Intertemporal job-risk choices and the theory of Compensating Wage Differentials

by

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ABSTRACT
This paper presents a model in which individuals, as they age and accumulate wealth, may change jobs in order to re-optimize the balance between remuneration and risk (expected life is thus an endogenous variable). The relationship between willingness to pay and compensating wage differential measures for the value of a statistical life are then explored within this framework.

KEYWORDS: Willingness to pay, Compensating wage differential, Value of life.

JEL CLASSIFICATION: D61, J17, J28, J31.
I. INTRODUCTION

The willingness to pay (WTP) approach to the value of statistical life (VOSL) requires that an individual’s willingness to pay for a specified risk reduction be somehow measured. Whilst estimates can be elicited simply by asking individuals to respond to hypothetical questions, or through the study of behavior in artificial laboratory settings, economists have generally preferred to look at the decisions individuals actually take in order to infer their WTP for risk reduction. In particular, labor markets have for a long time been a major source of empirical evidence regarding this risk-return trade off (for example, see Marin and Psacharopoulos [1982], Rosen [1986], Viscusi [1986], Leigh [1995]). Ceteris paribus, wages tend to be higher in riskier jobs; according to the theory of compensating wage differentials (CWD), the gradient of the estimated wage-risk frontier can be used to provide a simple estimate of the VOSL.

Perhaps the seminal article expounding the equilibrium analysis of the wage-risk trade off is that of Thaler and Rosen [1976], although the general idea can be traced back over two hundred years (to Adam Smith [1776] at least). In this approach, the workers job-risk decision is typically viewed as an essentially atemporal one. However, it is clear that aging (and endogenously determined mortality rates) could have a significant impact on the demand for jobs of varying levels of job risk and hence upon the structure of the equilibrium wage-risk frontier. This observation motivates the present paper, which focuses on the theoretical relationship between the WTP and CWD measures of the VOSL when individuals are potentially long lived and are able to optimize job risk over an endogenously risky lifetime.
In practice of course, individuals choose to move between jobs for many reasons other than purely to re-optimize the level of risk they face. For example, individuals accumulate some forms of human capital (education, skills..) but perhaps lose others (physical strength, dexterity...) and these may influence the jobs they wish to hold (or are able to hold) over time. Such factors need to be controlled for in an empirical context; however, the focus of the present paper is limited to that of clarifying the risk-remuneration aspect of the individual’s labor market decision.

Fairly clearly, if individuals live only one period, there is unlikely to be any discrepancy between the CWD and WTP measures. However, the scope for discrepancy is rather greater once it is recognized that individuals are potentially long lived. The individual faces the problem of trading off a bigger wage against the greater job risk associated with that wage. A higher job-risk reduces the individual’s expected future working life; one might therefore expect, *ceteris paribus*, an individual to move to jobs of increasing risk with age (taking a high risk job early in life jeopardizes future earning potential; later in life, this earning potential is smaller so it can be beneficial to take a higher risk job at this time).

Multi-period risk has of course been considered in the context of the WTP approach (for example, in Jones-Lee [1989], Shepard and Zeckhauser [1984], Ehrlich and Chuma [1990]), and the potential difference between willingness to pay out of wealth and willingness to pay out of income has been examined in this context (Ford, Pattanaik and Wei [1995]). Multi-period models have also been used in an examination of WTP and time preference rates (for example, in Viscusi and Moore [1989]). However, the inter-relationship examined here between WTP and CWD measures of the VOSL for the case
where individuals are potentially long lived has not, to the author’s knowledge at least, been seriously addressed.

In the case where there are zero transactions costs associated with changing jobs, one might expect individuals to continuously adjust the job-risk they face as they age; by contrast, with positive transactions costs, they will choose to periodically and discretely 'lurch' from job to job. In what follows, section II establishes the basic framework whilst section III discusses a polar case in which potentially long lived individuals are assumed to choose a 'job for life'. Given job-risk optimization and the availability of actuarially fair insurance (AFI), it is shown that the WTP and CWD approaches can give consistent estimates of the value of statistical life (VOSL). However, in the absence of AFI, the CWD estimate is generally incorrect. Furthermore, even with insurance, the CWD estimate is generally correct only at the time the individual makes a job selection. This makes sense; in the absence of transactions costs, the individual would wish to re-optimize job risk over time. Once an individual is 'locked' into a job, the WTP measure will systematically vary as the individual ages whilst the CWD measure, being invariant, becomes progressively 'out of date'. The ‘job for life case’ is thus a polar case in that it presumes that transactions costs are sufficiently high to preclude any subsequent job-risk re-optimization.

Section IV examines the other polar case, where there are zero transactions costs associated with changing jobs, such that individuals find it optimal to 'fine tune' job wage-risk on a continuous basis. In this case it is shown that the CWD measure is an appropriate measure of VOSL when considering projects which affect job risk, and that this estimate also coincides with the WTP measure of the VOSL for a 'short period'
project involving risk change. By contrast, the CWD measure is not the appropriate measure when considering projects which affect an individual’s level of risk over a longer period of time.

II. THE BASIC FRAMEWORK

The framework used throughout the paper assumes individuals have a preference ordering over joint distributions of present value wealth $W$ (defined to include discounted lifetime labor income) and time of death $t$ as summarized by a Von Neumann-Morgenstern (VNM) utility function $U: \mathbb{R}_+^2 \rightarrow \mathbb{R}$. This function is assumed to be smooth and bounded, with $\partial U / \partial W, \partial U / \partial t > 0$ whilst $\partial^2 U / \partial W^2 < 0$. This is a fairly natural and simple formulation, given the focus in this paper is on how job-risk influences wealth and longevity. It can of course be criticized on a variety of grounds, in particular because it abstracts from the individual’s inter-temporal consumption plans, and also because, since death typically occurs ‘unexpectedly’, some of an individual’s present value wealth will be enjoyed by his or her estate; no distinction is thus drawn regarding the wealth enjoyed by the individual and that by her progeny. The reader is referred to Jones-Lee [1989] for an extended discussion of these issues, and for a robust justification for the use of such a functional form when the object is to examine choices regarding wealth and longevity.\(^1\)

The CWD methodology assumes that individuals optimize over the wage-risk opportunities available in choosing jobs, that an equilibrium wage-risk frontier arises out of the demand/supply of jobs of varying degrees of riskiness, and that the gradient of the wage-risk function at a given level of risk provides an estimate of the VOSL at that level.
of risk (see e.g. Thaler and Rosen [1976]). Accordingly, it is assumed in this section that there is a continuum of jobs with different risks and that the equilibrium wage risk trade off is defined via a continuously differentiable, strictly increasing wage function, $w(p)$. This is taken as a given function here and in section III. In section IV, we consider a more general model in which the equilibrium wage function is endogenously determined. Thus $w'(p)>0$ is viewed as the CWD measure of the VOSL for an individual who chooses job risk $p$. Note that, since $w(p)$ is quite likely to be non-linear, the value of life may well vary with the risk faced.

