$

Without the profit constraint the first order conditions are that $P_e = MC_e = P_c = MC_c$ where MC denotes marginal cost (ie $MC_c = g(q_c)$). This then is the argument that price to the CHP producer should be set at marginal cost of central generation. Introducing the profit constraint in addition to $q = q_e + q_c$ implies the following first order necessary conditions:

$$P_e - MC_e = -\lambda(MR_e - MC_e) \quad (36)$$

$$P_e - P_c = -\lambda(MR_e - MR_c) \quad (37)$$

where MR denotes marginal revenue and λ is the multiplier associated with the profit constraint. Writing ϵ_e, ϵ_c as electricity final demand and CHP electricity supply elasticities then Equations (36) and (37) may be written as:

$$\frac{P_e - MC_e}{P_e} = \frac{\lambda}{1+\lambda}\,\frac{1}{\epsilon_e} \quad (38)$$

‡‡‡ This fact of course explains why the Marshall team found that the electricity export tariff was not too important in determining CHP economic viability.[19] In the former case it would be crucial.

§§§ See Refs 1–7 and Ref 20 for example.

♦♦♦ Burchnall's article was a contribution to the research effort of the Combined Heat and Power Group (Energy papers 20 and 35) – it's aim was to clarify the basis for pricing policy actually adopted by the ESI – because of the small numbers so far involved it could well be argued that the principles espoused were still only tentative.

¶¶¶ Since these are well understood they are not discussed here. There is an extensive literature on such issues: see for example Refs 22, 23 and 24.

$$P_c = P_e \left\{\frac{1 + \lambda(1 - (1/\epsilon_e))}{1 + \lambda(1 + (1/\epsilon_c))}\right\} \quad (39)$$

Equation (38) is the standard result[26] that price should deviate from marginal cost in inverse relation to the demand elasticity – the intuition is that the more inelastic the demand, the less welfare is lost in raising profits in that market. Equation (39) is the equivalent result for CHP producers: it suggests that the extent to which they should be required to contribute to ESI profits depends upon the final demand elasticity as well as their own supply elasticity, as one would expect.

If the CHP supply is elastic ($\epsilon_c = \infty$) the CHP producers should receive full marginal cost since Equation (39) collapses to $P_c = MC_e$ whilst if the supply schedule is more inelastic then the offer price falls ($P_c \to 0$ as $\epsilon_c \to 0$). This is intuitive – the more inelastic the CHP supply, the less, *ceteris paribus*, the welfare loss is and so the more the transfer can be. On this basis, then the welfare maximizing price to be set depends upon judgements about the long-run supply schedules for CHP generated electricity $g(q_d)$, something about which little is known. Such an analysis does suggest however that avoided cost would represent only an upper bound to the price, and that a price below this could well be justified if it was thought that CHP long run supply was inelastic to some extent. *A priori* one might expect it to be fairly elastic and so a price close to marginal avoided cost justified.

However, all the above discussion is in any case rooted in the notion of a uniform price schedule – once the possibility of non-uniform pricing schedules is admitted, the desirability of marginal prices being set at marginal costs is reinforced. Any desired transfers of wealth between consumers and the ESI can be effected through 'licence fees' or block tariffs (where inframarginal units are priced at above marginal cost) etc.

To summarize then, the above analysis suggests that avoided cost prices would be indicated if the profit constraint did not bind – or if CHP supply is elastic in the long run (something about which little is known, empirically). It may be also argued for because it has the property of being 'subsidy free' – the nature of cross-subsidization should perhaps be more rigorously dealt with in a game theoretic context but the following is a useful heuristic definition given by Faulhaber (p 966).[27]

> If the provision of any commodity (or group of commodities) by a multi-commodity enterprise subject to a profit constraint leads to prices for other commodities no higher than they would otherwise be then the price structure is said to be subsidy free.

