SHADOW PRICES, CONSISTENCY AND THE VALUE OF LIFE

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This paper examines the consequences of inconsistent shadow pricing. The principal example of such inconsistency is the value of life/safety. Second-best shadow pricing policy is shown to depend upon how public sector budgets are allocated and the principle that public sector decision-makers should agree to a consistent shadow price is shown to require information unlikely to be available. A weaker consistency principle requiring minimal information is then developed.

1. Introduction

Suppose that different public sectors use different (implicit or explicit) shadow prices for the same good. How then does the nature of the public sector budgeting system affect the choice of such a shadow price and what incentives does it create for 'incorrect' pricing? Furthermore, should all public sector decision-makers agree to some common shadow price (the so-called consistency principle)? The purpose of this paper is to investigate these questions and their relevance for public sector project appraisal under different assumptions about the nature of the budgeting process.

The principal example, and undoubtedly the most important, of shadow price inconsistencies is that of life and safety. The valuations explicitly or implicitly used here vary enormously, often by orders of magnitude, both within and across public sectors (health, transport, energy, water resources, etc.) — and this is true for most economies [see, for example, Jones-Lee (1976, 1982), Mooney (1977)].

Work done on the problem of second best has usually taken an aggregative view of the public sector,1 a view justified in part by the observation that production efficiency in the public sector is a necessary condition for the second-best optimum (see, for example, Hagen (1979), Guesnerie (1980)]. Clearly, by introducing inconsistent shadow prices and a multi-sector budgeting system, this will no longer be the case. However, the budget constraints

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1See, for example, Baumol and Bradford (1970), Boiteux (1956), Hagen (1979), and Guesnerie (1980). A recent exception is Marchand and Pestieau (1984).
here create the same type of trade-off and the results derived for a public sector's optimal choice of shadow price accords with what one would expect from previous work on the second best. These results (section 3) are therefore not really new in themselves, but they usefully focus attention on the role played by the budgeting system. Thus, shadow pricing and incentives are shown to be sensitive to the way budgets are allocated within the public sector.

The consistency principle, that independent decision-makers should agree to use a common shadow price, seems to be highly regarded by many practicing applied economists. Sections 2 and 3 provide a basis for analysing such principles. Section 4 shows that a full consistency principle is probably untenable given the kind of information available to decision-makers (although it does have some attractive features). However, within this framework at least, a more limited version of the consistency principle is shown to be valid; that decision-makers should agree to reduce the overall dispersion of their (explicit or implicit) shadow price valuations.

For simplicity, the focus is upon the shadow price of a single good which is called 'lives saved or lost' since life and safety is the pre-eminent exemplar of the problem to be discussed.

2. Public sector budget allocation and decision-making

Changes in assumptions as to exogenous constraints and available policy instruments normally affect second-best solutions. It is therefore desirable to investigate the robustness of results to variations in assumptions [Guesnerie (1980)]. In this section, the underlying framework is developed whilst in the following analysis alternative assumptions about decision rules are considered.

The consistency principle seems to be quite widely supported — the following quotes are illustrative:

In general, whenever different decisions are being taken by reference to different valuations of the same good there will be inefficiency. That is, it will be possible to produce unambiguous improvements in the allocation of resources. It is, then, more efficient for a number of independent decision-makers to take decisions by using a common agreed set of valuations of goods than for each to use his own valuations [Sugden and Williams (1978, p. 190)].

Given the difficulties surrounding the probability method and the need for further research, we would argue that in the meantime consistency, as opposed to hysterical reaction, is required for public policy towards life saving investment. Then it will be possible to ensure that similar values of life are being used to evaluate public projects without systematic biases in particular safety areas such as road safety or health [Cullis and West (1979, p. 213)].

In an important sense, it simply does not matter what positive value is put on human life. The problem again correctly identified is put as 'allocating scarce resources'. In order to do this it is more likely that a better allocation of resources will be achieved if a necessarily arbitrary but consistent figure is used in decision-making [Hockey (1983, p. 11)].
Within the hierarchy of public sector decision-making, four levels may be usefully distinguished (table 1). The choice of overall budget at level 1 is taken to be exogenous and is not examined in this paper, the concern being with decision-making at levels 2, 3 and 4.

What then are the likely information flows within this structure? Some variation of the following kind of process is envisaged:

(a) Shadow prices are set at level 3 (where inconsistencies between sectors may arise because of the independence of decision-making). They might however be set at level 4 (where inconsistencies would be even more likely).
(b) Level 4 decision-makers undertake cost-benefit analyses (CBA) and return these to level 3.
(c) Level 3 decision-makers compile investment programmes based on CBA returns from level 4, returning the former to level 2.
(d) Level 2 decision-makers decide upon sector budgets after comparison of level 3 investment programmes.
(e) Given the sector budget, each sector (level 3) implements its investment programme.