The individual is viewed as inheriting wealth $W_0$ at time 0; following this, the individual may move from job to job as time passes. Finally, at some point in time $t$ the individual dies; the present value (at time zero) of the initial wealth plus discounted lifetime wage income up to time $t$ is denoted $W^0(t)$. Viewed from time zero, the time of death is a random variable (except when we consider the special case where there is just one period). Let $V_w$ denote the value of statistical life as computed using willingness to pay out of initial wealth, whilst $V_{CWD}$ denotes the value of statistical life as computed using the gradient of the wage risk frontier. That is, $V_{CWD} = w'(p)$.

Before considering models in which the time of death is a random variable, it is useful to review the simplest one period case. The scenario is as follows:

The individual chooses a job; immediately following this the wage is paid and then the uncertainty resolved. The individual either lives to the end of the period (time 1, with probability 1-$p$), or dies at the beginning (time 0, with probability $p$).
Thus, whether the individual dies at 0 or 1, wealth is \( W_0 + w(p) \). Expected utility for such an individual is

\[
EU = pU(W_0 + w(p),0) + (1 - p)U(W_0 + w(p),1).
\]  

(1)

It is assumed that the individual optimizes job selection at time 0. The first order necessary condition is that

\[
\frac{\partial EU}{\partial p} = U(W_0,0) - U(W_0 + w(p),1)
+ \left( pU_i(W_0 + w(p),0) + (1 - p)U_i(W_0 + w(p),1) \right) w'(p) = 0.
\]  

(2)

The individual's choice of \( p \) thus satisfies

\[
V_{cWD} = w'(p) = \frac{U(W_0 + w(p),1) - U(W_0,0)}{(1 - p)U_i(W_0 + w(p),1) + pU_i(W_0 + w(p),0)}.
\]  

(3)

With \( p \) representing the probability of death, the right hand side represents the ratio of the difference in utility arising from ‘being alive rather than being dead’ to the expected marginal utility of the compensation. This is a fairly standard result in the literature (see e.g. Viscusi [1986]). Now consider WTP out of initial wealth to secure a risk reduction: this is computed by asking what variation \( dW_0 \) will just compensate for a change in risk \( dp \) (such that expected utility remains constant). It is assumed that the individual is not allowed to re-optimize job-risk. It can be shown that allowing individuals to simultaneously re-optimize job risk makes no difference to the estimate of \( V_w \) (the first order conditions for job selection imply the additional terms sum to zero and have no effect - this is explicitly demonstrated in a more general multi-period case - see footnote 5 below). Thus
\[
d EU = \left[ (p + dp)U(W_0 + dW_0 + w(p),0) + (1 - p - dp)U(W_0 + w(p) + dW_0,1) \right] \\
- \left[ pU(W_0 + w(p),0) + (1 - p)U(W_0 + w(p),1) \right] \\
= \left[ U(W_0 + w(p),0) - U(W_0 + w(p),1) \right] dp \\
+ \left[ pU_1(W_0 + w(p),0) + (1 - p)U_1(W_0 + w(p),1) \right] dW_0 = 0 \\
\]

Rearranging, this implies that

\[
V_w = \frac{dW_0}{dp} = \frac{U(W_0 + w(p),1) - U(W_0,0)}{pU_1(W_0,0) + (1 - p)U_1(W_0 + w(p),1)}. \\
\]

Hence the expected result, that \( V_w = V_{CWD} \), obtains.

The above analysis makes clear there is no real problem with the CWD measure of the VOSL for the one period case. The following two sections explore whether this observation holds up in the case where the individual lives an uncertain life; section III deals with the polar ‘Job for life’ case where transactions costs preclude intermediate job re-optimization, whilst IV deals with the other polar case, namely that of zero transactions costs.

III. THE JOB-FOR-LIFE CASE

If job-switching transaction costs are large enough, the individual will choose a job at time zero and remain in that job for his or her remaining life. This section examines a simple model for this case. It is shown that, at the time of job selection, there is a potentially significant discrepancy between the value of life measured via willingness to pay and that via the compensating wage differential. This arises simply because, with time and wealth dependent utility, risk aversion means that individuals do not value a certain sum (an addition to wealth 'up front') as the same as the equivalent actuarially
fair annuity (the increment in the wage they receive over an uncertain future period of time). It follows that this discrepancy is resolved if actuarially fair insurance is available.

Unfortunately, this consistency generally disappears at points in time different from that of job selection; the CWD measure is only correct, in general, at the time of job selection. The willingness to pay measure for an individual at any other time will typically diverge, and possibly significantly, from the time zero CWD measure. The CWD estimate becomes 'out of date' but the individual is locked in to it due to the presence of the transactions costs associated with re-optimizing job risk.

In this section, the WTP question posed is that of WTP for a 'constant per period' change in conditional risk which is itself a constant over time. Furthermore, for simplicity, it is assumed that individuals are potentially infinitely lived and they work until they die (the analysis carries through to the finite horizon case, with some additional notational clutter). Of course, in practice, the underlying (non-job related) conditional risk faced by an individual will be time varying; section IV, *inter alia*, deals with these points (by allowing for time varying non-job related risk and also a finite potential maximum life span).

Conditional non-wage death risk, denoted $\psi$, is assumed constant over time. The additional conditional job risk selected is denoted $p$. This is also assumed constant over time (workers do not move from job to job as they age). Unconditional death risk thus has density function $(\psi + p)e^{-(\psi + p)t}$ whilst the conditional death risk at time $t$ is simply $(\psi + p)$. Job risk defines the wage paid, via the wage function $w(p)$ as in section II.
The present value of wealth for an individual who survives until time $t$ is denoted $W^0(t)$, this is defined as

$$W^0(t) = W_0 + \int_{t=0}^{\tau=t} w(p)e^{-r\tau}d\tau = W_0 + \frac{w(p)}{r}(1-e^{-rt}). \quad (6)$$

Expected utility for the individual is

$$EU = \int_{0}^{\infty} U(W^0(t),\psi)(\psi + p)e^{-(\psi+p)t}dt. \quad (7)$$

In what follows, two cases will be examined:

**Case A: No actuarially fair insurance available.**

Here, the only way of affecting wealth, conditional on living to $t$, is through the selection of the job risk $p$ at time $0$. It is shown that there is no discrepancy between the CWD and WTP measures if utility is additively time separable and the individual is risk neutral (but not in general otherwise).

**Case B: Actuarially fair insurance is available.**

Here, it is shown that there is no discrepancy between the WTP and CWD measures at the time of job selection, but that a discrepancy emerges as time passes.

**Case A: No Insurance**

The optimal choice of $p$ requires the first order necessary condition $\frac{\partial EU}{\partial p} = 0$ to be satisfied: that is

$$\frac{\partial EU}{\partial p} = \int_{0}^{\infty} \left[U(W^0(t),\psi)[1-t(\psi + p)]e^{-(\psi+p)t}\right]dt + \int_{0}^{\infty} \left[U_i(W^0(t),\psi)(\psi + p)e^{-(\psi+p)t}(w^1\tau + \psi)(1-e^{-rt})\right]dt = 0 \quad (8)$$

According to the WTP approach, an individual is asked for willingness to pay out of initial wealth $W_0$ for a given reduction in conditional risk $\psi$. Consider then the change in wealth, $dW_0$, which an individual would be willing to pay in order to secure a risk
reduction of amount $d\psi$. The variations just compensate, thus $dW_0$, $d\psi$ satisfy the condition

$$dEU = \frac{\partial EU}{\partial \psi} d\psi + \frac{\partial EU}{\partial W_0} dW_0 = 0.$$ \hspace{1cm} (9)

Note that, again, job risk re-optimization is not allowed (it makes no difference, as previously remarked, whether the individual is allowed to simultaneously re-optimize job risk as the additional terms cancel by virtue of the first order condition (8)).