The marginal avoided cost pricing approach to CHP output would thus be Pareto-superior to its non-provision although it would not necessarily be 'welfare maximizing' in the sense used throughout this paper. It is perhaps worth noting that such an approach would avoid cross-subsidization on the electricity *production* side; this might be deemed important if more extensive decentralization of generation were being considered.

The ability to administer more complex tariff structures increases the desirability of setting avoided cost prices for marginal units – a natural approach would be to have marginal cost pricing for generation units and to merge the license free transfer with the contribution to capacity element. That is, CHP generation implies that less capacity needs to be held by the ESI. A payment for this contribution should be made, from which could be deducted the 'license fee'. If the CHP long-run supply schedule is elastic then the level of such 'license fees' would be small and so close to avoided cost pricing would also apply for the CHP capacity contribution.

A priori, the CHP output decision is fairly price sensitive (see above). Whether long-run supply is elastic is empirically unknown but seems likely to be the case. At least, for those who argue for a 'fair price' it provides some underpinning for justifying pricing at avoided costs.

The avoided costs

As far as CHP is concerned these fall into three categories:

- Operating costs;
- Capacity costs; and
- Standby.

Central system marginal operating costs avoided when CHP replaces central generation are in principle well defined. Capacity and standby costs pose interesting questions and have been the subject of some debate (see for example Dobbs[28] and Burchnall[29]. The capacity credit issue will be considered first.

If CHP exports electricity to the ESI at times close to maximum demand this should in principle reduce the level of central capacity that needs to be installed. What this avoided cost is depends essentially upon two factors:

- CHP generation reliability
- defining the relevant avoided cost.

Burchnall,[30] representing the ESI viewpoint, is critical of CHP reliability and especially of the idea that the credit should be based upon the producers *average* contribution to capacity (which is analogous to the 'after-diversity' demand underlying area boards calculations for electricity tariffs). He suggests that CHP contributions to capacity would have to be allowed for in the industry's plant programme for the seventh year ahead; to have guaranteed life and reliability; to be readily incorporable into the central system without undue waste of resources (in standby, distribution strengthening etc); that averaging implies cross-subsidization as between 'firm' and 'less-firm' CHP supplies; and that in calculating retail tariffs, the area boards apply a different (not average) diversity to different load factors to reflect the use made of capacity.

This position may be criticized: the relevance of requiring a 7-year advance warning appears dubious – the uncertainty associated with forecasting private generation load growth is unlikely to be significantly greater than in making other load forecasts; such forecasts are essentially broad brush in nature. In any case, private generators can be conceptually regarded as the absence of extra consumers/additional load-consumers after all are not required to give a 7-year capacity requirement notice to the ESI. Similarly with the requirement of 'guaranteed reliability and life'; the ESI does not reserve capacity in one-to-one correspondence with individuals possible demands but allows for diversity – neither do consumers guarantee 'reliable' consumption or consumption life. Averaging of course would involve cross-subsidization if it were applied over the class of CHP producers since as a group they could be expected to be rather heterogeneous in their output patterns (however tariff structures would be likely to have a significant impact here) and a tariff which reflects the benefits of capacity contribution to the ESI should also promote CHP group homogeneity precisely because they are likely to be fairly price sensitive as regards their output decisions. It is thus quite likely that Burchnall's observation that CHP supplies are 'non-firm' and that, 'Many contribute their kWh equally if not mainly at night', is simply a reflection of the tariff structure they face.

It is an interesting point that if CHP producers are regarded as a group and if the ESI assess the contribution to capacity upon the basis of 'that contribution which can be guaranteed with a level of reliability equal to that of the ESI system' then there is a kind of scale barrier[32] in that the credit for capacity contribution increases with the number of contributors until in the limit it reaches that of the average value. On this basis one might argue for the average value to overcome the scale barrier.

Whether in fact a full average credit for capacity contribution should be adopted is thus debatable – it does however seem that a tariff could be appropriately structured to encourage more consistent and reliable contributions to ESI capacity – some system of credits/debits for *ex poste* contributions might well suffice to influence CHP generators *ex ante* decisions.

