RISK AVERSION, GAMBLING AND THE LABOUR–LEISURE CHOICE

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The expected utility hypothesis continues to remain the most important tool available to economists wishing to analyse problems involving risk and uncertainty. Within this framework it is common to assume that the cardinal utility function involved features diminishing marginal utility of wealth/income as wealth/income increases. This concavity assumption is intuitive, mathematically convenient and coherent (in that concave or quasi-concave utility functions are used in a wide range of economic models). However, the stylised empirical fact that individuals often purchase insurance and indulge in actuarially fair or less than fair gambles seems to conflict with this concavity assumption.

The apparent contradiction may be resolved in various ways. For example, it may be that the individual obtains enjoyment from the act of gambling per se, or that there are external effects involved (for instance, where some part of a lottery's proceeds go to a charity). A difficulty with these explanations is that it is usually possible to point to instances of gambling where there seems to be little enjoyment gained from the act per se and where externalities do not seem to be present (e.g. the holding of premium bonds). For these or other reasons, it seems to be regarded as a basic requirement that any theory of uncertainty should be capable of accounting for the coexistence of gambling and insurance (Kim, 1973, p. 154).

Perhaps the most famous explanation of the coexistence of insurance and gambling is that due to Friedman and Savage (1948) who proposed a utility function composed of two strictly concave segments separated by a strictly convex segment. However, economists have been reluctant to give up on the diminishing marginal utility assumption and the Friedman–Savage resolution of the paradox has come to be seen as too "ad hoc". Much of the subsequent work in this area has thus been concerned with enabling the co-existence of gambling and insurance without dropping the concavity assumption: Indivisibility of expenditures (Ng, 1975), borrowing restrictions (Hakansson, 1970) differential interest rates (Kim, 1973) and transactions costs (Flemming, 1969) have been proposed as reasons why individuals may gamble notwithstanding the diminishing marginal utility assumption. This note offers an alternative and appealing simple induced-utility explanation which is particularly relevant to the case where small stake/large prize gambles are being considered (premium bonds, the pools, etc.).

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The basic model involves a labour leisure choice. Let \( l \) denote leisure; \( L \), Labour; \( w \), the wage rate; \( c \), consumption expenditure; \( y \), interest income and \( W \), wealth. \( U(c, l) \) denotes the individual's von Neumann-Morgenstern utility function which is assumed to be strictly concave; \( U_c, U_l > 0, U_{cc}, U_{ll} < 0, \) and \( U_{cc} U_{ll} - U_{cl}^2 > 0 \). With \( U_{cc} < 0 \), the individual is risk averse with respect to consumption expenditure. Kim (1973) suggests that individuals value wealth for the income it generates. An easily generalisable assumption is that \( y = \alpha W \) where \( \alpha \) is some positive constant. The budget constraint is thus \( c \leq wL + y = wL + \alpha W \). Normalising total time available to unity, so that \( l = 1 - L \), the choice problem becomes

\[
\text{Maximise} \quad U(c, 1 - L) \\
\text{Subject to} \quad c \leq wL + \alpha W \\
\quad 0 \leq L \leq 1
\]

It is straightforward to show that the induced utility of wealth function associated with this problem is strictly concave, notwithstanding the possibility that the constraints on \( L \) may bind. Furthermore, the concavity result proves to be robust to the introduction of overtime rates (see appendix). However, many individuals cannot easily or freely vary their choice of working hours; the existence of such institutional rigidities generates a potentially significant non-concavity in the induced utility of wealth function. The simplest case is where choice is restricted to \( L = 0 \) or \( L = \tilde{L} \) where \( \tilde{L} \) is some fixed level. That is, the individual faces the stark choice between either working or not working.

With \( L = 0 \),

\[
U = U(\alpha W, 1)
\]

and with \( L = \tilde{L} \),

\[
U = U(w\tilde{L} + \alpha W, 1 - \tilde{L})
\]

The individual's problem, for given \( W \), is to choose the larger of these. The solution will typically switch from \( L = \tilde{L} \) for low values of \( W \) to \( L = 0 \) for high values. Analytically, the existence of a switchpoint may be established through entirely reasonable assumptions about the utility function: consideration of subsistence suggests that

\[
U(w\tilde{L} + \alpha W, 1 - \tilde{L}) > U(\alpha W, 1) \quad \text{for low } W
\]

whilst assuming

\[
\sup_c U(c, 1 - L_1) > \sup_c U(c, 1 - L_2)
\]

for \( L_1, L_2 \) such that \( 0 \leq L_1 < L_2 \leq 1 \), guarantees that

\[
U(w\tilde{L} + \alpha W, 1 - \tilde{L}) < U(\alpha W, 1)
\]
for sufficiently large $W$. That a switchpoint $\bar{W}$ exists for which

$$U(w\bar{L} + \alpha\bar{W}, 1 - \bar{L}) = U(\alpha\bar{W}, 1)$$

then follows by continuity.