From (7) this implies

$$dEU = \left( \int_0^\infty \left[ U_i(W^0(t),t)(\psi + p)e^{-\psi^+}\psi^+\right] \right) dt dW_0$$

$$+ \left( \int_0^\infty \left[ U(W^0(t),t)[1-t(\psi + p)]e^{-\psi^+}\psi^+\right] \right) dt d\psi = 0$$ \hspace{1cm} (10)

However, using (8), this can be written as

$$\frac{dW_0}{d\psi} = w'(p) \frac{w'(p)}{r} \left( \frac{\int_0^\infty \left[ (1-e^{-\psi^+})U_i(W^0(t),t)(\psi + p)e^{-\psi^+}\psi^+\right] \right) dt \right)$$ \hspace{1cm} (11)

A risk reduction of amount $d\psi$ holds over the life of the individual whilst the payment $dW_0$ is 'up front'. However, the value $dW_0$ can be converted to an actuarially fair annuity which is paid over the individual's life. Let this annuity payment rate be denoted as $da$. It applies for the life of the individual, as does the increase in risk $d\psi$, hence the VOSL may be identified as $V_w = da / d\psi$. Now, if the individual lives until time $t$, the annuity will have accumulated to $da(e^\psi - 1)/r$ or, as a present value, $da(1-e^{-\psi^+})/r$. To be actuarially fair, the annuity $da$ is thus determined by the following equation;

$$dW_0 = \int_{t=0}^\infty \left( da / r \right) \left( 1-e^{-\psi^+} \right) (\psi + p)e^{-\psi^+}\psi^+ dt = da \int (\psi + p + r)$$ \hspace{1cm} (12)
so

\[ da = (\psi + p + r)dW_o \]  \hspace{1cm} (13)

The value of life according to WTP is therefore given as

\[ V_w = \frac{da}{d\psi} = (\psi + p + r)w'(p) \left( \frac{1}{r} \int_0^\infty \left\{ e^{-\eta U_i(W^0(t),t)(\psi + p)e^{-\psi p t}} \right\} dt \right) \]

Thus \( V_w \) is not in general equal to \( w'(p) \). Whether \( V_w > w'(p) \) or \( V_w < w'(p) \) rather depends on the behaviour of \( U_i(W^0(t),t) \) over time. For example, in the special case where \( U_i(W,t) = k \), then it is clear that (14) simplifies to give

\[ V_w = \frac{da}{d\psi} = w'(p). \]

Thus, if the individual has additively separable utility (\( U_{i2}(W,t) = 0 \) for all \( W,t>0 \)) and is risk neutral with respect to wealth (\( U_{i1}(W,t) = 0 \) for all \( W,t>0 \)), the discrepancy between WTP and CWD measures of the VOSL disappears.

Unfortunately, in the general case it is unlikely that \( U_i(W,t) \) will be constant independent of \( t \). Clearly, from (6), \( W^0(t) \) is increasing over time, so with risk aversion, such that \( U_{11}(W^0(t),t) < 0 \), this gives some tendency for \( U_i(W^0(t),t) \) to fall.

It also seems reasonable to assume that \( U_{12}(W^0(t),t) > 0 \) (that is, the longer the life, the greater the marginal utility gained from a given level of wealth). This factor operates to increase \( U_i(W^0(t),t) \) as time passes. Thus the variation of \( U_i(W^0(t),t) \) over time is ambiguous and hence so too the issue of whether \( V_w > w'(p) \) or \( V_w < w'(p) \).
**Case B: Actuarily Fair Insurance available**

Given the above observations, the existence of actuarily fair insurance can obviously be expected to eliminate any discrepancy between the alternative measures if individuals are risk averse but have additively separable utility. In fact, it does so for the general case of non-separable utility. To see this, suppose that actuarily fair insurance is available which allows the individual to contract at time 0 to obtain present value wealth level \( W^a(t) \) if the individual lives until time \( t \). For the program \( \{W^a(t)\} \) to be actuarily fair, it must be the case that

\[
\int_{t=0}^{\infty} [W^a(t) - W^0(t)](\psi + p)e^{-(\psi + r)t}dt = 0.
\]

(16)

The individual chooses \( \{W^a(t)\} \) and \( p \) so as to maximize

\[
EU = \int_{0}^{\infty} U(W^a(t), t)(\psi + p)e^{-(\psi + r)t}dt
\]

subject to (6) and (16). Forming the Hamiltonian

\[
H(W^a(t), p, t) = \{U(W^a(t), t) + \lambda(W^a(t) - W^0(t))\}(\psi + p)e^{-(\psi + r)t}.
\]

(18)

the necessary conditions are that

\[
\frac{\partial H}{\partial W^a} = 0 \quad \text{for all } t \geq 0
\]

(19)

and

\[
\int_{0}^{\infty} \left[ \frac{\partial H(W^a(t), p, t)}{\partial p} \right] dt = 0.
\]

(20)

From (19),

\[
\left\{U_1(W^a, t) + \lambda\right\}(\psi + p)e^{-(\psi + r)t} = 0
\]

(21)

so

\[
U_1(W^a(t), t) + \lambda = 0.
\]

(22)

In (20), since from (6), \( \partial W^0 / \partial p = w'(p)\left[1 - e^{-rt}\right]/r \), it follows that
\[
\frac{\partial H}{\partial \psi} = \left[ U(W^a, t) + \lambda(W^a - W^0) \right] \left[ 1 - t(\psi + p) \right] e^{-(\psi + p)t} + (\psi + p)e^{-\psi t} \left[ -\lambda w'(p)(1 - e^{-\alpha}) / r \right]
\]  \tag{23}

so using (16), equation (20) becomes

\[
\int_{t=0}^{\infty} \left( \frac{\partial H}{\partial \psi} \right) dt = \int_{t=0}^{\infty} \left\{ \left[ U(W^a, t) + \lambda(W^a - W^0) \right] \left[ 1 - t(\psi + p) \right] e^{-(\psi + p)t} + (\psi + p)e^{-\psi t} \left[ -\lambda w'(p)(1 - e^{-\alpha}) / r \right] \right\} dt
\]

\[
= \int_{t=0}^{\infty} U(W^a, t) \left[ 1 - t(\psi + p) \right] - \lambda(\psi + p) \left[ t(W^a - W^0) + w'(p)(1 - e^{-\alpha}) / r \right] e^{-(\psi + p)t} dt = 0
\]  \tag{24}