Apart from the diversity/reliability issue, the other major questions seem to revolve around what the avoided cost really is. Lucas suggests that the CHP credit for capacity could be arguably zero if excess capacity exists in the ESI. Conceptually, in a programming format, the shadow prices associated with the capacity constraints will be zero if excess capacities are present – under uncertainty they will tend to be positive in so far as there is some probability of call on capacity.

To take the extreme case where zero credit is indicated, this appears to base prices on 'short' run avoidable costs; such an approach seems reasonable if the consumer is made aware of the future course of such prices; the absence of such information would involve 'faulty' decision making on the part of consumers contemplating durable investment problems. The argument for prices based upon long-run costs thus usually revolves around the expected greater price stability this will imply and hence the improvements in consumers long-run planning decisions.[34,35]

Whether long-run based prices would be more stable is presumably an empirical question as is indeed the question of the costs and benefits associated with the different approaches. Kay[36] makes the point that if short-run based pricing did in practice imply fluctuations in prices this would not support a case for long-run marginal cost pricing but rather that the price calculations be subject to the constraint that prices remain stable over the relevant period; ie that prices should be essentially based upon short-run considerations. It seems reasonable however that the relevant cost basis should correspond to the time period involved in the consumer's decisions. For electricity consumption this is presumably primarily the short-run (given the preponderence of the 'captive' demand element over the 'free' demand element) whilst for capacity credit the period is much longer (several years in fact).

If the argument in favour of a long-run cost basis is accepted there would then be a positive credit for CHP contributions to capacity even in periods where, as arguably at present, over-capacity exists on the central system.

'Standby' refers to the reserving of ESI capacity as an insurance against local CHP outage; the issues involved in assessing the level of charge for the ESI provision of this service essentially mirror those involved in assessing the credit for capacity contributions and so are not discussed further here. A question of consistency may be raised however: the standby 'tariff' and the 'buyback capacity credit tariff' should be set on an equal footing whether the pricing basis be long- or short-run; for example, if short-run principles are applied such that a situation of ESI excess capacity implies a low credit for CHP capacity contributions, then *mutatis mutandis* the standby tariff should also be relatively cheap.

Concluding comments

We have examined the economic and technical structure of the CHP joint production system within the context of a peak-load pricing model. The pricing implications of assuming different market structures for electricity and heat were investigated. The policy relevance of this analysis lies in the fact that different policies would lead to different institutional structures and hence different modes of CHP operation. For electricity, CHP may be an integral part of the central ESI system, or decentralized but linked to the ESI, or completely isolated – for heat the system may compete with other fuels, or act as a local monopoly supplier, or supply heat to a national heat grid and so on. Some of the more interesting effects arising out of the interaction between the two markets and the joint-production system were analysed in more detail including the possibility of peak period switching.

In the potentially important case where CHP is linked to a national electricity grid (important in the UK as it is seen as the main alternative to CHP units being run as an integral part of the ESI system) but operates as a decentralized and independent decision-making unit, the tariff offered by the ESI is crucial to the capacity, pricing and output decisions of the CHP producer.

In view of the importance of this tariff interface this was examined in greater detail. The commonly held view on this is typified in the most recent and most important study of CHP undertaken, that of Marshall *et al* (Energy Papers 20 and 35):[37]

> the ESI (Electricity Supply Industry) should therefore give more weight in its negotiations to reflecting the production costs it avoids, rather than the current practice of an equal sharing of any difference which may exist between the industrialists' production costs and the ESI's marginal production cost.

Such a view can always be justified from the viewpoint of a 'first best' world where a pound is a pound to whoever it may accrue. In a second (or worse) best world there are many complicating factors. We have indicated that the long-run supply elasticity for CHP is a significant factor: that assuming this to be elastic is an important element in justifying marginal cost pricing for the ESI tariff offered to CHP. We have further discussed the identification of production costs and the impact of reliability and concluded with a consistency argument for setting the tariff structure for standby (where the ESI provides an electricity supply back up in the event of CHP failure) on an equal footing with the tariff for the purchasing of electricity from CHP units.

