DEFINING MARKETS FOR EX ANTE REGULATION USING THE HYPOTHETICAL MONOPOLY TEST

by

Ian M. Dobbs†

† The University of Newcastle upon Tyne Business School, University of Newcastle upon Tyne, NE1 7RU, UK. Email address: I.M.Dobbs@ncl.ac.uk

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ABSTRACT

The EU is gradually systematising its approach to ex ante regulation (notably with the publication of the recent regulatory framework for Electronic communications). Market definition is viewed as an essential first step in this, with the ‘Hypothetical Monopoly Test’ a useful conceptual tool for use in the assessment. This paper examines the use of this test for market definition when there are several differentiated goods or services under consideration. It sets out an analytic framework, discusses the pros and cons of using the test and illustrates its use in a Telecom context.
1. INTRODUCTION
The assessment of market boundaries by competition and regulatory authorities in the EU has until recently been a qualitative judgemental process. Recently however, some national regulatory authorities (NRA s) have moved toward a more quantitative approach based on the hypothetical monopoly test (HMT), notably Ofcom (2003 a-e), (2004), at least for the ‘single product’ case. However, the use of the HMT in a differentiated product setting has not been fully explored, and this motivates the present paper. A definition and computational procedure is provided, along with a review of various issues concerned with the test and an examination of recent practice in UK Telecoms regulation.

The overall context for this lies in the fact that the EU is gradually systematising its approach to ex ante regulation, notably with the publication of the recent regulatory framework for Electronic communications; see EC (2002a, 2002b). In considering whether ex ante regulatory intervention is warranted, this regulatory framework envisages a three step process:

(1) Market definition: the assessment of the extent of the market.
(2) Assessment of dominance, or significant market power (SMP), within the market.
(3) If dominance/SMP is found, imposition of ex ante regulatory control is considered.

The above three step procedure has been applied on a regular basis by national regulatory authorities (NRA s) in recent years. The concern of the present paper is with the first step in this process. Market definition is a critical step since, clearly, the narrower the market boundary is drawn, the higher concentration and market shares will appear to be (and hence the greater the chance that regulatory intervention is found to be justified). Market definition of course plays a role in other types of
enquiry – notably concerning merger proposals and competition law enquiries
concerning a range of abuses (such as predatory pricing, margin squeeze). The issues
raised in this paper apply equally to any form of intervention, whether ex ante or ex
post, for which market boundaries are a pre-requisite.

Market definition using the HMT first came to prominence in the US department of
Justice horizontal merger guidelines, DOJ (1984), where it was explained as follows:

“When formally, a market is defined as a product or a group of products and a
geographical area in which it is sold such that a hypothetical, profit
maximising firm, not subject to price regulation, that was the only present
and future seller of those products in that area would impose a ‘small but
significant and non-transitory increase in price’ above prevailing or likely
future levels.’” (italics and bold added)

The HMT is often referred to, acronymically, as a SSNIP test (as per the underlined
italics above). Although not explicit in the above quote, it is explained later in that
document that a relevant market for regulatory purposes is defined as the smallest set
of products for which the price increase (usually set at 5 or 10%) is profitable. A
‘price increase’ clearly presupposes an initial or benchmark price level for the test. In
merger investigations this is usually taken as ‘existing price levels’; by contrast, for
regulatory purposes, it is usually argued that this should be the ‘competitive level’.
The issues this raises are discussed in more detail in section 4 below.

If there are good substitutes, a price increase is more likely to be unprofitable, and the
market can be extended to include these substitute products. The HMT has also been
viewed more as a conceptual tool, one to be applied ‘intuitively’, rather than as a
mechanistic scheme. Unfortunately, whilst in some cases it may appear ‘self-evident’
what the conclusion of such an intuitive test should be, when there is significant
product differentiation, this can strain intuition too far. There is merit then in moving to a more quantitative approach, and it is interesting to note that some regulators are now moving in this direction; see Ofcom (2003 a-e). These merits are worth emphasising:

(i) for cases where the relevant data are available, because it facilitates a clear cut assessment (and facilitates sensitivity analysis and robustness testing – see section 5 below).
(ii) for cases where data are currently not available, it directs effort towards the gathering and assessment of the relevant information (through econometric analysis and survey work – and in the latter case it gives a clear indication of the appropriate questions that need to be asked – see section 6 below).
(iii) even where it is not possible to gather the relevant information, it makes clear what the relevant variables are over which ‘judgement’ or ‘guesstimation’ is required.
(iv) where there is a problem with the assessment of the key parameters, it is also possible to ‘reverse’ the analysis and identify critical values or combinations of critical values for key parameters (see sections 3 and 6 below). Decision makers can then ponder the likelihood that parameter values are likely to lie above or below such values.
(v) it shows how intuitive or judgemental assessments about key elements should be combined.

The HMT has been used in the US primarily in merger investigations, and the academic literature on it since its inception has likewise taken this focus. A useful review of earlier work, focused on the single product case, is given in Massey (2000). However, whilst the single product HMT can be a useful tool, in many/most regulatory enquiries, the market under investigation features a range of more or less close but differentiated substitute products or services. Although work on merger investigations has considered differentiated product applications, it has been largely focused on the direct assessment of market power (given that the merger under investigation is already specified). The EU proposal to use the HMT as a basis for assessing market boundaries for ex ante regulation when there are differentiated goods does not seem to have received an explicit treatment, and this motivates the present work.
The information requirements to operationalise a computational approach to market boundary assessment for ex ante regulatory purposes is similar to that required for merger analysis – namely, elasticity information (own price and cross price) and price cost margins. Clearly, in a particular application, this information may prove hard to assess, particularly in the case of communication markets which are often rapidly developing or emergent. Apart from econometric analysis based on historical data, a major source of information in such markets often comes from survey work (typically undertaken by regulators). Although the information garnered from such sources is always to a degree problematic (due to the hypothetical nature of the questions involved), it at least offers the prospect that an appropriate form of analysis can be undertaken (and sensitivity testing of assumptions then followed through). Section 6 below discusses the way this information has been used in recent (UK) NRA assessments, and makes some suggestions regarding how such surveys might be improved for the future.

Section 2 presents the analytics of the HMT for market definition purposes in differentiated good markets. Section 3 then relates this to some old single product ‘critical value’ concepts (such as ‘critical sales loss’) and some new, including that of a ‘critical price increase’, a concept which also proves useful in the multi-product context. Having outlined the structure of the test, section 4 addresses key issues associated with the use of the test in practice (notably, the so called ‘cellophane fallacy’). Section 5 gives a 5-product numerical example and illustrates the value of sensitivity analysis, given there are always uncertainties associated with the data inputs. Section 6 then discusses market definition in UK internet services, an area of
current concern for the UK regulator Ofcom. Finally, section 7 draws together the main conclusions.

2. THE HYPOTHETICAL MONOPOLY TEST AND MARKET DEFINITION

In order to study product groupings under the *HMT*, the starting point is to select the largest set of candidate products and/or services (hereafter, simply ‘products’) which potentially needs to be considered in the investigation. This may not always be entirely straightforward; for example, a market which might at first sight appear to be ‘homogenous’ may in fact be segmentable in various ways (for example, by geography or customer type). However, it is hard to imagine how one could proceed to any analysis of whether different products belong to the same market or not if one is unable to identify them in the first place. The products under investigation are labelled 1,..,\(n\), such that \(N = \{1,\ldots,n\}\) denotes the set of product identifiers (hereafter referred to simply as a ‘set of products’). Conceptually, the demand for each product can be thought of as a function of all the prices \(p'_1,\ldots,p'_n\), so that the \(i^{th}\) demand can be written as \(q_i(p')\) for \(i \in N\). In what follows, it is assumed that these demand functions are twice continuously differentiable.

The *HMT* looks at the smallest sub-set of products for which a hypothetical monopolist, if in control of that sub-set, could profitably raise prices by some specified amount, denoted \(\alpha\) (\(\alpha = 0.05\) or 0.1 for a 5% or 10% test, respectively). This test is referred to as an \(\alpha\%\text{SSNIP}\) test in what follows. Since the focus is on relatively small price changes, it is reasonable to assume that, for the range of outputs
under consideration, the marginal cost $c_j$ of producing an additional unit of output of each product indexed $j \in N$ does not vary with output.

**Definition:** (Market boundary under the $\alpha\%SSNIP$)
For a set of products $K$ to be a “market” according to the hypothetical monopoly test, it must be the case that

(i) for the set $K$ of products, there exists a price increase of $\alpha\%$ or more which increases profitability and

(ii) for any proper subset $L \subset K$ of products, any price increase of $\alpha\%$ or more is not profitable.

This definition merely codifies the description of the HMT given in the DoJ guidelines described in section 1 above. Part (i) of the above definition is straightforward enough; however, notice that (ii) specifies not only that, for all sub-sets, price increases equal to $\alpha\%$ must be unprofitable, but that any larger price increases must also be unprofitable. This part of the definition ensures that set $K$ is a smallest sub-set on which a price increase of at least $\alpha\%$ is profitable.