(Note that \( W^a, W^0 \) are time varying functions whilst \( W_0, w'(p) \) are constants). As before, we consider WTP out of initial wealth \( W_0 \). The value of life on this measure, \( V_{W} \), is given using (13) as

\[
V_{W} = \frac{da}{d\psi} = (\psi + p + r) \frac{dW_0}{d\psi}.
\]  \tag{25}

Now, consider variations in wealth, \( dW_0 \), and risk, \( d\psi \), allowing for re-adjustment of insurance, but holding \( p \) constant. Then \( dW_0 \), \( d\psi \), \( dW^a \) must satisfy (9), so

\[
dEU = \int_{0}^{\infty} \left\{ (\psi + p)U_i(W^a, t) + U(W^a, t) \left[ 1 - t(\psi + p) \right] \right\} e^{-(\psi + p)t} dt = 0 \]  \tag{26}

where from (16), noting that \( dW^0(t) = dW_0 \) from (6),

\[
\int_{0}^{\infty} \left\{ (dW^a - dW_0)(\psi + p) + (W^a - W^0) \left[ 1 - t(\psi + p) \right] \right\} e^{-(\psi + p)t} dt = 0 \]  \tag{27}

Given from (22) that \( U_i(W^a, t) = -\lambda \), equation (27) can be used to replace the term

\[
dW^a(\psi + p)e^{-\psi t} \]  \tag{28}

in (26) whilst (24) can be used to substitute for the term

\[
U(W^a, t) \left[ 1 - t(\psi + p) \right] e^{-(\psi + p)t} \]  \tag{29}

Equation (26) then becomes
Using (16), performing some integrations similar to that in (12), and further simplifying, (28) collapses to yield the result

$$\frac{dW_0}{d\psi} = \frac{w'(p)}{\psi + p + r}. \quad (29)$$

So from (25), this implies

$$V_w = \frac{da}{d\psi} = (\psi + p + r) \frac{dW_0}{d\psi} = w'(p). \quad (30)$$

Thus there is no discrepancy when actuarially fair insurance is available; $V_w = V_{cwd}$.

To sum up, when individuals are locked into choosing a single job 'for life', if fair insurance is available, the gradient of the observed wage risk frontier provides a satisfactory measure of the individual's value of statistical life at the point in time when job selection occurs.

Unfortunately, the above equivalence result holds only at $t=0$ (since the marginal conditions which determine $p$ hold only at $t=0$). Clearly, for any time $\tau > 0$, $V_{cwd}$ will no longer coincide with $V_w$ and will in general give an incorrect measure of such an individual’s VOSL. The point is that for a job with given job risk $p$, the VOSL measured by $w'(p)$ applies only to new recruits into the industry. It follows that using $w'(p)$ as a measure of the VOSL for projects which alter job risk is theoretically incorrect. The same observation can be expected to hold true in the intermediate case where the individual changes jobs discretely a finite number of times; at points in time
other than the times of job-risk selection, there will be some degree of bias in the estimates of VOSL afforded by the CWD approach.

IV. FRICTIONLESS LABOR MARKETS

The lower the transaction costs associated with searching for and changing jobs, the more frequently individuals can be expected to shift between jobs (and the shorter the period spent in any job of given job risk). Whilst section III examined the polar case where transactions costs were sufficiently high to preclude any subsequent job-risk re-optimization, this section examines the other polar case, where the transactions costs incurred in changing job (and hence job risk) are assumed to be zero. Individuals are thus assumed to be well informed about job risks, rational in their occupational choices, and, given zero transactions costs, are able to benefit from continuous adjustment in the level of job risk they face as time passes.

The aim is to explore the nature of the equilibrium wage frontier, the characteristics of individual job-risk behavior and the relationship between WTP and CWD measures of VOSL when individuals endogenously adjust job risk over time and live uncertain lives. For relative simplicity therefore, a partial equilibrium view of the demand and supply of hours of work in jobs of different risk is taken. On the demand side, industries are viewed as having a derived demand for hours of work at each level of risk (over a specified range) such that the demand at a given risk level is a strictly decreasing function of the wage at that level. On the supply side, individuals are taken to be identical (having the same initial wealth and preference functions); such individuals age, move between jobs and die (the time of death being a random variable). We imagine a steady state equilibrium in which the total population of workers is a constant as indeed
is the age distribution of these workers, given that entry and exit from the workforce is in balance. The model is a significant extension to that discussed in section III in that here the individual’s level of conditional base risk is allowed to vary arbitrarily, the conditional job risk may be continuously optimized over the individual's maximum potential life span (which is both finite and endogenous) and the wage risk frontier is also endogenously determined.

One of those unfortunate facts of life (or death) is that individuals can die at any time, even if working in jobs which are 'perfectly safe' (jobs which add no additional conditional risk). This is referred to here as 'base' risk; if the individual never takes a risky job at any time, the probability of dying on the time interval \([0, t], t > 0\) is

\[
\int_0^t \psi(\tau) d\tau
\]

where \(\psi(\tau) > 0\) denotes the unconditional density function associated with dying at time \(\tau\). The function \(\psi(t)\) is assumed continuous, with support \([0, T^m]\) where \(T^m\) denotes the maximum conceivable life span of an individual. Thus

\[
\int_0^{T^m} \psi(\tau) d\tau = 1.
\]

Denote the unconditional job-associated death risk density function as \(p(t)\). Then the conditional death risk density, \(p_c(t)\) is defined as

\[
p_c(t) = p(t) / \left[1 - \int_{\tau=0}^{t} \left[\psi(\tau) + p(\tau)\right] d\tau\right]
\]

Thus \(p_c(t)\) is the 'instantaneous' risk faced, conditional on having survived up to time \(t\). If an individual takes a job with conditional risk \(p_c\) and holds the job for a short interval \(dt\), then he/she incurs a probability of death on this time interval of \(p_c dt\).
A continuum of jobs is assumed to be available on a specified interval for conditional job risk, $[0, p_c^m]$, where $p_c^m$ denotes the risk associated with the most risky job available. It is assumed that workers are identical in all respects; that is, they have the same initial wealth $W_0$ and the same VNM utility function $U(W, t)$ where $W$ denotes present value wealth and $t$ denotes time of death. It is assumed that a steady state equilibrium arises in which workers who die are replaced by 'new-born' workers in the population. By assumption, this equilibrium gives rise to a wage risk function $w(p_c)$ defined on $[0, p_c^m]$: $p_c$ here denotes the conditional risk associated with a job and $w(p_c)$, the wage associated with it. The function $w(p_c)$ is assumed to be a twice continuously differentiable, strictly increasing function on $[0, p_c^m]$.