The induced utility of wealth function defined as

$$V(W) = U(wL + \alpha W, 1 - L) \quad \text{for} \quad W \leq \bar{W}$$

$$V(W) = U(\alpha W, 1) \quad \text{for} \quad W \geq \bar{W}$$

is clearly strictly concave on $[0, \bar{W}]$ and on $(\bar{W}, \infty)$, but the corner at $\bar{W}$ renders the function non-concave on $[0, \infty)$ (see Figure 1).

The utility function will retain this property so long as there are lower bound restrictions on the choice of hours worked. For example, admitting overtime, and a choice set $\{L : L = 0 \text{ or } L \leq L \leq 1\}$ will modify the function over the wealth range where overtime is worked (and $V(W)$ will be concave here—see appendix) but, as $\bar{W}$ increases, $L$ will tend to fall (assuming leisure is a normal good) until, at $W_0$ say, the level $L$ is reached. This will be maintained for the interval $[W_0, \bar{W}]$ with $\bar{W}$ being the switchpoint as in Figure 1. Thus, again, $V(W)$ will be strictly concave for $W \leq \bar{W}$ and for $W \geq \bar{W}$, but not concave over the whole interval $[0, \infty)$.

The switch point from labour to leisure, $\bar{W}$, will generally vary across individuals as will the initial wealth position in relation to $\bar{W}$. There is little need to dwell at length on the gambling-insurance implications of induced utility as in Figure 1 as these have been extensively analysed in previous work. Since the kink at $\bar{W}$ can be quite pronounced, this provides an
explanation of individuals enthusiasm for purchasing small stake/large prize lottery tickets even where initial wealth is far below $\bar{W}$. The point is that lotteries offer the possibility of an alternative life style—the life of leisure. Introspection suggests this may be the principal reason why such lotteries seem so popular. Certainly, casual empiricism suggests that many large prize lottery winners do indeed make the shift from a regime of work to a life of leisure.

APPENDIX

To show that the induced utility of wealth function, $V(W)$, is strictly concave when the individual can freely choose hours of work, first consider the problem

$$\begin{align*}
\text{Maximise} & \quad U(c, 1 - L) \\
\text{Subject to} & \quad c = wL + \alpha W \\
& \quad 0 \leq L \leq 1
\end{align*}$$

For an interior solution, necessary conditions are that

1. $wU_c - U_l = 0$  
2. $H = w^2U_{cc} - 2wU_{ct} + U_{tt} \leq 0$

Assume the latter holds with strict inequality so that (i), (ii) constitute sufficient conditions (it is common to assume $U_{ot} > 0$ which guarantees this result). Defining $V(W) = U(c^*, 1 - L^*)$ where $c^*, L^*$ denote the optimal solution values in the above problem, a routine comparative statics exercise establishes that

$$V'(W) = \alpha U_c(wL^* + \alpha W, 1 - L^*) > 0$$

and, suppressing arguments,

$$V''(W) = \alpha^2[U_{cc}U_{tt} - U_{ct}^2]/H < 0$$

Hence $V(W)$ is strictly concave if, for all $W$, there is an interior solution. Now suppose the constraint $L \leq 1$ never binds but that there is a level of wealth, $W_0$, above which the individual chooses $L = 0$. Clearly $V(W) = U(\alpha W, 1)$ is concave in wealth for $W \geq W_0$, and from the above, $V(W)$ is concave for $W \leq W_0$. At $W_0$, $V(W_0) = V(W_0^*) = V(W_0^*)$ since, by assumption, as $W \to W_0^-$, $L \to 0$ and $L = 0$ for $W \geq W_0$. Referring to (iii), we also have $V'(W_0^-) = V'(W_0^+) = \alpha U_c(\alpha W_0, 1)$ (and, additionally, $V''(W_0^+), V''(W_0^-) < 0$, though these are not in general equal). Hence, $V(W)$ is continuously differentiable on $[0, \infty)$ with a positive but strictly decreasing first derivative—it is thus strictly concave on this interval. A similar argument applies if the constraint $L \leq 1$ should bind.

Introducing overtime rates does not affect this concavity result (so long as hours can be freely chosen). Thus, suppose there is a wage rate $w_1$ for $0 \leq L \leq L_1$ and a wage rate $w_2 (w_2 > w_1)$ for $L_1 < L \leq 1$. Assuming leisure is a
normal good (which it certainly is if $U_{it} > 0$, since $dL^*/dW = -\alpha(wU_{it} - U_{it})/H$), the solution is likely to involve overtime for low values of $W$, then a region where $L_1$ hours are worked, followed by a region where the individual works less than $L_1$ hours, as depicted in Figure 2.

It is straightforward to check that $V(W)$ is strictly concave on $[0, W_1]$, $[W_1, W_2]$, $[W_2, W_0]$, and $[W_0, \infty)$ and that the limit argument used above may be applied to show that $V(W^-) = V(W^+) = V(W)$ and $V'(W^-) = V'(W^+)$ for $W = W_1, W_2$ and $W_0$. Hence it follows that $V(W)$ is continuously differentiable on $[0, \infty)$ with $V'(W)$ positive, continuous and strictly decreasing on this interval. Hence $V(W)$ is strictly concave on $[0, \infty)$.

REFERENCES