Let $p^0$ denote the initial benchmark price vector and $q(p^0)$ the associated quantities. Computationally, the test can be conducted using *any* initial price vector; for market definition for merger analysis, it is typically appropriate to use the current price level whilst for ex ante regulatory purposes, the appropriate benchmark is the competitive price level. The issues raised by this are discussed further in section 4 below; for the moment, the focus is on the ‘mechanics’ of the test. Consider then the set of products $K$, and a proportionate price change from $p^0$ to $\bar{p}$ such that

$$p_j = (1+z)p^0_j, \quad j \in K,$$

for some $z>0$, and

$$\bar{p}_j = p^0_j, \quad j \notin K \quad (1)$$

In the process of ‘grouping’ a set of products in order to test whether there is market power with respect to that set, no change is allowed in the way these products are manufactured and distributed; that is, cost structures are assumed to be unaffected by
the process of ‘grouping’. A further point to note is that prices outside the set \( K \) are held constant; this is explicit in the US DoJ (1993) guidelines. The profit function for the product set \( K \) is thus

\[
\Pi^K(p) = \sum_{k \in K} p_k q_k(p) - C(q(p))
\]

(2)

where \( C(q) \) denotes the total cost of producing the output vector \( q \). The change in profit induced by the proportionate price increase is thus

\[
\Delta \Pi^K(z) = \Pi^K(\bar{p}) - \Pi^K(p^0)
\]

(3)

where the change in profit is viewed as a function of the extent of the price increase, \( z \).

It is useful, in what follows, to define cross price elasticities, at the current price vector \( p^0 \), using the standard notation

\[
\varepsilon_{ij} = \left( \frac{\partial q_i(p^0)}{\partial p_j} \right) \left( \frac{p^0_j}{q_i(p^0)} \right)
\]

for all \( i,j \in K \), and to define the price cost mark-up on the \( k \)th product as

\[
m_k = \left( p_k^0 - c_k \right) / p_k^0.
\]

(4)

It is shown in the appendix that, assuming local linearity in demand functions, and defining the functions

\[
\Delta^z_k = \sum_{k \in K} R_k \left\{ 1 + (m_k + z) \sum_{i \in K} \varepsilon_{ki} \right\}
\]

(5)

where \( R_k \) denotes the revenue earned on the \( k \)th product, and

\[
\psi_K = \left( \sum_{k \in K} R_k \left( \sum_{i \in K} \varepsilon_{ki} \right) \right),
\]

(6)

then (3) can be written as

\[
\Delta \Pi^K(z) = \Delta^z_k z = \Delta^0_k z + \psi_K z^2
\]

(7)

(where \( \Delta^0_k \) denotes \( \Delta^z_k \) with \( z = 0 \)). That is, the profit impact is a quadratic function of the extent of the price increase \( z \).
Empirically, it is usually reasonable to assume that the own price effect outweighs (in absolute value) the sum of the cross price effects. This *dominant diagonal* assumption implies the Jacobian matrix $\begin{bmatrix} \partial q_i(p^0) \\ \partial p_j \end{bmatrix}$ is negative definite and hence that the term $\nu_k < 0$ (see appendix). This in turn implies the function in (7) is strictly concave in $z$. The assumption allows the following necessary and sufficient conditions to be stated for the $\alpha\%SSNIP$ test (where $\alpha = 0, 0.05, 0.1$ for the 0%, 5% or 10% test):

**Result 1:** Assuming constant marginal costs and linearity of demand functions in the neighbourhood of $p^0$ and assuming own price effects outweigh in absolute value the sum of cross price effects, then a set of products $K$ is a ‘relevant market’ according to the $\alpha\%SSNIP$ hypothetical monopoly test if and only if

(i) $\Delta_k^\alpha > 0$, and  
(ii) $\Delta_L^\alpha \leq 0$ for all $L \subset K$

**Proof:** See appendix

Note the above proof works because of the assumptions of linearity and diagonal dominance guarantee strict concavity of the $\Delta \Pi$ functions for all product groupings.

For small price increases, the linear approximation is likely to be reasonable. For larger price increases, this assumption is less appealing. However, in practice, where the small price increases ($\alpha \%$) are unprofitable, it can usually be reasonably assumed that larger price increases will also be unprofitable.

Given benchmark prices, quantities, estimates of marginal costs and the matrix of cross price elasticities, it is possible to check result 1 conditions (i) and (ii), and hence to establish market boundaries according to the $HMT$. For more than 3 products, this is a fairly tedious albeit computationally straightforward task. The numerical example
discussed in section 5 below is analysed using a computational program developed to operationalise result 1 conditions (i) and (ii). The program involves computing $\Delta^a_k$ for all possible subsets of the total product set $N$ - namely \{1\}, \ldots, \{n\}, \{1,2\}, \{1,3\}, \ldots, \{1,n\}, \{2,3\}, \{2,4\}, \ldots, \{1,2,3\}, \{1,2,4\}, \ldots, \{1,2,\ldots,n\}$ working upwards. Note that, whenever a sub-set $K$ is found which satisfies $\Delta^a_k > 0$, then all sets $M$ for which $K \subset M$ can be eliminated (as they cannot be smallest sub-sets).

The following corollary to result 1 is worth remarking:

**Result 2:** Any grouping of products $K$ cannot be a ‘relevant market’ according to the HMT if the products within that grouping have non-negative price-cost margins and are independent and/or complements ($\eta_{ki} \leq 0$ for all $k, i \in K, k \neq i$).

**Proof:** See Appendix

This makes intuitive sense since it indicates that there can be no increase in market power associated with an enlargement in the set of products unless the products included in the set are to some extent economic substitutes.

3. CRITICAL SALES LOSS, CRITICAL ELASTICITY, CRITICAL MARK-UP AND CRITICAL PRICE CHANGE

This section relates the above analysis to some old and some new single product ‘critical values’, and suggests two useful multi-product ‘critical value concepts’. For the single product case, **Critical Sales Loss (CSL)** is defined as the % change in sales that would make the $\alpha\%SSNIP$ test just marginal. Again in the single product case, the **critical elasticity** is the value of the own price elasticity of demand that would make the $\alpha\%SSNIP$ test just marginal. These concepts have been discussed and applied in recent work; see e.g. Werden (1998), Hausman, Sidak and Singer (2001),
Massey (2000), Langenfeld and Li (2001). On the same line of reasoning, three additional concepts are also worth defining, namely the critical mark-up, the critical price increase \((\text{CPI})\) in what follows), and the maximum profit price increase. The critical mark-up is defined as the level of mark-up which would just make the test marginal, while the critical price increase would be the value of \(\alpha\) that just makes the test marginal. The maximum profit price increase is the proportionate price increase which would lead to a maximal profit increase. The last three concepts have not been much discussed in the previous literature, but are at least as useful as critical elasticity and critical sales loss. This is so because uncertainty is rarely confined solely to the elasticity estimates; marginal costs, and hence mark-ups are often of equal concern.

The above ‘critical concepts’ reflect a standard and useful form of sensitivity analysis in which each parameter is unilaterally adjusted so as to make the result marginal. Such calculations are helpful in assessing the robustness of HMT test results. In the case where there are several products being investigated, it is possible to undertake similar forms of calculations for critical sales loss or elasticity (and this can be extended to consideration of the role of cross price elasticities etc.) although the usefulness of the exercise rapidly diminishes. By contrast, a variation always worth exploring in both single and multi-product cases is that of the critical price increase \((\text{CPI})\), the value of \(\alpha\) that just makes the test marginal. This concept does not seem to have had much (any) attention, and yet it is a concept that naturally generalises to the multi-product case and is useful as a measure of the robustness of a market boundary test (for example, for a 5\%SSNIP test, the greater the \(\text{CPI}\) is above 5\%, the more robust the market boundary finding will be). Furthermore, it greatly facilitates
comparisons across alternative product groupings, as will be seen in section 5 below. The rest of this section gives brief derivations for the above critical values.

In the single product case, the α%SSNIP test in result 1 above simplifies as follows. Firstly, setting \( z = \alpha \), the function \( \Delta^z_K \) in (5) simplifies to \( \Delta^\alpha \equiv R \{ 1 + (m + \alpha) \varepsilon \} \) where \( p \) is price, \( q \) sales, \( R=pq \) revenue, \( m \) the mark-up and \( \varepsilon \) the own price elasticity for this single product. Hence (7) can be written as

\[
\Delta \Pi(\alpha) = \alpha R \left[ 1 + (m + \alpha) \varepsilon \right].
\]

**Single product critical Elasticity**

The critical elasticity, denoted \( \varepsilon_{\text{crit}} \), is the value of the elasticity which just makes the test marginal \( \Delta \Pi = 0 \). Hence

\[
\Delta \Pi(\alpha) = \alpha R \left[ 1 + (m + \alpha) \varepsilon_{\text{crit}} \right] = 0 \quad \Rightarrow \quad \varepsilon_{\text{crit}} = \frac{-1}{m + \alpha}
\]

**Single product critical Sales Loss**

The critical sales loss, denoted \( \text{CSL} \), is that percentage of sales loss that would make the test marginal. By definition, it is related to the price increase and elasticity by

\[
\varepsilon_{\text{crit}} = \frac{\text{CSL}}{\alpha},
\]

so it follows that

\[
\text{CSL} = \alpha \varepsilon_{\text{crit}} = \frac{-\alpha}{m + \alpha}.
\]

**Single product critical Mark-up**

The critical mark-up, \( m_{\text{crit}} \), is defined by

\[
\Delta \Pi(\alpha) = \alpha R \left[ 1 + (m_{\text{crit}} + \alpha) \varepsilon \right] = 0 \quad \Rightarrow \quad m_{\text{crit}} = -\left( \alpha + \frac{1}{\varepsilon} \right)
\]
Single product critical price increase

The critical percentage price increase $\alpha_{\text{crit}}$ is defined by

$$\Delta \Pi (\alpha_{\text{crit}}) = \alpha_{\text{crit}} R \left[ 1 + (m + \alpha_{\text{crit}}) \varepsilon \right] = 0 \Rightarrow \alpha_{\text{crit}} = -\left( m + \frac{1}{\varepsilon} \right)$$

(12)

so long as $-\left( m + \frac{1}{\varepsilon} \right) > 0$; if this is negative, $\alpha_{\text{crit}} = 0$; no price increase is profitable.