An individual is viewed as selecting a lifetime 'career profile', that is, a job risk profile \( \{ p(t); 0 \leq t \leq T \} \), or equivalently, \( \{ p_c(t); 0 \leq t \leq T \} \), where $T$ denotes the maximum life span implied by such a program. Clearly $T$ is endogenously determined, since

$$\int_0^T [\mu(\tau) + \psi(\tau)] d\tau = 1. \quad (34)$$

Given $w(p_c)$ and the choice of \( \{ p(t); 0 \leq t \leq T \} \), an individual who dies at time $t$ has accumulated (present value of) wealth $W^0(t)$ according to the equation

$$W^0(t) = W_0 + \int_{\tau=t}^{\tau=T} w(p_c(\tau))e^{-\tau} d\tau. \quad (35)$$

As before, $W_0$ denotes the individual's initial inheritance of wealth whilst $W^0(t)$ is the present value of wealth accumulated up to the time of death, $t$. 
Given the existence of actuarially fair insurance, the individual can re-allocate present value wealth such that the individual 'enjoys' present value wealth \( W^o(t) \) in the event of dying at time \( t \). The choice of \( W^o(t) \) to be actuarially fair, must satisfy
\[
\int_0^T \left[ W^o(t) - W^o(t) \psi(t) + p(t) \right] dt = 0. \tag{36}
\]
The individual's objective function is
\[
EU = \int_0^T U(W^o(t), t) \left[ \psi(t) + p(t) \right] dt . \tag{37}
\]
To set this up as a control problem, first define the variable
\[
z(t) = \int_0^t \left[ \psi(\tau) + p(\tau) \right] d\tau , \tag{38}
\]
so that the constraint (34) becomes the transversality condition
\[
z(T) = 1 \tag{39}
\]
and, from (33),
\[
p_c(t) = p(t) / \left[ 1 - z(t) \right] . \tag{40}
\]
Given the range of jobs available, \( p_c \) must also satisfy
\[
p_c(t) \geq 0 , \tag{41}
\]
\[
p_c^m - p_c(t) \geq 0 . \tag{42}
\]
The problem of maximizing (37) subject to (36), (38)-(42), can thus be seen as a problem involving the two state variables, \( z(t) \) and \( W^o(t) \) and two controls, namely \( p(t) \) and \( W^o(t) \). The state variable transition equations are, differentiating (38) and (35) respectively;
\[
z(t) = \psi(t) + p(t) \tag{43}
\]
\[
\dot{W}^o(t) = w(p_c(t)) e^{-\alpha t} \tag{44}
\]
and the Hamiltonian is
\[
H = U(W^a(t),t)[\psi(t) + p(t)] + \lambda_1(t)[\psi(t) + p(t)] \\
+ \lambda_2(t)[W(p_c(t))e^{-\alpha} + \Theta(W^a(t) - W^0(t))[\psi(t) + p(t)] \\
+ \varphi_1(t)p_c(t) + \varphi_2(t)[p_m - p_c(t)]
\] (45)

The transversality conditions being that \(z(0) = 0\), \(z(T) = 1\), \(W^0(0) = W_0\), and \(W^0(T)\), \(T\) both free.

The necessary conditions are (dropping time subscripts to avoid notational clutter);
\[
\dot{\lambda}_1 = -\partial H / \partial z = -\lambda_2 w'(p_c) p_c e^{-\alpha} / (1 - z),
\] (46)
\[
\dot{\lambda}_2 = -\partial H / \partial W^0 = \Theta(\psi + p),
\] (47)
\[
\partial H / \partial W^a = 0 \implies [U_1(W^a,t) + \Theta][\psi + p] = 0,
\] (48)

which, given \(\psi(T) + p(T) > 0\), implies
\[
U_1(W^a,t) = -\Theta.
\] (49)
\[
\partial H / \partial p = U(W^a,t) + \Theta(W^a - W^0) + \lambda_1 + \lambda_2 w'(p_c) e^{-\alpha} + \frac{\varphi_1 - \varphi_2}{1 - z} = 0,
\] (50)
\[
\partial^2 H / \partial p^2 = \lambda_2 w''(p_c) e^{-\alpha} / (1 - z)^2 \leq 0 \text{ if solution for } p \text{ is interior.}
\] (51)
\[
\varphi_1(t) \geq 0; \varphi_1(t)p(t) = 0.
\] (52)
\[
\varphi_2(t) \geq 0; \varphi_2(t)[p_m - p_c(t)] = 0,
\] (53)
\[
\Theta, \text{ constant.}
\] (54)

Transversality conditions:
\[
H(T) = 0,
\] (55)
\[
\lambda_2(T) = 0.
\] (56)
**Analysis of Necessary conditions:**

First consider the endpoint condition (55). Using (56), (52), (53), and noting that

\[ \psi(T) + p(T) > 0, \]

equation (55) implies that

\[ U(W^a(T), T) + \theta(W^a(T) - W^0(T)) + \lambda_1(T) = 0. \]  

(57)

Integrating (47) gives

\[ \int_{t_0}^{T} \dot{\lambda}_2(t) \, dt = \lambda_2(T) - \lambda_2(t) = \int_{t_0}^{T} \theta(\psi(t) + p(t)) \, dt. \]  

(58)

Since \( \lambda_2(T) = 0 \), and given (38), this implies that

\[ \lambda_2(t) = -\theta(1 - z(t)). \]  

(59)

Note, from (49) that \( \theta < 0 \). Thus \( \lambda_2(t) > 0 \) for \( 0 < t < T \).

Using (40) and (59), from (46),

\[ \dot{\lambda}_1 = \theta w'(p_c) p_c e^{-\beta}. \]  

(60)

From (50), using (59),

\[ U(W^a, t) + \theta(W^a - W^0) + \lambda_1 - \theta w'(p_c) e^{-\beta} + \frac{\varphi_1 - \varphi_2}{1 - z} = 0 \]  

(61)

As \( t \to T \), (57) holds, so

\[ \lim_{t \to T} \left[ -\theta w'(p_c(t)) e^{-\beta} + \frac{\varphi_1(t) - \varphi_2(t)}{1 - z(t)} \right] = 0. \]  

(62)

If there is an interior solution, (51) holds and, from (52), (53), \( \varphi_1(t) = \varphi_2(t) = 0 \). Thus if the solution is interior at \( T \), it must be the case that

\[ w'(p_c(T)) = 0 \]  

(63)

In this case, the CWD measure of the value of life is equal to zero at the 'end of road'.

Alternatively, if \( w'(p_c(T)) > 0 \), then clearly it must be that \( \varphi_2(T) > 0 \) and

\[ p_c(T) = p_c^m. \]  

(64)
That is, the individual, lucky enough to live to $T$ terminates whilst working in the most risky job available. In fact, both (63) and (64) hold in the equilibrium solution (see below).

**Characteristics of Labor Market Equilibrium**

In a steady state equilibrium, as workers die, they are replaced by 'newborn' workers entering the labor market; the rate of inflow of new workers into the labor markets is thus a constant over time and, given that these individuals will choose an identical job-risk profile, there is a fixed distribution for workers by age. Given this, we can examine demand, supply and equilibrium at the level of the representative worker.

A simple partial equilibrium specification of the demand for worker-hours is given as follows: Let

$$ t^d = \int_{0}^{\rho} g(\rho, w) d\rho, \quad (65) $$

where $t^d$ denotes the aggregate demand for hours by industries in the job-risk interval $[0, \rho_c)$. That is, 'industries' in the job-risk interval $(\rho_c, \rho_c + \delta\rho_c)$ have a demand for hours

$$ \delta t^d = g(\rho_c, w) \delta\rho_c \quad (66) $$

if a wage rate $w$ holds on this job interval (the wage rate will generally vary with $\rho_c$).

The function $g(\rho_c, w)$ is assumed to be continuously differentiable on $[0, \rho^m]$ with $g(\rho_c, w) > 0$, $g_2(\rho_c, w) < 0$ for all $\rho_c \in [0, \rho^m]$ and $w \geq 0$, although with $g(\rho_c, w) \to 0$ as $w \to +\infty$. Thus an increase in wage on any given risk interval diminishes the demand for hours on that interval, but at any finite $w$ there is still some positive demand. This
ensures that jobs exist for all $p_c \in [0, p_c^w]$; jobs are never completely priced out of the market. Since $g(p_c, w) > 0$, it follows that $t^d$ is also a strictly increasing function of $p_c$.