### The model

The overall government public sector investment budget ($B$) is allocated to sectors, the $j$th sector receiving $B_j$. Thus $\sum B_j = B$. Each sector is assumed to have a given opportunity set of projects it would like to undertake in the absence of a budget constraint, but the budget constraint is assumed binding in each sector. There is one good which requires a shadow price — call this 'life'. Each sector $j$ appraises projects using a value of life $v_j$ whilst $V^*$ denotes the 'true' value of life. $V^*$ can bear different interpretations; in

<table>
<thead>
<tr>
<th>Level</th>
<th>Location</th>
<th>Decision</th>
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<tbody>
<tr>
<td>1</td>
<td>Central government</td>
<td>Setting of overall public sector investment budget</td>
</tr>
<tr>
<td>2</td>
<td>Central government</td>
<td>Division of budget amongst public sectors (health, transport, etc.)</td>
</tr>
<tr>
<td>3</td>
<td>Within public sector</td>
<td>Construction of investment programmes</td>
</tr>
<tr>
<td>4</td>
<td>Within public sector</td>
<td>Decentralised cost-benefit analysis of individual project opportunities</td>
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discussing second-best pricing (section 3) it is the value considered to be
correct by the relevant decision-maker. In discussing the consistency principle
(section 4), a ‘deus ex machina true value’ is also considered.

Project $i$ in sector $j$ is characterised by three features: capital outlay ($K_{ij}$),
lives saved ($X_{ij}$) and other (net of outlay) benefits ($A_{ij}$). We abstract from
intertemporal issues and assume that sector $j$’s perceived net benefits are
maximised, in the absence of a budget constraint, by implementing all
projects which have a benefit-cost ratio $R_{ij} \geq 0$, where

$$R_{ij} = \frac{(A_{ij} + X_{ij}v_j)}{K_{ij}}.$$  

(1)

It is a ‘perceived’ maximisation because it is based upon sector $j$’s view of the
correct shadow price to be used. With a budget constraint, the perceived
maximisation is accomplished by rank ordering and implementing projects
until the budget is exhausted.\(^3\)

Denoting $A_j$, $X_j$, $K_j$ as $j$ sector net benefit, lives saved and outlays,
respectively, project $i$ in sector $j$ locates at a specific point in $(A_j, X_j, K_j)$
space. Defining $a_j = A_j/K_j$ and $x_j = X_j/K_j$, a bivariate histogram may be con-
structed as follows. Consider the region defined by $a_j, a_j + \Delta$ and $x_j, x_j + \Delta$
for some $\Delta > 0$. Sum the capital outlays of projects which locate in this region
and denote this as $k_j$. For a given set of projects with given characteristics,
$k_j$ will be a function of $a_j$ and $x_j$ (for example, $k_j \to 0$ as $a_j, x_j$ become large
since few projects earn very large rates of return). Transposing this discrete
description into continuous format, $k_j(a_j, x_j)$ becomes the bivariate density
function for project outlays: Project viability requires $R_j = a_j + x_jv_j \geq 0$ whilst,
with a binding budget restriction, there will be a higher cut off at $\bar{R}_j$; viability
requires that $a_j + x_jv_j \geq \bar{R}_j$.

For a given $\bar{R}_j$, total capital outlays on implemented projects would be

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} k_j(x_j, a_j)dx_j da_j,$$

so the budget constraint is

$$\psi_j = B_j - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} k_j(x_j, a_j)dx_j da_j \geq 0.$$  

(2)

Note that this defines a relation between $B_j$, $\bar{R}_j$ and $v_j$.