Multi-product critical price increase

The above critical values are readily computed and can be useful when assessing the robustness of results to variations in underlying parameter estimates. Whilst it is possible to define similar critical values in the multi-product case, the number quickly proliferates, and their relevance diminishes. However, the critical price increase $\alpha_{\text{crit}}$ remains a useful number to compute for the multi-product case. It is of particular interest because it is a number which can be usefully compared across different product groupings (see section 5 below). Note first that given the concavity assumption, if $\Delta_k^0 \leq 0$, no price increase is profitable, whilst if $\Delta_k^0 > 0$, then a profitable price increase will exist. In this latter case, the maximum price that is still profitable is denoted $\alpha_{\text{crit}}$; from equation (7), this is determined as

$$\left( \text{if } \Delta_k^0 > 0 \right) \text{then}$$

$$\Delta \Pi (\alpha_{\text{crit}}) = \alpha_{\text{crit}} \Delta_k^{\alpha_{\text{crit}}} \equiv \alpha_{\text{crit}} \left( \Delta_k^0 + \alpha_{\text{crit}} \psi_K \right) = 0$$

$$\Rightarrow \alpha_{\text{crit}} = -\Delta_k^0 / \psi_K \ (> 0) \tag{13}$$

As with (12), if $\Delta_k^0 \leq 0$, then no price increase is profitable. This analysis of $\alpha_{\text{crit}}$ is illustrated in figure 1 below. Clearly, if $\alpha < \alpha_{\text{crit}}$ then the price increase is profitable under the SSNIP test.

Figure 1 about here
The $\alpha_{\text{crit}}$ value is computed in the numerical work and sensitivity analysis reported in section 5 and 6 below. If the value is significantly above 5% (or 10%), it gives an indication of the robustness of the 5% (or 10%) test. Of course, where $\alpha_{\text{crit}}$ is very large, although this does suggest robustness for the test, not too much credence should be given to its precise numerical value – as clearly it is based on parameter estimates which are assumed to be invariant for the price change under consideration – and on the assumption that multi-product demands are to good approximation linear for this price perturbation.

**Multi-product maximum profit price increase**

Having identified the percent price increase, $\alpha_{\text{crit}}$, below which the price increase is profitable, it is also worth remarking that it is possible to also compute the level of proportionate price increase which would attain maximum profit. Defining this as $\alpha_{\text{max}}$, this is also illustrated in figure 1; given the quadratic nature of (7), clearly so long as $\Delta K^0 > 0$,

$$\alpha_{\text{max}} = \arg \max_{\alpha, \alpha \geq 0} \Delta \Pi^k(\alpha) = \Delta K^0 / (2\psi_k) = \frac{1}{2} \alpha_{\text{crit}}. \quad (14)$$

The estimate of $\alpha_{\text{max}}$ is an estimate of the ‘cellophane ceiling’, the profit maximising price level (see figure 1). For the single product case, $\alpha_{\text{max}}$ gives an estimate of how far away the current price is from that which would maximise profit. For the multi-product case, it is an estimate of how far away current prices are from the profit maximising point under the constraint that prices are raised proportionately from existing levels.
4. ISSUES CONCERNING MARKET DEFINITION USING THE HMT

Having set out the definition and computational procedure, this section briefly reviews the principal issues that confront the use of this test for market definition purposes. These concern (a) the binary nature of the test, (b) the appropriate prices (and price cost margins) (c) the appropriate time horizon and (d) its relevance in dynamic or emergent markets. The first two of these issues has been fairly well discussed in the literature, but the latter two have not, until recently.

(a) The binary nature of the test

The test, mechanically applied, gives a result that individual products are “in” or “out” of the market for regulatory purposes. The drawback with this is that, as Fisher (1979, p. 16) has observed, it “converts a necessarily continuous question into a question of yes or no. The temptation is to regard products which are in as all counting equally and products which are out as not counting at all.” One might wish to dispute the meaning that might be ascribed to Fisher’s suggestion that products that are “in” may tend to be treated “equally”, but the real force of this observation lies in the fact that products which are excluded are given ‘zero weight’ in the subsequent assessment of dominance and the harm that may spring from that. To put it another way, market definition can only have a useful role to play if it facilitates the computation of market shares which then prove to be a useful index of the market power of a firm or group of firms.

Essentially, any drawing of a boundary can be thought of as imparting an upward bias to measures of dominance or concentration in differentiated product settings. However, without a boundary, it is not possible to compute any of the traditionally used measures. Rather than abandoning market definition, this counsels rather that
caution should be taken in the interpretation of such measures. A finding that there is no evidence of dominance (individual or collective) is a strong finding in these circumstances. By contrast, a finding of dominance has to be treated more cautiously. A natural strategy here is to examine whether the dominance finding is sensitive to the market boundaries drawn. This kind of robustness or sensitivity analysis is illustrated in section 5 below.

(b) The appropriate price level

The level of price increase used in the test (usually 5% or 10%) is intrinsically arbitrary, as indeed is the fact that the price increase is an across the board fixed percentage amount. Clearly a ‘real’ monopolist would more generally wish to vary individual prices non-proportionately (and might even wish to reduce some, depending on spillover effects). In considering the impact of price increases, it is usually argued that the appropriate benchmark is the competitive price level. Observed prices may already reflect the exercise of monopoly power – in the extreme case where the current price is the monopoly price, clearly no further price increase will increase profitability. Some commentators, e.g. White (1999), have argued that, because the competitive price level is ‘unknown’, this renders the HMT non-operational. By contrast, the EU guidelines sidestep the issue by proposing that, in currently unregulated markets, existing prices should proxy for competitive prices unless there is convincing evidence to the contrary; EU (2002b at para. 42).

Clearly, where there is uncertainty regarding the appropriate level for the price benchmark, it is useful to conduct sensitivity analysis to establish the robustness of the proposed market grouping. This approach is outlined in section 5 below.

Referring to (5), notice that the choice of price level has implications for revenues, for
mark-ups, and for elasticities. The most sensitive of these will usually be the price cost mark-ups. Mark-ups should in theory be those that would arise in a competitive market. In perfect competition, price is equal to marginal cost and there is a zero mark-up. However, in nearly all practical applications, there are significant fixed and sunk costs; in such circumstances the relevant pricing benchmark is that in which price is equal to average cost (a ‘quasi competitive price level’ as would occur in a contestable market). It follows that there can be a positive mark-up over marginal cost.

To see how important the assumed mark-up is, consider the single product case and two scenarios (a) of pricing at marginal cost (‘perfect competition’) and (b) of pricing at average cost but above marginal cost (a contestable market scenario). The former has a price cost margin of zero and (9) indicates that market power will be found so long as demand is more inelastic than the ratio $1/\alpha$. For example, with a 10% test, $\alpha = 0.1$, and demand merely needs to be more inelastic than -10 (something which is likely to be found in many applications). By contrast, in (b), if there are significant fixed costs, the price cost margin is positive and so this makes finding a price increase profitable less likely. In the extreme case where all of the costs are fixed, with zero marginal costs, the price cost margin is unity, and (9) indicates that the 10% price increase will be profitable only if demand is inelastic (actually, more inelastic than $1/(1+\alpha)$; with $\alpha = 0.1$ for a 10% test, more inelastic than 0.91). Given the difficulty of pinning down with any degree of precision these mark-ups, this clearly indicates why there is a need for sensitivity analysis and robustness testing.\textsuperscript{16}
(c) The appropriate time horizon

In applying the HMT, the time horizon involved also becomes critical not only because this will typically affect the assessment of elasticities, but also the assessment of marginal costs, and hence price-cost margins. The longer the period, the more elastic demands tend to be – and the more costs will be variable rather than fixed (that is, the higher the marginal cost assessment will tend to be, the longer the time horizon taken).\(^{17}\)

As a general rule it might be reasonable to assume that both price elasticities and cost to volume elasticities increase with the choice of time horizon. In the longer run, consumers are able to adapt their capital equipment (where it is needed to consume the service) and to research alternative products for suitability. Similarly on the supply side, capital equipment needs replacing over time; as a consequence, long run marginal costs are often assumed to be higher than short run marginal costs.\(^ {18}\)

New products and services also cause further difficulties; the HMT is essentially a short run static assessment, but the nature of the market (and its boundaries) may be changing rapidly. Market definition then involves making forecasts over time of the likely evolution of the key variables – but these are by nature likely to be unstable and difficult to pin down with any confidence.

(d) The use of the test in dynamic/emergent markets

Regulation of emergent markets is fraught with difficulties. Firstly, it is precisely in such markets that the standard approach outline in this paper is likely to prove most difficult (because of the difficulties associated with parameter estimation). Secondly, in this type of market, regulatory risk is at its greatest. Such markets are typically
characterised by Schumpeterian competition in which innovative firms gain temporary monopoly power as a reward (when successful) for earlier R&D expenditure. Such monopoly power can be transient, with new product development ousting previously dominant incumbents. In such markets, it can be argued that allowing monopoly power to be exercised is necessary for there to be any incentive to expend resources on R&D (with its beneficial long term impact on the development of the industry). The welfare benefits of such innovation can often far exceed the short term losses arising from the exploitation of monopoly power; see Lind et al (2002). This observation does not invalidate the use of the test to assess the extent of market boundaries but it does qualify how market definition and subsequent dominance findings should be used. Rather than being an automatic signal for the imposition of ex ante regulation, in an innovative industry, the decision involves an assessment of the relative importance of welfare losses that may arise from the exercise of short run monopoly power against the long term benefits associated with a faster pace of innovation in the industry.