Turning to the supply of hours by a worker on the risk interval $[0, p_c)$, this clearly depends on two things, namely the individual’s choice of job-risk over time (the function $p_c(\tau)$ defined on the interval $0 \leq \tau \leq T$) and the individual’s time of death, $t$ ($0 \leq t \leq T$). Since the latter is a random variable, the individual’s supply of hours is also a random variable. However, if we view workers as ‘atomistic’, then the relevant supply function is that of expected hours. We denote $t'(p_c)$ as the representative individual’s supply of expected hours on the risk interval $[0, p_c)$; the nature of this function is examined in some detail below; however, before doing so, consider the nature of the equilibrium given the functions $t'(p_c)$ for supply and $t^d(p_c)$ for demand. This requires

$$t^d(p_c) = t'(p_c) \text{ almost everywhere on } [0, p_c^w].$$ (67)

That is, the hours demanded, $\delta^d$, on any job risk interval $(p_c, p_c + \delta p_c)$ must equal the hours supplied on that interval, $\delta^s$:

$$\delta^d = \int_{p_c}^{p_c + \delta p_c} g(\rho, w(\rho))d\rho = \delta^s = \int_{p_c}^{p_c + \delta p_c} dt' \text{ for all } p_c, p_c + \delta p_c \in (0, p_c^w).$$ (68)

The expected hours supply function is determined by the individual’s choice of job risk over time - the function $p'_c(t)$, where the superscript 's' is added to emphasize that the individual’s choice of job-risk, $p_c(t)$, through time determines hours supplied. This is explained in more detail below.

Figure 1 here
Consider the job risk profile illustrated in figure 1. This is shown covering the full range of values for $p_c$ on $[0, p_c^m]$. The profile depicts a jump on the risk interval $(p_2, p_3)$ and a 'flat segment' on the time interval $(t_8, t_9)$. It also features intervals on which job risk is increasing and intervals on which it is decreasing, along with satisfying both (63) and (64) (that these both hold is established below). First consider an interval such as $(p_5, p_3 + \Delta p_3)$; hours supplied on this interval would be $\delta_{10}$ if there was no risk of death. However, the probability of a worker being alive at time $t_{10}$ is $1 - \delta_{10}$. Thus the expected hours supply on this interval is $1 - \delta_{10}$. Similarly, the expected number of hours supplied on a risk interval such as $(p_1, p_1 + \Delta p_1)$ is clearly $\sum_{i=1,4,6} [1 - \delta_{1i}] \delta_{1i}$.

In what follows, given the assumption that for any given risk, the wage rate adjusts to realize equilibrium, it is established that there can be no jump-discontinuities or 'flat spots' in the graph of the function $p_i(t)$. That is, the function must be continuous almost everywhere and be composed of intervals on which the function is strictly increasing and/or intervals on which it is strictly decreasing.

Firstly, note that hours demand on any risk interval $(p_c, p_c + \Delta p_c) \in [0, p_c^m]$ is strictly positive for $\Delta p_c > 0$, and that $\delta_d \downarrow 0$ as $\Delta p_c \downarrow 0$. Now consider the jump at time $t_2$ from $p_2$ to $p_3$. Given that $H$ is continuous on $[0, T]$, clearly $H(t_2) = H(t_3^*)$ and also, by (50), $\partial H(t_2) / \partial p = \partial H(t_3^*) / \partial p = 0$. Analysis of these conditions shows that if there is a jump from some $p_2$ to some $p_3$, then the individual will never optimally select a $p \in (p_2, p_3)$ at any other point in time (a contradiction can be demonstrated using the
necessary conditions). It then follows that the individual's job risk function would feature a zero supply of hours on such a risk interval (as depicted in figure 1). However, by assumption, there is positive demand for hours on this interval at any finite wage rate and hence demand exceeds supply on such an interval. Thus, the existence of equilibrium rules out such jumps in $p_t^*(t)$ (wage rates would increase on such intervals to call forth positive supply).

Now consider the 'flat spot' on the interval $(t_k, t_{k+1})$ associated with risk level $p_k$ and examine the risk interval $(p_k - \delta p, p_k + \delta p)$ as $\delta p \downarrow 0$. Hours supply on this risk interval converges to $\int_{t_k}^{t_{k+1}} [1 - z(t)] dt$, which, given $z(t) < 1$ on this interval, is a strictly positive number. By contrast, hours demand, $\delta d \downarrow 0$ as $\delta p \to 0$. Thus for some value of $\delta p > 0$, demand is not equal to supply on the interval, which clearly contradicts the assumption of market equilibrium. Thus, the existence of equilibrium allows turning points but precludes 'flat spots', there can be no non-degenerate intervals on which $p_t^*(t) = 0$. This in turn also rules out finite intervals on which $p_t^*(t) = 0$ or $p_t^*(t) = p_c^m$.

Again, if the constraints (41), (42) bind, it can only be instantaneously. Thus $p_t^*(t)$ is interior almost everywhere on $[0, T]$ and, this in turn implies, from (52), (53), that $\phi_1(t) = \phi_2(t) = 0$ almost everywhere. Notice also that, since $p_t^*(t)$ is continuous, it follows that $\phi_1(t), \phi_2(t)$ are also continuous, so in fact $\phi_1(t) = \phi_2(t) = 0$ everywhere on $[0, T]$. Furthermore, since equilibrium holds for all $p_c \in (0, p_c^m)$, the second order condition (51) holds for all $p_c \in (0, p_c^m)$. This in turn implies that

$$w''(p) \leq 0 \quad \text{for all } p_c \in (0, p_c^m)$$  \hspace{1cm} (69)
It thus follows that, in equilibrium, \( w(p) \) is concave on \((0, p^\infty_c)\). Finally, note that since 
\[
\varphi_1(t) = \varphi_2(t) = 0 \quad \text{everywhere on } [0, T],
\]
this implies that from (62),
\[
w'(p_c(T)) = 0
\] (70)
also holds. Furthermore, since \( w(p_c) \) is a strictly increasing and concave function, it also follows that
\[
p_c'(T) = p^\infty_c.
\] (71)
(since equilibrium requires positive hours supply everywhere on \((0, p^\infty_c)\)). Thus, from (71), individuals take to the riskiest jobs at the end of their maximum potential life span and, from (70), the CWD measure of the value of life goes to zero at this point.

Furthermore, notice that at high levels of risk, the prediction is that relatively large increases of risk do not require much compensation in the way of increased wages, whilst at low levels of risk, the value of statistical life is high and small increases in risk require large increases in wage to compensate (as illustrated in figure 2).