\(^3\)In an inter-temporal context, $A_{ij}$, $X_{ij}$ could be regarded as present-value equivalents. With
just a single period budget constraint the $B-C$ ratio ranking method is still valid [see, for
example, Weinstein and Zeckhauser, (1973)] but with inter-temporal constraints this no longer
holds. Few simple or general principles hold in this context, although see Cantor and Lippman
(1983) for a recommendation of IRR techniques when the inter-temporal constraints are of a
non-negative liquidity type.
From an overall social context, the concern is with the 'true' (as viewed by some particular decision-maker) social value of the projects which are implemented (rather than the sector j’s perception of such benefits). This is given by

$$T_j \equiv \int_{-\infty}^{\infty} \int_{\frac{1}{V_j}(\bar{R}_j-a_j)}^{\infty} (a_j + x_j V^*) k_j(x_j, a_j) dx_j da_j.$$  \hspace{1cm} (3)

Expositionally, it proves useful to define the following functions:

$$F_j(v_j, \bar{R}_j, a_j) \equiv \int_{\frac{1}{V_j}(\bar{R}_j-a_j)}^{\infty} k_j(x_j, a_j) dx_j,$$ \hspace{1cm} (4)

$$G_j(v_j, \bar{R}_j, a_j, V^*) \equiv \int_{\frac{1}{V_j}(\bar{R}_j-a_j)}^{\infty} (a_j + x_j V^*) k_j(x_j, a_j) dx_j.$$ \hspace{1cm} (5)

3. Sector 1: Shadow pricing decisions

Suppose sectors j = 2, . . . , n use inconsistent shadow prices. How then does budget allocation policy at level 2 affect the sector 1 (level 3) choice of v1? Three possible assumptions are considered.

Assumption 1. Sector budgets allocated arbitrarily.

There are various reasons why this could be regarded as realistic: (i) there may be insufficient information flow from level 3 to make a more informed allocation; (ii) inertia and historical precedent may dominate in such decisions, in the short run at least; and (iii) budgets may be allocated by non-economic processes.

With an independent budget, the shadow price rule is to set $v_1 = V^*$. Analytically, the problem is:

$$\max_{v_1, \bar{R}_1} T_1$$

subject to $\psi_1 \geq 0$.

That is, choose a shadow price and cut-off benefit–cost ratio to maximise net benefits. It is straightforward to show that the first-order necessary conditions are satisfied if $v_1 = V^*$ and not, in general, otherwise.

Assumption 2. Full information budget allocations.
Level 2 has full information on $k_j, v_j, j = 1, \ldots, n$. This assumption seems decidedly unrealistic — it is most unlikely that level 2 could have full information on $k_j(x_j, a_j)$ (for all $x_j, a_j$). It is included only to point out that if sector 1 agrees with level 2 about the value of $V^*$, whilst sectors $j = 2, \ldots, n$ fix $v_j, j = 2, \ldots, n$, exogenously, then the shadow pricing rule is again $v_1 = V^*$.

Formally, level 2 chooses $B_j, j = 1, \ldots, n$, and hence $R_j, j = 1, \ldots, n$, whilst sector 1 chooses $v_1$; the overall budget constraint is

$$\psi = B - \sum_{j=1}^{n} \int_{-\infty}^{\infty} F_j(\cdot) da_j = 0$$

and social welfare is given by

$$T = \sum_{j=1}^{n} \int_{-\infty}^{\infty} G_j(\cdot) da_j.$$  

The problem is:

$$\max_{v_1, R_1, \ldots, R_n} T$$

subject to $\psi = 0$.

Again, under some reasonable technical assumptions, it is straightforward though tedious to show that the first-order conditions are satisfied if $v_1 = V^*$ and not in general otherwise. Given the agreement by sector 1 and level 2 on $V^*$ this result is to be expected — in view of the Diamond–Mirrlees (1971) results. Where government can control for distortions, public sector firms should set (shadow) prices at marginal costs [see also Boadway (1975)].

Intuitively, suppose $v_1 \neq V^*$ and level 2 chooses $B_1, \ldots, B_n$ optimally. Now change $v_1$ to $V^*$, holding $B_1, \ldots, B_n$ fixed: there is an increase in welfare (as under assumption 1). Allowing the authority to readjust $B_1, \ldots, B_n$ can only increase this welfare gain. Thus $v_1 = V^*$ is a necessary condition for optimality.

If, however, sector 1 level 3 decision-makers disagree with level 2 as to the correct value for $V^*$, then one would expect that, in general, optimality would require sector 1 to set $v_1 \neq V^*$; changing the shadow price allows it to manipulate the budget allocations.

**Assumption 3.** Uniform cut-off benefit–cost ratios.

A sensible and feasible alternative decision rule to assumption 1 is to assume that budgets are allocated according to a uniform cut-off benefit–cost
This decision rule merely requires that level 2 decision-makers have knowledge of investment schedules [as functions of the cost–benefit ratio (CBR)] from each sector. This kind of information, it could be argued, is available — perhaps not in detail — but central government must have some feeling for project rates of return in the various sectors. Consider for example a sector whose allocated budget is small relative to its project opportunities. It will be required to reject marginal projects which would earn relatively high returns. It is likely that those concerned would be able to communicate this fact in some way to the central authority. If this is so, the central authority could be expected to respond to these demands, at least in the longer run, by increasing such sectors’ budgets and correspondingly diminishing the budgets of other sectors. Such a process would realise (approximately) the rule of the uniform cut-off benefit–cost ratio.