To sum up, the issues raised in (a)-(c) above do not negate the rationale for market definition, but they do indicate the importance of sensitivity analysis and robustness testing when conducting this form of assessment. This type of robustness testing should be conducted not only for the assessment of market boundaries, but also for the subsequent measure of dominance and market power. The issues raised in (d) above are concerned less with market boundary assessment and/or dominance per se, but rather with the decision over what should be done if dominance is subsequently found. The point being that in Schumpeterian dynamic competition, it is less likely that static monopoly power should be viewed as a problem to be remedied.
5. A FIVE PRODUCT EXAMPLE AND SENSITIVITY ANALYSIS

This section examines a stylised version of a 5-product ‘light beers’ market assessment; it illustrates the mechanics of the procedure and shows how various forms of sensitivity analysis can be pursued. The following section then examines the market definition exercise for internet markets in the UK.

The relevant information for the implementation of the test is given in Table 1 and comprises the initial prices, quantities, marginal costs and the matrix of cross price elasticities. The cross price elasticity information used in Table 1 panel (1a) is taken from econometric work on the US beer market reported in Hausman, Leonard and Zona (1994). Unfortunately, no price, cost, or volume data were reported in that study so the present analysis does not constitute a ‘live’ case study – but it (a) deals with a significant number of products and illustrates how the process can be applied in this case, and (b) demonstrates the value of sensitivity analysis when conducting such analysis. In what follows, the case where prices, costs and volumes are all set equal is examined – the study then explores the impact of variations in the price cost margin on the assessment of market power and market groupings.

Table 1 panel (1b) reports $\Delta^0_k$ and $\Delta^{0.05}_k$ ($\alpha = 0.05$ for a 5% SSNIP test), and $\alpha_{crit}$ (as a %) for all product groupings when prices are set at $p_i^0 = 1.4$. Panel (1c) then gives the smallest sub-set product groupings for the 5% test based on panel 1(b) results. Notice that no grouping is a market under the 5% SSNIP test other than the whole collection \{1,2,3,4,5\}. Also notice that the conclusion would be the same under a 10% SSNIP test, in view of the fact that $\alpha_{crit} = 10.8\%$. for \{1,2,3,4,5\}. This makes clear how the computation of $\alpha_{crit}$ is a useful element for sensitivity analysis as it
gives an indication of how robust market determinations are. For example, if \( \alpha \) was set at 4.5\%, then referring to Table 1 panel 1(b), this would make for a smaller relevant market, as \{1,2,3,4\}, whilst a 4\% test would give two market groupings, namely \{1,2,3,4\} and \{1,2,4,5\}. This latter result illustrates a well known fact which has been discussed at an ‘intuitive’ level in previous work - see OFT (1992) for example - namely that the HMT does not always identify unique product groupings. Non-uniqueness is not necessarily a problem – but it is clearly useful to have explored the range of possible product groupings and their robustness, since as previously remarked, the choice of market boundary may materially affect the assessment of SMP and the assessed need for regulatory intervention.

To illustrate a simple form of sensitivity analysis, consider the effect of uniformly varying the assessment of the competitive price level; panels (1c) through to (1e) illustrate the impact of varying the price level (from price cost margins of 40\%, to 30\% to 20\%) whilst holding the values of other parameters constant. In Panel (1d) where the price cost margins are 30\%, this changes the values for \( \alpha_{crit} \) and also the conclusions regarding which product groupings constitute ‘smallest sub-sets’ under the 5\%SSNIP test - there are now 5 product groupings under a 5\% test. Note also that none of these would be ‘relevant markets’ under a 10\% test (all \( \alpha_{crit} \) values are less than 8\%). In panel (1e), where the price cost margins are 20\%, this results in \{1\}, \{2\} and \{3,4,5\} as relevant markets under a 5\% test. Again notice none of these remain so under a 10\% test. This again reveals why sensitivity analysis can be useful – in indicating the extent of robustness of market determinations.
To sum up, the SSNIP test may uniquely and robustly identify a single ‘relevant market’. The above forms of sensitive analysis help to establish whether this is the case or not. Whilst the SSNIP test does not always or necessarily identify unique market boundaries, the information generated by performing the above calculations, and the type of results reported in Table 1, are clearly useful in any given regulatory enquiry. Notice also that quantitative magnitudes for $\Delta_k^0, \Delta_k^\alpha$ and $\alpha_{crit}$, reported in panel (1b) might be of interest in their own right as they give a measure of the extent of market power in the region of benchmark prices.\(^{20}\)

6. USE OF THE HMT FOR INTERNET SERVICE MARKET ASSESSMENTS

This section briefly examines how the UK Telecoms regulator Ofcom has approached the problem of market definition in its recent work on internet services. In its earlier work, Ofcom used the HMT in an ‘intuitive and qualitative’ manner. More recently, it has also essayed a quantitative approach to market definition in Ofcom (2003a-e), (2004), through the use of the critical sales loss concept (explained in section 3 above). This quantitative approach has been based on survey data.\(^{21}\) Although survey data is necessarily problematic (regarding the significance of individuals’ responses to hypothetical questions – in this case regarding whether they would switch – and if so to what service/product – in response to an x% increase in the price of a given service or set of services), this is a useful source of information, and one that is relatively easy to gather.

The recent Ofcom surveys focussed on raising the price of a single service (and estimating the extent of switching that this would induce). However, in order to
assess market boundaries, it is often necessary to examine various different potential product groupings – to examine sub-sets of an overall collection of products or services, as explained in section 2 above. That is, there is a need to assess, for an increase in the price of an individual service, the extent of switching to each of the other services under consideration (as well as the extent of switching outside for this grouping, the extent of ‘exit’). Furthermore, this information has to be gathered for a price increase for each of the services in turn. If this is done, it is then straightforward computationally to implement the form of analysis outlined in section 2 above. As will be seen below, the surveys undertaken by Ofcom are deficient in this respect, and this is an area where information gathering can be improved.

**TABLES 2,3,4,5 ABOUT HERE**

In order to discuss in detail Ofcom’s market analysis for retail internet services, Table 2 summarises the ‘raw data’ gathered by Ofcom concerning demand switching propensities. Table 3 is then derived from Table 2 as follows. Firstly, as in Ofcom’s work, the “don’t knows” are allocated proportionately; this is equivalent to simply dropping the don’t knows and scaling up the other percentages so they sum to 100. The values in column 1 in Table 3 are derived as the negative sum of all the other columns. That is, total sales loss % is equal to the sales loss going to each of the other services or exiting the market completely. A final minor revision concerns Table 3 panels 2 line 3 and panel 4 line 3. Notice in both these cases that the Ofcom latest surveys have unfortunately aggregated some of the switching outcomes. This aggregation prevents testing for some groupings of services. One option is to go back to earlier survey evidence (lines 1 or 2 for example). However, in order to utilise the latest survey evidence, these merged cells have been subsequently disaggregated (in line 4 in each case) in the same proportions as the next earlier survey that does have
Table 4 is then derived from Table 3, and summarises the % sales loss and implied elasticities based solely on the latest survey evidence for the case of Residential customers (panel 1) and SMEs (panel 2) respectively. Notice that, with a price increase of 10%, elasticities are simply 1/10 the sales loss figures in each case.

The “?”s in Table 4 arise because the original surveys did not ask the appropriate questions to elicit this information. Table 4 shows that the survey evidence is sufficient to conduct single product HMT assessments of whether the following are relevant markets under the HMT: (a) residential BB, (b) residential uNB, and (c) SME BB, (d) SME uNB and also (e) SME BB+uNB. There is no way of checking whether mNB (metered narrowband) or ISDN/LL are separate markets – or markets in combination with any of the above services.

**Single Service Market Assessments**

Table 5 shows the ranges of $MC/P$ and $CSL$ values actually used by Ofcom (2003a-e, 2004) in their market reviews for Internet services along with our revised assessments (based on the same survey evidence). Only the latest survey evidence is used in Table 5 (i.e. the last lines in each of the panels in tables 2,3). These figures are also presented with a $\pm 10\%$ error margin in Table 5. Two key issues arise in this analysis, namely

(a) the use of long run assessments of marginal cost by Ofcom and (b) the often erroneous measurement of own price elasticity of demand. These are dealt with in turn.
**Cost Analysis**

Throughout the assessments (Table 5), Ofcom uses fairly high assessments for the marginal cost to price ratio. That is, $MC_i / p_i$ is generally taken to be in excess of 40%, with upper bound values of up to 80%. Midpoints estimated are often around 50-60%. Ofcom explicitly argues that the appropriate cost concept for use in this analysis is long run incremental cost ($LRIC$). However, $LRIC$ includes the imputed costs of capital assets with lives well in excess of five years. This is of course, a time horizon significantly longer than the time horizon explicitly stated for the market review (which is 1-2 years). It can also be argued that the appropriate cost concept is that of long run average avoidable cost ($LRAAC$), simply because the concern is with an output decrement rather than increment. The latter is generally (and often significantly) lower than $LRIC$ because it excludes sunk costs. In practice, on that shorter time horizon, for output reductions (consequent on the hypothetical price increases), clearly marginal cost could be really quite low. Accordingly, the ‘new assessment’ of the ratio of marginal cost to average cost is taken as around 30% for the time horizon of 1-2 years, with 40% regarded as an upper bound. Given the figures for $MC_i / p_i$, the associated CSL can then be calculated (for example, in Table 5, when $MC_i / p_i = 0.3$, $m_i = 0.7$ which implies from (10) that $CSL = 0.1/(0.7 + 0.1) = 0.125$, or 12.5%).