**Figure 2 here**

Differentiating (50) (and noting that \( \varphi_1(t) = \varphi_2(t) = 0 \) on \([0, T]\)) gives
\[
\left[ U_1(W^a, t) + \theta \right] \dot{W}^a + U_2(W^a, t) - \theta \dot{W}^\theta \\
+ \dot{\lambda}_1 - \theta \left[ -rw'(p_c)e^{-\eta} + w''(p_c)e^{-\eta}\dot{p}_c \right] = 0
\] (72)
This simplifies, in view of (44), (46), and (49), to yield
\[
\dot{p}_c = \left( \frac{1}{w'(p_c)} \right) \left( U_2(W^a, t) \right) \left( \frac{\theta}{\theta} e^{-\eta} - w(p_c) + w'(p_c)[p_c + r] \right)
\] (73)
if \( w''(p_c) < 0 \) or
\[
U_2(W^a, t)e^{-\eta} + \theta \left[ -w(p_c) + w'(p_c)[p_c + r] \right] = 0
\] (74)
on intervals for which \( w''(p_c) = 0 \). Suppose then that (74) holds on an interval \((t_a, t_b)\); differentiating (74) and noting that \( w'(p_c) = 0 \) on this interval, it follows that

\[
\frac{d}{dt} \left[U_2(W^a, t)e^\alpha\right] + \theta[-w'(p_c) + w'(p_c) + w''(p_c)(p_c + r)\dot{p}_c] = \frac{d}{dt} \left[U_2(W^a, t)e^\alpha\right] = \left[U_{12}(W^a, t)W^a + U_{22}(W^a, t)\right] + rU_2(W^a, t)e^\alpha = 0 \tag{75}
\]
on the interval \((t_a, t_b)\). Now, if \( U_2(W^a, t)e^\alpha \) is constant over time, then, from (75), this implies that \( p_c \) is constant on this interval. However, as already established, in equilibrium there are no non-degenerate intervals on which \( p_c \) is constant. Hence it must be that (73) holds on \([0, T]\) and that \( w(p_c) \) is strictly concave, with \( w''(p_c) < 0 \) on \((0, p_c^w)\).

Notice that, for \( t \) sufficiently close to \( T \), job risk \( p_c \) is definitely increasing with age; since \( w'(p_c(T)) = 0 \) and \( \theta < 0 \) in (91), clearly \( \dot{p}_c > 0 \) as \( t \uparrow T \). Indeed, the riskiest jobs are selected as \( t \uparrow T \). However, (73) is non-autonomous, and it is possible to have turning points and intervals on which risk is decreasing as well as intervals on which it is increasing.

A special case of some interest is that where \( U_2(W^a, t)e^\alpha \) is constant over time; for example, this would be true if the utility function was additively separable (so \( U_{12} = 0 \)) and if \( U_2(W^a, t) = ke^{-\alpha t} \) on the interval. The solution would then be one in which job risk is strictly increasing over time. To see this, note that in this special case, (73) can be written as
\[ \dot{p}_c = \left( \frac{1}{w''(p_c)} \right) f(p_c) \]  

(76)

where \( w''(p_c) < 0 \) and

\[ f(p_c) = (k / \theta) - w(p_c) + w'(p_c)[p_c + \theta]. \]  

(77)

Now, \( k / \theta < 0 \) and at \( T, w'(p_c(T)) = 0 \), so \( f(p_c(T)) < 0 \) and hence \( \dot{p}_c(T) > 0 \). Given the differential equation (76) is autonomous, it must then be that \( \dot{p}_c(t) > 0 \) on \([0, T]\). In this case then, the individual continuously increases job risk as time passes.

**Interpretation of \( w'(p) \).**

On the optimum path, the Hamiltonian is maximized at each point in time \( t \). The solution is interior almost everywhere, so, a small pulse of amount \( \delta p \) on a short interval \((t, t + \delta t)\), leads to a change in conditional risk of amount \( \delta p_c \) (‘change of job’ and hence job risk) on this interval. The pulse induces a change in present value wealth through this interval of amount \( w'(p_c) \delta p_c \delta t \) and a change in the risk of death on the same interval of amount \( \delta p_c \delta t \). However, by virtue of the first order condition \((\partial H / \partial p = 0)\), there is no impact of the optimal value, \( EU^* \). It follows that \( w'(p_c) \)
bears the interpretation of the instantaneous value of statistical life. For an individual in an industry with risk level \( p_c \), the gradient of the job-risk function, \( w'(p_c) \), measures the value of a small reduction in the risk of death. This point generalizes; even if individuals are different (within the present context, this would have to be through having different inherited wealth, different utility functions, albeit with the same arguments \( W \) and \( t \)), it remains true that in frictionless labor markets, individuals found in a job with given job risk will have an 'instantaneous' VOSL equal to \( w'(p_c) \). That is,
\( w'(p_r) \) indicates the value of making a small change in the risk of an occupation of given risk.

Thus the CWD approach provides useful information about changes in occupational risk. A project might reduce risk in a given occupation over several periods - but this would not alter the validity of using the CWD measure. The point is that, when individuals are in the job, they all have the same (instantaneous) VOSL.\(^{12}\) By contrast, if the project involves a multi-period occupational risk reduction, it would be inappropriately (theoretically at least) to ask individuals in such jobs what they would be WTP for a risk reduction over several periods (since they are in a given job only 'instantaneously'). In principle, asking such individuals a 'short period' WTP question regarding risk changes would also elicit a WTP figure appropriate for the valuation of long lived projects concerned with occupational risk (ignoring here all the problems associated with getting a truthful revelation of WTP).

To sum up, the above model of the equilibrium wage-risk trade off for the case where individuals are potentially long lived and there are zero transactions costs associated with job-switching has the following characteristics:

1. Individuals systematically vary their level of job risk and in doing so, also choose their own expected life span (and maximum potential life span).

2. It is possible for the job-risk selected to feature periods of increase and decrease in the initial stages of working life, but eventually individuals start to move to jobs with increasing job-risk (ending up in the most risky jobs if they
live to the end of their maximum potential life span).

3. The value of statistical life associated with an individual worker may fluctuate upward or downward in early life, but eventually the VOSL must decline with age, reaching zero at the end of the maximum potential life span.

4. The equilibrium wage-risk function $w(p)$ is strictly concave; the statistical value of life as measured by the CWD is thus strictly decreasing with risk. The riskier the occupation, the lower the VOSL of individuals in that occupation.

5. The gradient of the equilibrium wage/risk function, $w'(p)$ provides a correct measure of the VOSL for a job with a level of conditional risk $p$. It thus provides a measure useful for assessing the value of risk reductions in such jobs.

6. Multi-period WTP gives an estimate of VOSL which is effectively a 'time weighted average' for an individual (since the individual's base risk and job risk change over time). The multi-period WTP-VOSL thus is an inappropriate measure for valuing risk reductions in jobs (although the 'single period' WTP question would be appropriate). The multi-period WTP measure may however provide information useful in other contexts; for example, for projects which affect the risk faced by an individual over a period in his or her life (for example, improvements in air quality etc.).

V. CONCLUDING COMMENTS

If there are subjective or objective transactions costs associated with individuals adjusting occupational risk over time, and in the absence of actuarially fair insurance, then the compensating wage differential (CWD) estimate of the value of statistical life (VOSL) for any given individual will generally be biased. If actuarially fair insurance is available, then the model suggests that CWD gives a correct estimate of the VOSL, but that it is correct for the individual only at the time of job selection. The transactions
costs 'lock the individual into a fixed level of job risk' for which there is a fixed CWD-VOSL; this clearly becomes out of date as time passes. If, empirically, individuals seem to lock into jobs for substantial periods, then this suggests that they may face significant subjective/objective transactions costs which effectively prevent them re-adjusting the occupational risk they face. If so, the extent of bias in the CWD-VOSL could be substantial.