In this case, even if sector 1 decision-makers agree with level 2 decision-makers about $V^*$, the optimal shadow price will generally be $v_1 \neq V^*$. In this case $\bar{R}_j - \bar{R}, j=1,\ldots,n$. Sector 1 chooses $v_1$. The problem is:

$$\max_{v_1, R} \sum_{i=1}^{n} \int_{-\infty}^{\infty} G_j(v_j, R, a_j, V^*) \, da_j$$

subject to

$$\psi = R - \sum_{j=1}^{n} \int_{-\infty}^{\infty} F_j(v_j, \bar{R}, a_j) \, da_j.$$ 

The first-order conditions in this case tell us little. However, the following special case (which proves useful in the analysis of consistency principles in section 4) establishes the fact that $v_1$ is not in general equal to $V^*$.

Restrict candidate projects to those which earn net benefits and save lives: $x_j \geq 0, a_j \geq 0$ for all $j$. Assume a uniform project outlay density function:

$$k_j(x_j, a_j) = k_j > 0 \quad \text{for } 0 \leq x_j \leq \beta_j,$$

$$0 \leq a_j \leq \alpha_j,$$

$$k_j(x_j, a_j) = 0 \quad \text{elsewhere}.$$ 

The upper bounds, $\alpha_j, \beta_j$, make the uniform density function a sensible approximation to what in reality would be a non-uniform density function. It is assumed that $\beta_j > \bar{R}/v_j$ and $\alpha_j > \bar{R}$ for $j=1,\ldots,n$ (see fig. 1).

The $n$-sector budget constraint for this case simplifies as follows:
whilst total benefits are given by

\[ T = \sum_{j=1}^{n} \int_{0}^{\bar{R}} \int_{(1/v_j)(\bar{R}-a)}^{\beta_j} (a+\alpha v^*) k_j dx da + \sum_{j=1}^{n} \int_{0}^{\bar{R}} (a+\alpha v^*) k_j dx da \]

\[ = \sum_{j=1}^{n} k_j \left[ \beta_j \alpha_j \left( \alpha_j + \beta_j V^* \right) - \frac{R^3}{6v_j} \left( 1 + \frac{V^*}{v_j} \right) \right]. \quad (9) \]

The problem is to maximise (9) subject to (8) with choice variables \( v_1 \) and \( \bar{R} \). Denoting \( \lambda \) as the multiplier on the constraint, the first-order conditions give

\[ \lambda = \frac{\bar{R}}{3} \left( 1 + \frac{2V^*}{v_1} \right) \quad (10) \]

\[ \sum_{j=1}^{n} \frac{Rk_j}{v_j} \left[ \frac{\bar{R}}{2} \left( 1 + \frac{V^*}{v_j} \right) - \lambda \right] = 0. \quad (11) \]

Now suppose

\[ 0 \leq v_j < V^*, \quad j = 2, \ldots, n, \quad (12) \]
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and

\( k_j > 0, \ j = 1, \ldots, n. \) \hspace{1cm} (13)

Then suitable manipulation of (10) and (11) yields:

\[ V_{\text{min}} < v_1 < V^*, \] \hspace{1cm} (14)

where \( V_{\text{min}} \) represents the lowest shadow price used in sectors \( j = 2, \ldots, n. \)

Apart from furnishing a counter-example (to \( v_1 = V^* \)), the assumption (12) is motivated by the observation that, in the United Kingdom at least, most sectors currently adopt shadow valuations of life significantly below the kind of estimates proposed in recent research [see, for example, Jones-Lee (1982)]. Since many of these sectors do not explicitly use a cost benefit analysis (CBA) methodology, it seems reasonable to regard their (implicit) shadow prices as exogenous. Sectors (such as road transport in the United Kingdom) which explicitly use a CBA methodology and currently use low shadow price estimates relative to recent revaluations face the question of how far to revise their shadow prices. The above results suggest a significant compromise between the full estimated value of \( V^* \) and values used elsewhere — if budgets respond to the quality of projects within sectors.