**Own Price Elasticity of Demand/% Sales Loss**

Ofcom emphasises that what binds products into the same market is the extent to which they are substitutes (have high cross price elasticities of demand). However, in assessing whether a single service is a relevant market under the $HMT$, the relevant measure is not that of substitution to another service, but the totality of all
substitution, including *exit* from the set of telecom services in its entirety. Ofcom, in focusing purely on switching to another single service, has often significantly underestimated the overall level of sales loss. The errors primarily manifest themselves in panels 1, 2 and 4 of Table 5. In panel 1 line 1, the Ofcom (2003c, p.131) assessment of 14% can be traced back to Table 2 panel 1 line 2 col. 2, where the 14% is simply the number of customers of uNB who said they would switch to broadband if uNB price increased by 10%. However, as explained in this paper, the relevant sales loss is the totality of all who switch; that is in Table 4 panel 1 line 2, it is 14+7+5+2+6%+an allocation for the ‘don’t knows’ (at 4.2%), making a total of 38.2%, equivalent to an own price elasticity of -3.82 (see table 3 panel 1 line 2).

The same problem arises in Table 5 panel 2 line 1, where the Ofcom figure of 3.45% is likewise purely for those who switch from uNB to BB (see table 3 panel 3). The correct figure, including all the other services to which individual might switch is 44.8%! (i.e. from table 3, panel 3, the sum of 3.45+16.09+18.39+0.00+6.90). Finally, in panel 4 of Table 5, the Ofcom figure of 4.3% is likewise erroneous. This comes from table 2 panel 4, columns 2&3 line 3, with an allocation of “don’t knows”, equivalently from the sum of Table 3 panel 4 column 2&3 line 3. However the correct figure is the total for *all* who switch is in fact 32.3% (from Table 3 panel 4 line 4 column 1, equal to the negative sum of the column 2,3,4,5,0 figures).

Given the survey evidence, and the new (correct) assessments of sales loss (own price elasticity) figures, the conclusions are reasonably clear-cut – for unmetered narrowband services, neither SMEs nor Residential customers constitute a relevant economic market under the *HMT*. The same holds true for *SME* broadband users.
The only case where there is any evidence for a service being a relevant economic market under the HMT lies with residential Broadband users. Here, in Table 5 panel 3, the survey result of 10.4% switching was correctly assessed by Ofcom; however, notice that the error margin on this is 10% and this gives a confidence band of 0.4%-20.4%; see Collins (2004). This sales loss has to be compared with a central estimate of 12.5% CSL with a confidence interval of 10-14%. A quick comparison suggests that the result is not at all statistically significant. Note also the previous two Ofcom consumer surveys on broadband would have revealed no relevant economic market under the HMT (with sales losses of 15.8% and 14.9% in Table 3 panel 2 col. 1). Overall the survey evidence does show something of a trend toward inelasticity for residential BB but the evidence so far does not appear to substantiate a finding that this is a market on its own under the HMT.  

Multi-Service Testing

Table 5 makes clear that, given the survey information gathered by the NRA, nothing can be said about ISDN/leased Lines or about metered narrowband services. Naturally, it would have been preferable if the surveys had been designed to gain a more complete range of information. However, some investigation of results for BB and uNB is possible, and this is of some interest, given that, for residential customers, BB is marginal whilst uNB is not a relevant economic market under the single service tests. To undertake the test requires market shares and mark-ups information. Market share evidence at the time of the surveys is available from Ofcom, in terms of the percentage of households; around 10% of households have BB and 50% have NB, whilst prices in 2003 were around £25 p.a. for BB - Ofcom (2003e, p. 158) and £13-
Using subscripts ‘1’ and ‘2’ to denote BB and uNB respectively, and using the midpoint of the latter range (£15), this allows rough revenue shares to be calculated as

\[ R_1 = \frac{25 \times 0.1}{25 \times 0.1 + 15 \times 0.5} = 0.25 \]
\[ R_2 = \frac{15 \times 0.5}{25 \times 0.1 + 15 \times 0.5} = 0.75 \]

**TABLE 6 ABOUT HERE**

Table 6 gives some results for both single service and multi-service tests. In fact, as indicated in the Table, on the existing figures, the point estimate sales response of 10.4% (own price elasticity of -1.04) for BB makes this a marginally relevant economic market. However, the confidence interval was from 0.4-20.4% (in Table 5, panel 3). In fact, for any sales response greater than 12.5% (the CSL), BB is not a relevant economic market under the **HMT**. Suppose that BB alone is marginal – that is, suppose \( \varepsilon_{i1} = -1.25 \). Then it becomes of interest to ask whether the two services combined constitute a relevant economic market under the **HMT**. Setting out the calculation makes clear that this is not the case. First, since

\[ \frac{MC_1}{p_1} = \frac{MC_2}{p_2} = 0.3 \], then \( m_1 = m_2 = 0.7 \), so that \( (m_i + \alpha) = 0.8 \) when \( \alpha = 0.1 \).

Then, from (5), using the elasticity data from Table 4 but setting \( \varepsilon_{i1} = -1.25 \),

\[ \Delta^{0.1}_{i(1,2)} = 0.25 \left( 1 + 0.8 \times (-1.25 + 0.372) \right) + 0.75 \left( 1 + 0.8 \times (1.573 - 3.82) \right) = -0.524 \]

(this calculation gives the value in the bottom right cell in table 6). Clearly, the cross price effects help to draw the services into the same market, but the key feature is that uNB has very elastic demand; the own price effect on this service swamps the contributions of the cross price effects and renders the price increase across the two services unprofitable.
To sum up, the survey evidence generated by the NRA was rather incomplete and of variable quality (in terms of sample size and degree of aggregation) and that there were various errors manifest in the computational analysis. Overall, the quantitative approach taken by the NRA to market boundary assessment for UK internet services was shown to be suspect, with various areas where the analysis could be improved.

7. CONCLUDING COMMENTS

This paper has focused on the use of the HMT for market definition purposes, with some emphasis on how a quantitative approach might be implemented. It has to be recognised that market definition is not an end in itself, and further that there are dangers associated with an ‘uncritical’ or ‘mechanical’ use of the test. However, it has been argued that if the HMT is to be used as a basis for market definition, there is merit, where feasible, in moving beyond intuitive/qualitative application to a more quantitative application of the test. In the one or two product cases, it is often not too hard to provide rough ball park figures for the elasticities and price cost margins, so the test can be operationalised (this was illustrated by the application to internet services in section 6 above). Things are less straightforward where many (>2) products are involved. However, in any regulatory enquiry into market power, there is a prima facie case for trying to establish the relevant parameter values. That means there is a prima facie case for designing appropriate forms of customer survey and a serious assessment of the feasibility and potential value of a formal econometric study of demand in such enquiries. When such information is available, the debate can move on to the adequacies and inadequacies of different models and modelling choices, to the robustness or otherwise of the key parameters (elasticities, price cost
margins). Such assessments (including assessments of robustness) can then be built into the study of market boundaries using the procedures outlined in this paper.

APPENDIX

Consider a price change from \( p^0 \) to \( \bar{p} \) (not necessarily a proportional price change) for a set of products \( K \). Then

\[
\Delta \Pi^K = \Pi^K(\bar{p}) - \Pi^K(p^0)
\]  
(A.1)

Take an exact third order expansion for \( \Pi^K(\bar{p}) \) about \( p^0 \). Writing the price changes as \( \Delta p_i = \bar{p}_i - p^0_i \) for \( i \in K \),

\[
\Pi^K(\bar{p}) = \Pi^K(p^0) + \sum_{i \in K} \frac{\partial \Pi^K(p^0)}{\partial p_i} \Delta p_i \\
+ \frac{1}{2} \sum_{i \in K} \sum_{j \in K} \frac{\partial^2 \Pi^K(p^0)}{\partial p_i \partial p_j} \Delta p_i \Delta p_j \\
+ \frac{1}{6} \sum_{i \in K} \sum_{j \in K} \sum_{h \in K} \frac{\partial^3 \Pi^K(\bar{p})}{\partial p_i \partial p_j \partial p_h} \Delta p_i \Delta p_j \Delta p_h
\]  
(A.2)

where \( \bar{p} = \theta p_0 + (1 - \theta)\bar{p} \) for some \( \theta \in (0,1) \). Now

\[
\frac{\partial \Pi^K(p^0)}{\partial p_i} = q_i(p^0) + \sum_{k \in K} \left( p_k^0 - c_k \right) \left( \frac{\partial q_k(p^0)}{\partial p_i} \right)
\]  
(A.3)

and

\[
\frac{\partial^2 \Pi^K(p^0)}{\partial p_i \partial p_j} = \frac{\partial q_i(p^0)}{\partial p_j} + \frac{\partial q_j(p^0)}{\partial p_i} + \sum_{k \in K} \left( p_k^0 - c_k \right) \left( \frac{\partial^2 q_k(p^0)}{\partial p_i \partial p_j} \right)
\]  
(A.4)

since by local linearity, \( \frac{\partial^2 q_k(p^0)}{\partial p_i \partial p_j} = 0 \) for all \( i, j \in K \). The third order terms are zero for the same reason: that is,

\[
\frac{\partial^3 \Pi^K(\bar{p})}{\partial p_i \partial p_j \partial p_h} = \frac{\partial^2 q_i(\bar{p})}{\partial p_j \partial p_h} + \frac{\partial^2 q_j(\bar{p})}{\partial p_i \partial p_h} = 0.
\]  
(A.5)