By contrast, if labor markets can be viewed as approximately frictionless, it would appear that even where individuals are potentially long lived, the compensating wage differential approach (CWD) may afford useful information on the value of statistical life (VOSL). In this case, the gradient of the wage-risk function, for any given level of risk, measures the value of improving occupational risk. This is so because all individuals in a job of given risk have, in equilibrium, the same instantaneous VOSL and this holds true for all points in time. Thus even where a project leads to an occupational risk change which lasts for an extended period, from a theoretical perspective, the compensating wage differential measure of the value of statistical life is the correct measure to use in any discounted cash flow appraisal of such a risk change.

In this zero transactions cost case, the CWD measure is also consistent with a WTP question for an 'instantaneous' (or 'single period') risk reduction, but not with a WTP question relating to multi-period risk. Thus a multi-period WTP measure would be generally inappropriate as a measure of value for a project involving changing job risk over a significant period of time - this is so because such a measure necessarily conflates the individual's changing base and job risk over time and individuals do not, in this model, stay in the same jobs as they age. However, multi-period WTP does of course
become appropriate in other applications. For example, if the concern is with a project which will reduce non-job related risk to specific individuals over a specific period of time in their life, asking the multi-period WTP question is entirely appropriate.

Finally, it is worth pointing out that the model in section IV makes a variety of predictions regarding behaviour which might be worth pursuing in empirical research. Thus, for an individual, ceteris paribus, the VOSL should decrease with age, individuals should be seen to be moving to increasingly risky jobs as they age, the VOSL should be decreasing with risk and, at the risky end of the job spectrum, relatively large increases in job risk are compensated for by relatively small increments in wage rates. Of course, there are many factors other than job risk which affect an individual’s occupational choices through time which would need to be controlled for - clearly, individuals also may accumulate some forms of human capital (education, skills..) and perhaps lose others (physical strength, dexterity...) as they age.14 Many risky jobs require the latter rather than the former types of human capital, so the overall effect regarding age versus risk, for example, is likely to be ambiguous. There is some evidence from past studies that, as predicted here, the VOSL does seem to fall with job risk, although there is much debate about how robust this result is.15 Finally, the notion that discrepancies in WTP and CWD measures of VOSL may linked to labour market characteristics (with higher discrepancies where there are higher transactions costs) might be explored through cross section studies.
REFERENCES


FOOTNOTES

1 The lifetime consumption and bequest model (e.g. Yaari [1965], Shepard and Zeckhauser [1984], Ehrlich and Chuma [1990]) provides a potentially more flexible formulation of an individuals inter-temporal objective function (although in practice this usually gets specialised in various ways - for example the above models all ignore the bequest motive and use constant elasticity functional forms at some point). There are thus pros and cons to alternative formulations - the present formulation, it can be argued, captures adequately the objects of choice we are primarily concerned about (namely wealth and longevity).

2 Much of the CWD literature makes an (implicit) assumption, as here, that a given job is associated with a given risk level, independent of the individual in the job. In practice, job risk is likely to depend not only upon the nature of the job, but also on the characteristics of the individual in the job. In the context of the present model, age could be a factor; for example, younger workers might on average be more agile and have faster reactions (ceteris paribus reducing the risk faced), but may also lack skills and experience (so increasing the risk they face). Thus one might expect self-selection to lead to a form of ‘clientele’ effect, in that riskier jobs attract the less risk averse, the more agile etc.

3 This one period case is discussed in Dobbs [1999].

4 To spell this out, the probability of being alive at time $t$ is

$$1 - \int_0^\tau (\psi + p)e^{-(\psi + p)^r} d\tau = e^{-(\psi + p)^r},$$

so the conditional density is the unconditional density divided by this; $$(\psi + p)e^{-(\psi + p)^r} / e^{-(\psi + p)^r} = (\psi + p).$$

The conditional probability of living until $t$ and then dying on the interval $(t, t + \delta t)$, for small $\delta t$, is approximately $(\psi + p)\delta t$.

5 In the case where job risk can be optimally adjusted, the changes in $dW_0$, $d\psi$ induce a change in $p$ of amount $dp$. Equation (29) then becomes
\[ dEU = \frac{\partial EU}{\partial \psi} d\psi + \frac{\partial EU}{\partial W_0} dW_0 + \left[ \frac{\partial EU}{\partial p} \left( \frac{dp}{d\psi} d\psi + \frac{dp}{dW_0} dW_0 \right) \right], \]

but in view of (25), \( \partial EU / \partial p = 0 \) so the term in brackets is zero and the outcome is identical to that in the text.

\(^6\) The remark regarding re-optimisation of job-risk \( p \) again applies.

\(^7\) The assumption that \( w(p_c) \) is continuous clearly entails that there is always some positive demand for jobs of given risk whatever the wage rate, so jobs are not completely priced out of the market. Clearly, given this and the fact that no one would willingly take a job that paid the same as one involving less risk, the equilibrium wage function must be strictly increasing.

\(^8\) Of course, the probability of surpassing an age \( T - \delta T \) goes to zero as \( \delta T \downarrow 0 \).

\(^9\) For example, let \( \tilde{t}^i(p_c) \) denote an individual’s random number of hours contributed to jobs on the risk interval \([0, p_c]\), where \( E[\tilde{t}^i(p_c)] = \mu(p_c) \) and \( \text{var}[\tilde{t}^i(p_c)] = \sigma^2(p_c) \). Now suppose there are \((k / \theta)\) such individuals, where \( k \) is a constant and \( \theta \) denotes an efficiency or ‘contribution per worker’ parameter such that the aggregate supply of efficiency hours is \( \left\{ \theta \sum_{i=1}^{k/\theta} \tilde{t}^i(p_c) \right\} \). Then clearly \( E\left\{ \theta \sum_{i=1}^{k/\theta} \tilde{t}^i(p_c) \right\} = k \mu(p_c) \) whilst \( \text{var}\left\{ \theta \sum_{i=1}^{k/\theta} \tilde{t}^i(p_c) \right\} \to 0 \) as \( \theta \to \infty \).

\(^{10}\) Suppose at some other \( t \), \( p \in (p_2, p_3) \) is selected; then it must be that \( H(p, t) \geq H(p_2, t) \) and \( H(p, t) \geq H(p_3, t) \) and that \( \partial H(p, t) / \partial p = 0 \). Further analysis of these conditions establishes a contradiction.

\(^{11}\) Which can be interpreted as the case where each extra year of life adds a constant ‘nominal’ increment to utility.

\(^{12}\) Thus the present value of a risk reduction lasting a period of time \( \tau \) would be given as \( w'(p)[1 - e^{-\tau r}] / r \).
Apart from a set of measure zero, workers die before this, with a positive VOSL at the time of death.

They also acquire other responsibilities and ties (especially children) which are likely to affect their attitude to job risk.

See Leigh [1995] for an interesting review of past work and an extended discussion of the potential problems associated with data sets widely used in US research.