Naturally, if government cannot control for the various distortions, as in this case, (shadow) price should generally not equal 'marginal cost' [here \( V^* \); cf., for example, Baumol and Bradford (1970)]. In this case the use of a higher value of life, ceteris paribus, increases the attractiveness of all projects in a sector and so increases that sector's claim upon the central budget at the expense of other sectors. The intuition is that if other sectors use lower values of life than \( V^* \), their projects will be misranked to some extent. However, this does not justify using \( V^* \) in sector 1. Using \( V^* \) in sector 1

\[ (10) \text{ and } (11) \text{ imply that:} \]

\[ \left( 1 - \frac{V^*}{v_1} \right) \frac{\sum_{j=2}^{n} \frac{k_j}{v_j} + \frac{\sum_{j=2}^{n} k_j}{\sum_{j=2}^{n} v_j}}{\frac{\sum_{j=2}^{n} k_j}{\sum_{j=2}^{n} v_j}} = 3 \frac{\sum_{j=2}^{n} k_j}{\sum_{j=2}^{n} \frac{v_j}{v_j}} \left( 1 - \frac{V^*}{v_j} \right). \]

With \( k_j > 0 \) and \( 0 < v_j < v^*, \ j = 2, \ldots, n, \) it follows that \( 0 < v_1 < v^*. \) Denote \( V_{\text{min}} = \min (v_j) \ (j = 2, \ldots, n). \) Then

\[ 3 \sum_{j=2}^{n} \frac{k_j}{v_j} \left( 1 - \frac{V^*}{v_j} \right) > 3 \left( 1 - \frac{V^*}{V_{\text{min}}} \right) \sum_{j=2}^{n} \frac{k_j}{v_j}, \]

hence, the result, \( V_{\text{min}} < v_1 < V^* \)

The above equation allows direct computation of \( v_1 \) given \( V^*, k_j, j = 1, \ldots, n, \) and \( v_j, j = 2, \ldots, n. \) To illustrate: if there are just two sectors and \( k_1 = k_2, \) then, with \( v_2 \) set at 0.1\( V^* \), 0.33\( V^* \), and 0.5\( V^* \), optimal \( v_1, \) is, respectively, 0.15\( V^* \), 0.44\( V^* \), and 0.62\( V^* \). This kind of calculation is useful because it suggests that what other sectors do may considerably influence \( v_1 \). A personal view is that the shadow price chosen for use in road transport in the United Kingdom should be heavily influenced by implicit and essentially exogenous shadow prices in the health sector — if budgets respond to the quality of projects as in assumption 3. If, however, it is deemed that assumption 1 is a better description of the budgeting process, then \( V^* \) should be chosen.
would yield a correct intra-sector rank ordering of projects but it would also, in the long run, mean that too many of these projects would be undertaken. Optimality requires that sector 1 accept a degree of misranking (by setting $v_1 \neq V^*$) in order to release funds to other sectors where, in spite of their project misranking, net welfare improvements can be made.

**Discussion**

The above results are not particularly surprising but they have the merit of pointing out the relevance of budget processes in shadow pricing decisions. Assumption 1 seems more appropriate in a short-run context, yet assumption 3 seems more appropriate in the longer run. In the short run, budgets seem fixed and largely historically determined; in the longer run, budget shares seem to shift in response to the various claims of sectors. In this latter context, an incentive problem arises, for if budgets do respond to the perceived quality of projects in the various sectors and if the details of project analysis are not available at level 2, then an increase in shadow prices at level 3 increases a sector's share of the overall budget. In so far as 'empire-building' enters into level 3 decision-maker utility functions, this might give some cause for concern.

Of course, it would be desirable for shadow prices to be determined centrally rather than by individual sectors. The problem discussed here remains, however, so long as there are sectors which do not operate the CBA methodology. In such a case sector 1 conceptually includes all sectors using CBA — and there is agreement between these sectors and level 2 about $V^*$. But implicit valuations in other sectors diverge from $V^*$, so the problem is essentially that discussed under assumptions 1 and 3.

To sum up, if it is believed that sector budgets are essentially independent and exogenously fixed, shadow pricing in this framework by one sector is independent of the behaviour of other sectors and so $V^*$ should be used. Alternatively, if it is believed that sector budgets do respond to sector demands for funds, account must be taken of other sectors behaviour. If other sectors adopt shadow prices below what is considered to be the correct value ($V^*$) it is probable (but not certain) that sector 1 should also adopt a lower value than $V^*$.

4. **The principle of consistent valuations**

If individual decision-makers adopt different valuations for the same good, the outcome is inefficient. The principle of consistency holds that an
unambiguous efficiency gain may be obtained if those involved adopt a common agreed set of values for such goods. This consistency principle seems especially attractive because its application seems to require minimal information requirements. The object of this section is (i) to show that the full consistency principle requires more information than will usually be available and (ii) to show that a weaker consistency principle can still be advocated with minimal information requirements.