Hence
\[
\Delta \Pi^K = \sum_{i \in K} \{ q_i(p^0) + \sum_{k \in K} \left( p_k^0 - c_k \right) \left( \frac{\partial q_k(p^0)}{\partial p_i} \right) \Delta p_i \\
+ \frac{1}{2} \sum_{i \in K} \sum_{j \in K} \left( \frac{\partial q_i(\tilde{p})}{\partial p_j} + \frac{\partial q_j(\tilde{p})}{\partial p_i} \right) \Delta p_i \Delta p_j \}
\]  
(A.6)

Now consider the proportionate price increase; in this case, \( \tilde{p}_i = (1 + z) p_i^0 \) for \( i \in K \)

and \( p_i = p_i^0 \) for \( i \notin K \) so \( \Delta p_i = zp_i^0 \) for \( i \in K \). Then

\[
\Delta \Pi^K = z \sum_{i \in K} \left\{ p_i^0 q_i^0 + \sum_{k \in K} p_i^0 \left( p_k^0 - c_k \right) \left( \frac{\partial q_k(p^0)}{\partial p_i} \right) \right\} \\
+ \frac{1}{2} z^2 \sum_{i \in K} \sum_{j \in K} \left( \frac{\partial q_i(\tilde{p})}{\partial p_j} + \frac{\partial q_j(\tilde{p})}{\partial p_i} \right) p_i^0 p_j^0
\]  
(A.7)

and using the definitions for elasticities and mark-ups (and working on the summations a little), this can be rewritten as

\[
\Delta \Pi^K = z \left( \sum_{k \in K} R_k \left\{ 1 + m_k \sum_{i \in K} \varepsilon_{ki} \right\} \right) \\
+ z^2 \left( \sum_{k \in K} R_k \left( \sum_{i \in K} \varepsilon_{ki} \right) \right)
\]
(A.8)

or

\[
\Delta \Pi^K = z \sum_{k \in K} R_k \left\{ 1 + \left( m_k + z \right) \sum_{i \in K} \varepsilon_{ki} \right\}
\]
(A.9)

Defining

\[
\Delta \tilde{\varepsilon}_K = \sum_{k \in K} R_k \left\{ 1 + \left( m_k + z \right) \sum_{i \in K} \varepsilon_{ki} \right\},
\]
(A.10)

\[
\Psi_K = \left( \sum_{k \in K} R_k \left( \sum_{i \in K} \varepsilon_{ki} \right) \right),
\]
(A.11)

then, focusing on \( \Delta \Pi^K \) as a function of \( z \), this becomes

\[
\Delta \Pi^K(z) = \Delta \tilde{\varepsilon}_K z = \Delta \tilde{\varepsilon}_K z + \Psi_K z^2
\]
(A.12)

which is equation (7) of the paper. The assumption that own price outweigh cross price effects (the dominant diagonal assumption) implies that the Jacobian matrix
$$\left\{ \frac{\partial q_i(p^0)}{\partial p_j} \right\}$$ is negative definite – see for example Beavis and Dobbs (1990, p. 192).

Notice from (A.6) and (A.12) that $\psi_K$ is a quadratic form:

$$\psi_K = \frac{1}{2} \sum_{j \in K} \sum_{i \in K} \left( \frac{\partial q_j}{\partial p_i} + \frac{\partial q_i}{\partial p_j} \right) p_i^0 p_j^0 = \sum_{j \in K} \sum_{i \in K} \left( \frac{\partial q_j}{\partial p_j} p_i^0 p_j^0 \right)$$

Hence, because the Jacobian matrix is negative definite, $\psi_K < 0$.

**Proof for Result 1**

Definition 1 establishes that a set $K$ is a relevant market if (i) $\Delta \Pi^K(z) > 0$ for some $z \geq \alpha$ and that (ii) for all subsets $L \subset K$, $\Delta \Pi^K(z) \leq 0$ for all $z \geq \alpha$. Result 1 states that the necessary and sufficient conditions for this are that (a) $\Delta^x_K > 0$ and (b) $\Delta^x_L \leq 0$ for all subsets $L \subset K$. Now, (a) implies $\Delta \Pi^K(\alpha) > 0$ from (A.12) so it is sufficient for def. 1(i). It is also necessary since if $\Delta^x_K \leq 0$ then

$$\left( \Delta^0_K + \psi_K \alpha \right) \alpha \leq 0 \Rightarrow \Delta^0_K + \psi_K \alpha \leq 0 \quad \text{and hence} \quad \partial \Delta \Pi^K(z)/\partial z = \Delta^0_K + 2 \psi_K z < 0 \quad \text{for} \quad z \geq \alpha \quad (\text{recall} \quad \psi_K < 0).$$

That is, $\Delta \Pi^K(z)$ is strictly decreasing for $z \geq \alpha$. Hence $\Delta \Pi^K(z) \leq 0$ for all $z \geq \alpha$; no price increase is profitable. For Definition 1 (ii), the same analysis establishes that $\Delta^x_L \leq 0$ for all subsets $L \subset K$ guarantees $\Delta \Pi^L(z) \leq 0$ for all $z \geq \alpha$ so (b) is sufficient to guarantee all subsets are never profitable for price increases $z \geq \alpha$, and it is also necessary since if $\Delta^x_L > 0$ for some subset, then for this subset there would exist a profitable price increase of amount $\alpha$, by (A.12).

**Proof for Result 2:**

Result 2 states that a grouping of products $K$ cannot be a ‘relevant market’ according to the $HMT$ if the products within that grouping have non-negative price-cost margins and are independent and/or complements ($\eta_{kl} \leq 0 \quad \text{for all} \quad k, i \in K, k \neq i$).
To see this, first note from result 1, for $K$ to be a relevant market requires (i) $\Delta^a_k > 0$ and (ii) $\Delta^a_L \leq 0$ for all $L \subset K$. From (5), $\Delta^a_k = \sum_{k \in K} R_k \left(1 + (m_k + \alpha) \sum_{m \in K} \eta_{km}\right)$. If $\eta_{km} \leq 0$, for all $k \neq m$, then since $m_k > 0$ so $(m_k + \alpha) > 0$ for all $k$, it follows that $\Delta^a_k \leq \sum_{k \in K} R_k \left(1 + (m_k + \alpha) \eta_{kk}\right)$. However, note that $\Delta^a_{\{k\}} = R_k \left(1 + (m_k + \alpha) \eta_{kk}\right)$, and for all $k \in K$, it must be that $\Delta^a_{\{k\}} \leq 0$ from part (ii) of result 1. Hence $\Delta^a_k \leq \sum_{k \in K} \Delta^a_{\{k\}} \leq 0$ which contradicts the requirement that $\Delta^a_k > 0$.

FOOTNOTES

1 For example, Ofcom (2002, 2003b,c). See also Monti (2001)] and the reports published by the Office of Fair Trading; OFT (1992, 1998, 1999, 2001). Note that throughout this paper, the UK telecoms regulator is referred to as ‘Ofcom’, the Office of Communications regulation. In practice, earlier ‘Ofcom’ reports were produced by an previous incarnation of that NRA, namely Oftel, the Office of Telecommunications Regulation. See the Ofcom website at http://www.ofcom.org.uk/ for details of the re-organisation and its rationale.

2 See for example CAT (2004).

3 It is also possible to include supply side substitution in the consideration of profitability – if other potential producers can quickly enter the market consequent on a price increase, this will reduce or eliminate the profitability of it. In the short run however, the main focus tends to be on demand side substitution.

This assumption rules out the possibility that grouping products might facilitate alternative modes of production; clearly economies of scale or scope would tend to reduce costs. However, for many regulatory market boundary assessments, the time horizon for the test is as short as 1-2 years, and on this horizon, the assumption probably holds fairly well.

There is some logic to this as it attempts to define market boundaries independently of the actual nature of competition in the market. The idea being, presumably, that the latter is assessed in the subsequent SMP assessment. Earlier versions of the merger guidelines appeared to allow the possibility of induced price changes outside the set of prices under the control of the HM. See Elroy (1995) for further discussion of this issue.

Of course, this holds for Hicksian demands at the level of the individual consumer. However, here, the interest is in Marshallian demands, and in addition, the demand system under consideration comprises only a (probably fairly small) sub-set of ‘all products’. This is so because any regulatory or anti-trust investigation will be concerned with some ‘industry grouping’ (such as ‘telecommunications’). It follows that properties which arise at the individual level imply no useful restrictions on the structure of the above functions. Notwithstanding this, diagonal dominance can still be regarded as a reasonable empirical assumption – that is, one which is valid in most empirical applications.

If $\Delta_k^0 \leq 0$, then no price increase is profitable and $\alpha_{\text{max}} = 0$; no price increase is profitable.

This is discussed in detail in the next section. See also footnote 4 above.

There are a range of other issues not addressed here which have been discussed in the literature, notably the issue of competitive price reactions, the geographic
dimension and also the issue of price discrimination (market segmentation through tariff design - see Hausman et al. (1996). Also, space limitations preclude consideration of alternative qualitative or quantitative assessment approaches to market definition; these are usefully reviewed in OFT (1992, 1998, 1999, 2001), ICP-ANACOM (2003). See also Slade (1984), Massey (2000).


13 In merger analysis, it may be possible to avoid the need for market boundary analysis through a direct approach to the assessment of market power – by analysing the extent to which the merger would lead to price increases etc. post-merger - see e.g. Nevo (2001), Slade (2004).

14 Clearly if an $\alpha \%$ SSNIP test is passed, then the $HM$ can profitably raise each individual price by at least $\alpha \%$. If a proportionate price increase is profitable, this is sufficient to establish that a more general price adjustment could generate even greater profit. It is possible to revise the form of the $HMT$ to allow for non-proportionate price adjustments, although this is beyond the scope of the present work.