References

1 W. Marshall *et al, District Heating Combined with Electricity Generation in the United Kingdom,* Energy Paper 20, HMSO, London, UK, 1977.
2 *Ibid.*
3 W. Marshall *et al, Combined Heat and Electrical Power in the UK,* Energy Paper 35, HMSO, London, UK, 1979.
4 G. T. Carter, ed, *Total Energy Conference, Brighton, 1971,* Institute of Fuel, 1971.
5 Local Energy Centres Conference, Imperial College, 1977, Construction Industry Conference Centre Bookshop, PO Box 31, Welwyn, UK.
6 McLellan and Partners, *Combined Power and Space Heating in Industry,* Department of Energy, 1979.
7 J. A. Bleay and I. M. Dobbs, *Combined Heat and Power Schemes and the Public Supply,* SRC, 1978.
8 N. J. Lucas, 'The case for CHP in the UK', *International Journal of Energy,* 1978, pp 29–42.
9 I. M. Dobbs, *Combined Heat and Power: An Economic Analysis,* unpublished PhD thesis, Newcastle University, 1980.
10 R. Ferguson, *Comments on Papers at Total Energy Conference, Brighton, op cit,* Ref 4, Vol 2, p 18.
11 I. M. Dobbs, *op cit,* Ref 9.
12 J. A. Burchnall, *The Supply Industry and Private Generation,* Monograph, The Electricity Council, 1976.
13 *Ibid,* paragraph 4.
14 O. E. Williamson, 'Peak load pricing and optimal capacity under indivisibility constraints', *American Economic Review,* Vol 56, 1966, pp 810–27.
15 M. Crew and P. Kleindorfer, 'Reliability and public utility pricing', *American Economic Review,* Vol 68, 1978, p 31.
16 I. M. Dobbs, *op cit,* Ref 9, Chapter 3.4, pp 284–311.
17 *Ibid,* Chapters 2.1, 2.2, 3.4.
18 *Ibid,* Chapter 2.2.
19 W. Marshall *et al, op cit,* Ref 3, p 3.
20 N. J. Lucas, *An Alternative Appraisal of District Heating in Glasgow,* Mimeo, Imperial College, London, 1977.
21 Burchnall, *op cit,* Ref 12, 1976.
22 O. E. Davis and A. Whinston, 'Piecemeal policy in the theory of second best', *Review of Economic Studies,* 1967, pp 323–331.
23 W. J. Baumol and D. Bradford, 'Optimal departures from marginal cost pricing', *American Economic Review,* Vol 60, 1970, pp 265–83.
24 M. S. Feldstein, 'Equity and efficiency in public sector pricing: the optimal two part tariff', *Quarterly Journal of Economics,* 1972, pp 175–187.
25 Burchnall, *op cit,* Ref 12, paragraph 34.
26 Baumol and Bradford, *op cit,* Ref 23.
27 G. R. Faulhaber, 'Cross-subsidisation: pricing in public enterprise', *American Economic Review,* Vol 65, 1975, pp 966–77.
28 I. M. Dobbs, *op cit,* Ref 9, Chapter 3.1.
29 Burchnall, *op cit,* Ref 12.
30 *Ibid,* paragraph 40.
31 Dobbs, *op cit,* Ref 9.
32 *Ibid.*
33 Lucas, *op cit,* Ref 8.
34 R. Turvey, *Optimal Pricing and Investment in Electricity Supply,* George Allen and Unwin, 1968.
35 R. Turvey, 'Marginal cost', *Economic Journal,* Vol 79, 1969, p 288.
36 J. A. Kay, 'Recent contributions to the theory of marginal cost pricing: some comments', *Economic Journal,* Vol 81, 1971, pp 366–71.
37 W. Marshall *et al, op cit,* Ref 3, p 14.