Consistency has usually been recommended for the case where budgets are set independently [see Sugden and Williams (1978, p. 188)]; this implies that eq. (2) holds for each sector. For simplicity, assume there are just two sectors which currently use valuations $v_1, v_2$, where $v_1 > v_2$. It is assumed that the individuals concerned believe they are initially using the correct valuation and that any agreed consistent valuation will lie within the range of these original valuations.

The following interpretation of the consistency principle is generally correct:

[1] Where decision-makers use different valuations for the same good, an unambiguous efficiency gain may be obtained by using some particular intermediate common value.

A simple way to illustrate why this is generally correct is to construct output transformation functions for each sector; given the project opportunity set and a fixed budget, the choice of a sector shadow price determines which projects are undertaken and hence the total number of lives saved ($\sum x$) and other benefits ($\sum a$). An increase in shadow price increases the attractiveness of life saving projects and so $\sum x$ tends to increase whilst $\sum a$ tends to fall. If a convex transformation frontier is assumed for each sector, fig. 2 clearly shows why an unambiguous efficiency gain is possible: an intermediate shadow price exists which will generate a greater total (across both sectors) of both lives saved and other benefits (by moving to a point of mutual tangency). In general there will be a range of common values $v$ which will realise such an efficiency gain. Clearly not all intermediate common values yield such a gain; consistent values close to $v_1$ or $v_2$ will yield a smaller total of one of the outputs than in the original situation. It may be that, evaluated at a deus ex machina 'true' $V^*$, some of these will also be welfare improving, but this will generally depend upon the value of $V^*$ vis-à-vis $v_1$ and $v_2$ (this point is made explicit later).

In principle, a common shadow price which yields such an efficiency gain could be identified if the decision-makers identified their transformation frontiers (as in fig. 2) and if that form of analysis was undertaken. However,

6A typical example: if the bulk of CBA studies use, say, the test discount rate, then consistency demands its use in any new study.
it seems more realistic to assume that this level of knowledge and cooperative analysis will not take place and that a common shadow price must be agreed upon before the analysis and choice of projects is undertaken. This is, for example, an appropriate assumption for the scenario where project appraisal is decentralised but the shadow prices for use in such studies are set centrally. Clearly in such a case the allocative outcomes of the shadow price choice cannot be observed until after the shadow prices have been chosen. In such a scenario, the consistency principle, to be useful, must not require detailed information on sector project opportunities and budgets.7

In the light of the above discussion, it seems worthwhile to set out some candidate versions of the consistency principle and to examine their pros and cons ([1] is repeated for convenience).

4.1. 'Deus ex machina' principles

[1] A welfare improvement results from the use of some common value \( v, v_1 > v > v_2 \).

[2] A welfare improvement results from the use of any common value \( v, v_1 > v > v_2 \).

7[1] might be achieved using an iterative procedure. Initially a consistent shadow price that yields efficiency gains has to be guessed, but once applied, outcomes (lives saved, other benefits) may be observed. The consistent shadow price may then be adjusted until an efficiency gain is realised. Apart from technical difficulties, such a procedure would almost certainly be unworkable in practice.
4.2. 'Subjective' principles

[4] Both decision-makers would agree that a welfare improvement may be obtained by agreeing on some common value \( v \), where \( v_1 > v > v_2 \).

[5] Both decision-makers would agree that a welfare improvement may be obtained by agreeing on any common value \( v \), where \( v_1 > v > v_2 \).

[6] Both decision-makers would agree that a welfare improvement may be obtained by agreeing to a marginal increment in \( v_2 \), decrement in \( v_1 \).

[1]–[3] presume the existence of a true but unknown \( V^* \); the idea is that they must hold independently of the value of \( V^* \). It has already been explained how this can be true for [1]. In [4]–[6], there is no exogenous standard by which welfare is judged — the true shadow price is subjective (thus, for the decision-maker in sector 1, \( V^* = v_1 \), etc.). An argument similar to that for [1] can be used to established that [4] is also correct. A common \( v \) which yields an efficiency gain must satisfy [4]. However, to render [1] and [4] practical requires detailed information (in order to identify a particular consistent shadow price which yields an efficiency improvement). Suppose then we reject [1] and [4]. It may also be shown that [2], [3] and [5] do not hold and that only [6] is unambiguously correct. To refute [2], [3] and [5], it suffices to show they will not generally hold for some particular case; the simplified model of section 3 (assumption 3) may be used to provide the necessary counter-examples. [6] is then demonstrated for the general case.