16 Section 6 provides a good illustration of the problems raised here.

17 There is also an argument that the relevant costs should be short-run avoidable costs since the price increase induces a volume reduction.
Particularly for new services, where the norm is to ‘build ahead of demand’ in anticipation of a growing market, in which case short run marginal costs (associated with short run excess capacity) can be quite low (and persistently low).

Undertaking sensitivity analysis is computationally feasible using hand calculations, but for more than three products, this is singularly tedious and a computer program or spreadsheet is really needed to undertake the exploration for larger product groupings (the program used for this example is available at http://www.staff.ncl.ac.uk/i.m.dobbs/).

Only fairly simple forms of sensitivity analysis are discussed here. A possible extension would be to embed the market definition algorithm in a simulation model. Thus, suppose the interest is in a particular subset $L$. Given the covariance matrix associated with the elasticity estimates, and perhaps with distributional assumptions made for marginal costs, one could envisage taking drawings from these distributions, computing $\Delta L$ - and then repeating this process a number of times. The proportion of the runs for which $\Delta L > 0$ could thus be used to back up statements such as “$L$ is a relevant market at the 95% level of significance.”

No attempt has been made to gather econometric evidence – in part no doubt because of the emergent nature of the market in this case – however there is some scope to move in this direction; see Nankervis (2004) for example.

This section is based on Dobbs (2004), comments submitted to Ofcom as part of the consultation process during the wholesale Broadband market review. This material is available from Ofcom at web address http://www.ofcom.org.uk/consultations/past/wbamp/response/unt.pdf. The original Ofcom surveys are available at web address http://www.ofcom.org.uk/static/archive/oftel/publications/research/2003/.
Thus for example, Table 2 panel 1 columns 2, 3, 4, 5, 0 figures are divided by \(<1-(\text{col6}/100)\>\). For example, the Table 3 line 1 col. 2 figure of 12.66% is given by Table 2 line 1 col. 22 figure of 10% divided by (1-0.21).

For example, Table 3 panel 1 line 1 column 1 figure of -29.11% equals the sum of 12.66+10.13++0.00+0.00+6.33% and so on.

For example, in Table 3 panel 2 line 3, the 10.42% has been allocated to columns 2, 3, 0 in line 4 in the same proportions as were found in the earlier survey (at line 2). This process was repeated at panel 4 line 4.

For example, Table 4 panel 1 line 1 column 1 figure of -10.42% corresponds to table 3 panel 2 line 4 column 1 (and so on).


There are several other issues; for a more detailed commentary, see Dobbs (2004).

This is of course fairly contentious. No significant analysis has been undertaken by any of the parties involved; the above assessments are based on ‘first principles’ reasoning..

For other concerns regarding the surveys, see Collins (2004).


Another issue, discussed in section 4, but not addressed here, concerns whether these constitute competitive price levels.

Note that only relative shares are required in order to compute the measure \(\Delta x^a\), as only the sign matters, not the absolute figure.
The above calculation illustrates the fact that it is erroneous to simultaneously subscribe to the HMT methodology and to focus solely on cross price effects (ignoring own price effects) when deciding on whether services are in the same market or not (and Ofcom, whilst subscribing to the HMT methodology, continues to emphasise cross price effects rather than own price effects; see Ofcom (2004, e.g. para. 2.70).

REFERENCES


Ofcom (2002) Direction to resolve a dispute between BT, Energis and Thus concerning xDSL interconnection at the ATM switch, 21 June (available at Web-address: http://www.ofcom.org.uk).


Table 1: A 5-product example

Panel (1a): Elasticities†

<table>
<thead>
<tr>
<th>Product</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.763</td>
<td>0.464</td>
<td>0.397</td>
<td>0.254</td>
<td>0.201</td>
</tr>
<tr>
<td>2</td>
<td>0.569</td>
<td>-4.598</td>
<td>0.407</td>
<td>0.452</td>
<td>0.482</td>
</tr>
<tr>
<td>3</td>
<td>1.233</td>
<td>0.956</td>
<td>-6.097</td>
<td>0.841</td>
<td>0.565</td>
</tr>
<tr>
<td>4</td>
<td>0.509</td>
<td>0.737</td>
<td>0.587</td>
<td>-5.039</td>
<td>0.577</td>
</tr>
<tr>
<td>5</td>
<td>0.683</td>
<td>1.213</td>
<td>0.611</td>
<td>0.893</td>
<td>-5.841</td>
</tr>
</tbody>
</table>

Other data, not varied in panels (1b)-(1e) below: $c_i, q_i = 1.0, i = 1, ..., 5$
†These elasticity figures are actually drawn from the study by Hausman et al. [1999] for low alcohol beers.

Panel (1b): With prices $p_1, ..., p_5 = 1.4$

<table>
<thead>
<tr>
<th>$\Delta K^0$</th>
<th>$\Delta K^{a=5%}$</th>
<th>$\alpha_{crit}$ (%)</th>
<th>Product Grouping</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.11</td>
<td>-0.37</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.44</td>
<td>-0.76</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-1.04</td>
<td>-1.47</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>-0.62</td>
<td>-0.97</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>-0.94</td>
<td>-1.35</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>-0.13</td>
<td>-0.64</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.49</td>
<td>-1.07</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.42</td>
<td>-0.98</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.69</td>
<td>-1.30</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.93</td>
<td>-1.59</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.58</td>
<td>-1.17</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-0.70</td>
<td>-1.31</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-1.08</td>
<td>-1.76</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>-1.50</td>
<td>-2.26</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>-0.96</td>
<td>-1.62</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0.03</td>
<td>-0.70</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>0.03</td>
<td>-0.70</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>-0.04</td>
<td>-0.78</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.23</td>
<td>-1.01</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.60</td>
<td>-1.45</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.41</td>
<td>-1.22</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.50</td>
<td>-1.32</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-0.72</td>
<td>-1.58</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-0.25</td>
<td>-1.03</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-0.96</td>
<td>-1.86</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0.76</td>
<td>-0.08</td>
<td>4.5</td>
<td>1</td>
</tr>
<tr>
<td>0.59</td>
<td>-0.28</td>
<td>3.4</td>
<td>1</td>
</tr>
<tr>
<td>0.72</td>
<td>-0.14</td>
<td>4.2</td>
<td>1</td>
</tr>
<tr>
<td>0.24</td>
<td>-0.69</td>
<td>1.3</td>
<td>1</td>
</tr>
<tr>
<td>0.30</td>
<td>-0.63</td>
<td>1.6</td>
<td>2</td>
</tr>
<tr>
<td>1.92</td>
<td>1.03</td>
<td>10.8</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 1 (Continued)
Panel (1c): With Prices $p_1, ..., p_5 = 1.4$ 5% SSNIP Test

<table>
<thead>
<tr>
<th>$\Delta \alpha = 5%$</th>
<th>$\alpha_{crit}$ (%)</th>
<th>Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.03</td>
<td>10.8</td>
<td>1 2 3 4 5</td>
</tr>
</tbody>
</table>

Panel (1d): With Prices $p_1, ..., p_5 = 1.3$ 5% SSNIP Test

<table>
<thead>
<tr>
<th>$\Delta \alpha = 5%$</th>
<th>$\alpha_{crit}$ (%)</th>
<th>Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>5.7</td>
<td>1 2 3</td>
</tr>
<tr>
<td>0.10</td>
<td>5.7</td>
<td>1 2 4</td>
</tr>
<tr>
<td>0.03</td>
<td>5.3</td>
<td>1 2 5</td>
</tr>
<tr>
<td>0.31</td>
<td>6.8</td>
<td>1 3 4 5</td>
</tr>
<tr>
<td>0.36</td>
<td>7.1</td>
<td>2 3 4 5</td>
</tr>
</tbody>
</table>

Panel (1e): With Prices $p_1, ..., p_5 = 1.2$ 5% SSNIP Test

<table>
<thead>
<tr>
<th>$\Delta \alpha = 5%$</th>
<th>$\alpha_{crit}$ (%)</th>
<th>Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.222</td>
<td>9.9</td>
<td>1</td>
</tr>
<tr>
<td>0.005</td>
<td>5.1</td>
<td>2</td>
</tr>
<tr>
<td>0.245</td>
<td>6.6</td>
<td>3 4 5</td>
</tr>
</tbody>
</table>

Table 2: Ofcom Sales Loss Surveys - Consequences of a 10% Price Increase
(BB=Broadband, uNB=unmetered narrowband, mNB=metered narrowband, LL=leased lines)

Panel 1: Residential uNB

<table>
<thead>
<tr>
<th>Survey Date</th>
<th>Sample Size</th>
<th>% Continue to use uNB</th>
<th>% Switch to BB</th>
<th>% Switch to mNB</th>
<th>% Switch to ISDN</th>
<th>% Other</th>
<th>% Stop</th>
<th>% Don’t Know</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Feb 2003</td>
<td>364</td>
<td>56</td>
<td>10</td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>2 May 2003</td>
<td>308</td>
<td>55</td>
<td>14</td>
<td>7</td>
<td>5</td>
<td>2</td>
<td>6</td>
<td>11</td>
</tr>
</tbody>
</table>

Panel 2: Residential BB

<table>
<thead>
<tr>
<th>Survey Date</th>
<th>Sample Size</th>
<th>% Continue to use BB</th>
<th>% Switch to uNB</th>
<th>% Switch to mNB</th>
<th>% Stop</th>
<th>% Don’t Know</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Feb 2003</td>
<td>250</td>
<td>80</td>
<td>8</td>
<td>2</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>2 May 2003</td>
<td>193</td>
<td>80</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>3 August 2003</td>
<td>133</td>
<td>85</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
### Panel 3: SMEs on uNB

<table>
<thead>
<tr>
<th>Survey Date</th>
<th>Sample Size</th>
<th>% Continue to uNB</th>
<th>% Switch to BB</th>
<th>% Switch to mNB</th>
<th>% to ISDN/LL</th>
<th>% Other</th>
<th>% Stop</th>
<th>% Don’t Know</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Feb 2003</td>
<td>113</td>
<td>50</td>
<td>3</td>
<td>14</td>
<td>16</td>
<td>-</td>
<td>6</td>
<td>13</td>
</tr>
</tbody>
</table>

### Panel 4: SMEs on BB

<table>
<thead>
<tr>
<th>Survey Date</th>
<th>Sample Size</th>
<th>% Continue to use BB</th>
<th>% Switch to uNB</th>
<th>% Switch to mNB</th>
<th>% ISDN/LL</th>
<th>% Other</th>
<th>% Stop</th>
<th>% Don’t know</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Feb 2003</td>
<td>153</td>
<td>55</td>
<td>8</td>
<td>1</td>
<td>17</td>
<td>-</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>2 May 2003</td>
<td>196</td>
<td>71</td>
<td>2</td>
<td>16</td>
<td>-</td>
<td>7</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>3 August 2003</td>
<td>244</td>
<td>63</td>
<td>4</td>
<td>18</td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>7</td>
</tr>
</tbody>
</table>

### Table 3: Implied Sales Loss/Gain Figures

**Panel 1:** Residential Consumers on Unmetered Narrowband (response to price increase in unmetered NB)

<table>
<thead>
<tr>
<th>Survey Date</th>
<th>% change in use of uNB</th>
<th>% change to BB</th>
<th>% change to mNB</th>
<th>% change to ISDN/LL</th>
<th>% to Other</th>
<th>% Stop</th>
<th>% Don’t know</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Feb-03</td>
<td>-29.11</td>
<td>12.66</td>
<td>10.13</td>
<td>0.00</td>
<td>0.00</td>
<td>6.33</td>
<td></td>
</tr>
<tr>
<td>2 May-03</td>
<td>-38.20</td>
<td>15.73</td>
<td>7.87</td>
<td>5.62</td>
<td>2.25</td>
<td>6.74</td>
<td></td>
</tr>
</tbody>
</table>

**Panel 2:** Residential Consumers on Broadband (response to a price increase in BB)

<table>
<thead>
<tr>
<th>Survey Date</th>
<th>% change in BB</th>
<th>% Switch to uNB</th>
<th>% Switch to mNB</th>
<th>% Stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Feb-03</td>
<td>-15.79</td>
<td>8.42</td>
<td>2.11</td>
<td>5.26</td>
</tr>
<tr>
<td>2 May-03</td>
<td>-14.89</td>
<td>5.32</td>
<td>4.26</td>
<td>5.32</td>
</tr>
<tr>
<td>3 Aug-03</td>
<td>-10.42</td>
<td>-10.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Aug-03 revised</td>
<td>-10.42</td>
<td>3.72</td>
<td>2.98</td>
<td>3.72</td>
</tr>
</tbody>
</table>
**Panel 3:** SMEs on Unmetered Narrowband (response to price increase in unmetered NB)

<table>
<thead>
<tr>
<th>Survey Date</th>
<th>% change in use of uNB</th>
<th>% change in BB</th>
<th>% change in mNB</th>
<th>% change to mNB</th>
<th>% change to ISDN/LL</th>
<th>% change to Other</th>
<th>% Stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb-03</td>
<td>-44.83</td>
<td>3.45</td>
<td>16.09</td>
<td>18.39</td>
<td>0.00</td>
<td>6.90</td>
<td></td>
</tr>
</tbody>
</table>

**Panel 4:** SMEs Broadband (response to a price increase in BB)

<table>
<thead>
<tr>
<th>Survey Date</th>
<th>% change in BB</th>
<th>% change in uNB</th>
<th>% change in mNB</th>
<th>% to ISDN/LL</th>
<th>% to Other</th>
<th>% Stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb-03</td>
<td>-38.20</td>
<td>8.99</td>
<td>1.12</td>
<td>19.10</td>
<td>0.00</td>
<td>8.99</td>
</tr>
<tr>
<td>May-03</td>
<td>-26.04</td>
<td>2.08</td>
<td>16.67</td>
<td>7.29</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Aug-03</td>
<td>-32.26</td>
<td>4.30</td>
<td>19.35</td>
<td>8.60</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Aug-03 revised</td>
<td>-32.26</td>
<td>3.82</td>
<td>0.48</td>
<td>19.35</td>
<td>8.60</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 4: Implied Results from latest Surveys

**Panel 1: Internet Services – Residential - %sales loss/gain**

Elasticities in parentheses

<table>
<thead>
<tr>
<th>Price Change</th>
<th>BB</th>
<th>uNB</th>
<th>mNB</th>
<th>ISDN/LL</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Price increase in BB</td>
<td>-10.42</td>
<td>3.72</td>
<td>2.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Price increase in unmetered NB</td>
<td>15.73</td>
<td>38.20</td>
<td>7.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Price Increase in metered NB</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Panel 2: Internet Services – SMEs - %sales loss/gain**

Elasticities in parentheses

<table>
<thead>
<tr>
<th>Price Change</th>
<th>BB</th>
<th>uNB</th>
<th>mNB</th>
<th>ISDN/LL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Price increase in BB</td>
<td>-32.26</td>
<td>3.82</td>
<td>0.48</td>
<td>19.35</td>
</tr>
<tr>
<td>2. Price increase in unmetered NB</td>
<td>3.45</td>
<td>44.83</td>
<td>16.09</td>
<td>18.39</td>
</tr>
<tr>
<td>3. Price Increase in metered NB</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>4. Price Increase in ISDN/LL</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>
Table 5: Comparison of Critical Sales Loss (CSL) and Survey Switching Percentages using latest survey evidence (latest of the surveys reported in tables 2,3)

<table>
<thead>
<tr>
<th>Panel 1: Residential uNB</th>
<th>% Sales loss</th>
<th>MC/p</th>
<th>m</th>
<th>Implied Critical Sales Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Ofcom’s (incorrect) assessment</td>
<td>14</td>
<td>Ofcom assessment</td>
<td>0.41-0.81</td>
<td>0.59-0.19</td>
</tr>
<tr>
<td>2 Actual SS%</td>
<td>38.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Actual SS% with ± 10% confidence interval</td>
<td>28.2-48.2</td>
<td>New assessment</td>
<td>0.3 (0.1-0.4)</td>
<td>0.7 (0.9-0.6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel 2: SMEs on uNB</th>
<th>% Sales loss</th>
<th>MC/p</th>
<th>m</th>
<th>Implied Critical Sales Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Ofcom’s (incorrect) assessment</td>
<td>3</td>
<td>Ofcom assessment</td>
<td>0.41-0.62</td>
<td>0.59-0.38</td>
</tr>
<tr>
<td>2 Actual SS%</td>
<td>44.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Actual SS% with ± 10% confidence interval</td>
<td>34.8-54.8</td>
<td>New assessment</td>
<td>0.3 (0.1-0.4)</td>
<td>0.7 (0.9-0.6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel 3: Residential on BB</th>
<th>% Sales loss</th>
<th>MC/p</th>
<th>m</th>
<th>Implied Critical Sales Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Ofcom assessment</td>
<td>10.4</td>
<td>Ofcom assessment</td>
<td>0.38-0.58</td>
<td>0.62-0.42</td>
</tr>
<tr>
<td>2 Actual SS%</td>
<td>10.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Actual SS% with ± 10% confidence interval</td>
<td>0.4-20.4</td>
<td>Our assessment</td>
<td>0.3 (0.1-0.4)</td>
<td>0.7 (0.9-0.6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel 4: SMEs on BB</th>
<th>% Sales loss</th>
<th>MC/p</th>
<th>m</th>
<th>Implied Critical Sales Loss Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Ofcom (incorrect) assessment</td>
<td>4.3</td>
<td>Ofcom assessment</td>
<td>0.41-0.62</td>
<td>0.59-0.38</td>
</tr>
<tr>
<td>2 Actual SS%</td>
<td>32.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Actual SS% with ± 10% confidence interval</td>
<td>23.3-43.3</td>
<td>New assessment</td>
<td>0.3 (0.1-0.4)</td>
<td>0.7 (0.9-0.6)</td>
</tr>
</tbody>
</table>

Table 6: BB and uNB Residential Market HMTs

<table>
<thead>
<tr>
<th>Parameter/Variable</th>
<th>Service</th>
<th>Original Values</th>
<th>Revised Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{11}$</td>
<td></td>
<td>-1.04</td>
<td>-1.25</td>
</tr>
<tr>
<td>$\alpha(%)$</td>
<td></td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>$\Delta_{(1)}^\alpha$</td>
<td>BB</td>
<td>0.0416</td>
<td>0.0</td>
</tr>
<tr>
<td>$\Delta_{(2)}^\alpha$</td>
<td>uNB</td>
<td>-1.542</td>
<td>-1.542</td>
</tr>
<tr>
<td>$\Delta_{(1,2)}^\alpha$</td>
<td>Both</td>
<td>-0.482</td>
<td>-0.524</td>
</tr>
</tbody>
</table>
**Figure 1:** Change in profit as a function of the price increase, $z$.

Gradient $= \Delta K$

$\alpha_{\text{crit}} = -\Delta K / \psi_K$

$\Delta \Pi(z)$

$\psi_{\text{crit}} = -\psi_{\text{max}} / \psi_K$