With independent budgets and uniform sector project densities, benefits and constraints are given by [cf. (8), (9)]:

\[
T = \sum_j k_j \left[ \frac{\beta_j \xi_j}{2} (\alpha_j + \beta_j \beta_j V^*) - \frac{R_j^3}{6 \bar{v}_j} \left( 1 + \frac{V^*}{\bar{v}_j} \right) \right],
\]

\[
B_j = k_j \left[ \beta_j \xi_j - \frac{R_j^2}{2 \bar{v}_j} \right], \quad j = 1, 2.
\]

\( \bar{v}_j \) is used to denote an arbitrary valuation used in sector \( j \). It follows from (15) and (16) that

\[
\frac{\partial T}{\partial \bar{v}_j} = \frac{R_j^3}{12 \bar{v}_j^2} \left( \frac{V^*}{\bar{v}_j} - 1 \right), \quad j = 1, 2,
\]
where initially $v_j = v_j$, $j = 1, 2$. Thus, overall benefits depend upon $V^*, \tilde{v}_1, \tilde{v}_2$; $T = T(\tilde{v}_1, \tilde{v}_2, V^*)$.

Consider the case where $V^* > v_1 > v_2$. It follows from (15)-(17) that $T(v_1, v_1, V^*) > T(v_1, v_2, V^*) > T(v_2, v_2, V^*)$ and that $T(v, v, V^*)$ is monotonically increasing in $v$ for $v_1 \geq v \geq v_2$. It follows that [2] cannot be correct; for some $v, v_1 \geq v \geq v_2, T(v, v, V^*) < T(v_1, v_2, V^*)$.

Inter alia, it also follows that [1] is correct here; for some $v, v_1 \geq v \geq v_2, T(v, v, V^*) > T(v_1, v_2, V^*)$. However, the range of values for $v$ for which $T(v, v, V^*) > T(v_1, v_2, V^*)$ holds is not independent of $V^*$, as consideration of the case $v_1 > v_2 > V^*$ shows. Thus, if $V^* > v_1 > v_2, v$ must not be too close to $v_2$, whilst if $v_1 > v_2 > V^*, v$ must not be too close to $v_1$. As explained earlier, there is a subset of the above range of values for which $T(v, v, V^*) > T(v_1, v_2, V^*)$ for all $V^*$. The problems of identifying this range have already been discussed.

Relaxing the constraint that a common value $v$ must be chosen, [3] suggests that a marginal reduction in inconsistency leads to a welfare gain. This is always true in one sense: suppose $V^* > v_1 > v_2$, clearly a sufficiently large increment in $v_2$ and small decrement in $v_1$ will yield a welfare gain (and vice versa if $v_1 > v_2 > V^*$). In the absence of knowledge of $V^*$, a general principle must be adopted; [3] requires an equal marginal increment and decrement. However, (17) indicates that this will not always lead to an increase in welfare, and so [3] must be rejected.

In [4], [5] and [6] it is the decision-maker's subjective views of benefits which are of concern. Thus, from sector 1's viewpoint, benefits are given by (15) in which $V^* = v_1$, whilst from sector 2's viewpoint, $V^* = v_2$. [4] has already been discussed. [5] is straightforward to refute. From sector 1's view, a common value $v$ maximises $T(v, v, v_1)$ if $v = v_1$ but $T(v, v, v_1)$ decreases as $v \rightarrow v_2$. When $v = v_2, T(v_2, v_2, v_1) < T(v_1, v_2, v_1)$, so in moving towards $v_2$ there comes a point, denoted $\tilde{v}(v_1 > \tilde{v} > v_2)$, below which subjective welfare gains disappear. Similarly, from sector 2's viewpoint, $V^* = v_2$ and $T$ is maximised when $v = v_2$. In this case $T$ diminishes as $v$ increases from $v_2$ and similarly subjective gains will disappear at some value $\tilde{v}(v_1 > \tilde{v} > v_2)$. Thus, they would not agree that a welfare improvement exists for any $v$ lying between $v_1$ and $v_2$.

This leaves interpretation [6] which can be shown to be unambiguously correct for the general case. A proof is presented in the appendix. [6] implies that independent decision-makers who believe they hold the correct valuations will always agree that a welfare gain may be obtained by some reduction in the inconsistency between the values they use. To sum up, [6] is the only version considered here which is both correct and whose application requires minimal information requirements. It is, of course, a fairly weak principle because it is marginal in nature. It suggests that shadow price outliers should agree to move toward the centre — but it does not indicate
by how far consistency may be increased (that would require the same kind of information as for [1]).

5. Conclusions

The aim has been to clarify certain shadow pricing principles, taking into account the fact that resource allocation in the public sector is dominated by budget constraints. The problem of different decision-makers adopting different shadow prices has received virtually no attention in the literature, yet this problem, for cases such as life and safety, could be of considerable practical significance.

For a particular sector, if sector budgets are independent, the shadow pricing problem is an intra-sector problem. However, if sector budgets are affected by the choice of shadow prices, account must be taken of other sectors' valuations, a classic second-best interaction. It was noted that, if budget allocations can be affected in this way, there may be an incentive to suboptimal shadow pricing.

Finally, various consistency principles have been examined. An efficiency gain can usually be obtained through the adoption of an agreed common price — but the information requirements are substantial — only some consistent prices will do. However, a weaker principle which has minimal information requirements seems tenable: independent decision-makers should agree to reduce the extent of their shadow price inconsistencies.

Appendix

Given $v_1 > v_2$ let the changes be $dv_1(<0)$, $dv_2(>0)$. Net benefits $T$ change by $dT = (\partial T/\partial v_1)dv_1 + (\partial T/\partial v_2)dv_2$. There is a welfare gain if $dT > 0$ on the subjective view of each decision-maker.

First, an expression for $\partial T/\partial v_j$ is established. The $j$th sector benefits are given by eq. (3) and the $j$th sector budget constraint by (2). Differentiating (2) with respect to $v_j$ gives:

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial v_j} \left\{ \frac{1}{v_j} (\bar{R}_j - a_j) \right\} k_j(\cdot) d\alpha_j = \int_{-\infty}^{\infty} \left\{ -\frac{(\bar{R}_j - a_j)}{v_j^2} + \frac{1}{v_j} \frac{\partial \bar{R}_j}{\partial v_j} \right\} k_j(\cdot) d\alpha_j = 0. \tag{A1}$$

so

$$\frac{1}{v_j} \frac{\partial \bar{R}_j}{\partial v_j} \int k_j(\cdot) d\alpha = \frac{1}{v_j} (\bar{R}_j - a_j) \int k_j(\cdot) d\alpha_j. \tag{A2}$$

\footnote{However, if there are other kinds of inter-sector dependence, second best will again rear its head.}

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Differentiating (3):
\[
\frac{\partial T}{\partial v_j} = - \int_{-\infty}^{\infty} \left( a_j \left( 1 - \frac{V^*}{v_j} \right) + \frac{\bar{R}_j V^*}{v_j} \right) \frac{\partial}{\partial v_j} \left\{ \frac{1}{v_j} \left( R_j - a_j \right) \right\} k_j(\cdot) da_j. \tag{A3}
\]

From (A1), (A3) reduces to:
\[
\frac{\partial T}{\partial v_j} = - \left( 1 - \frac{V^*}{v_j} \right) \int \left\{ - \frac{1}{v_j^2} (a_j \bar{R}_j - a_j^2) + \frac{a_j}{v_j} \frac{\partial \bar{R}_j}{\partial v_j} \right\} k_j(\cdot) da_j. \tag{A4}
\]

Adding \( \bar{R}_j \) times (A1) to (A4) gives:
\[
\frac{\partial T}{\partial v_j} = - \left( 1 - \frac{V^*}{v_j} \right) \frac{1}{v_j^2} \int (a_j - \bar{R}_j - \mu)^2 k_j(\cdot) da_j, \tag{A5}
\]

where
\[
\mu = \frac{\int (a_j - \bar{R}_j) k_j(\cdot) da_j}{\int k_j(\cdot) da_j}. \tag{A6}
\]

The integral in (A5) is unambiguously positive, hence:
\[
\frac{\partial T}{\partial v_j} \geq 0 \quad \text{as} \quad \frac{V^*}{v_j} - 1 \geq 0. \tag{A7}
\]

From sector 1’s viewpoint \( V^* = v_1 \); hence from (A7) \( \partial T/\partial v_1 = 0 \) and since \( v_1 > v_2 \), \( \partial T/\partial v_2 > 0 \). From sector 2’s viewpoint \( V^* = v_2 \); hence from (A7) \( \partial T/\partial v_2 = 0 \) and, since \( v_1 > v_2 \), \( \partial T/\partial v_1 < 0 \). Hence, each decision-maker computes \( dT > 0 \).

References


