1. General design of instrumentation systems and error analysis

1.1 The measuring system

Any measuring system can be broadly represented by the diagram shown in Figure 1.1 containing three main elements:
1). a transducer;
2). a signal conditioner;
3). a recorder or indicator.

![Block diagram of a measurement system](image)

Figure 1.1 The block diagram of a measurement system

The transducer element is an energy converter which receives the physical quantity being measured and converts it into some other physical variables, for example, flow to pressure and speed to voltage. The transducer is the weakest link in a measuring system because the measured quantity is always modified by the presence of the transducer, making perfect measurement theoretically impossible.

The signal conditioner rearranges the transduced signal into a form which can be readily recorded or monitored.

The recorder or indicator is a recording or display device.

1.2 System performance

1.2.1 Static performance

1). Sensitivity

Static sensitivity is defined as the ratio of the change in output to the corresponding change in input under static or steady state conditions.

\[ K = \frac{\Delta y}{\Delta u} \]  

\( \Delta u \) : the change in input  
\( \Delta y \) : the corresponding change in output

Sensitivity may have different units depending on the instrument being considered. For example, the
platinum resistance thermometer gives a change of resistance with change of temperature and therefore its sensitivity would have a unit of ohms/°C.

Figure 1.2 shows a linear relationship between output and input, and sensitivity therefore equals the slope of the calibration graph and is constant. In the case of a nonlinear relationship shown in Figure 1.3, the sensitivity will vary according to the value of the output.

Figure 1.2 Linear static sensitivity       Figure 1.3 Nonlinear static sensitivity

If elements of a measuring system having static sensitivities of \( K_1, K_2, K_3, \ldots \), etc are connected in series, then the overall system sensitivity \( K \) is given by

\[
K = K_1 \times K_2 \times K_3 \times \ldots
\]  \hspace{1cm} (1.2)

If an element has input and output in the same form, for example the voltage amplifier, then the term gain is used rather than sensitivity.

**Example 1.1** A measuring system consists of a transducer, an amplifier, and a recorder. The three elements are connected in series with individual sensitivities as follows:
- transducer sensitivity: 0.3 mV/°C
- amplifier gain: 2.5 V/mV
- recorder sensitivity: 4.0 mm/V

Find out the overall system sensitivity.

Since the three elements are connected in series, the overall system sensitivity is

\[
K = K_1 \times K_2 \times K_3 \times \ldots = 0.3 \text{ mV/°C} \times 2.5 \text{ V/mV} \times 4.0 \text{ mm/V} = 3.0 \text{ mm/°C}
\]

2). Accuracy and precision

The accuracy of a measuring system is normally stated in terms of the errors introduced

\[
\text{percentage error} = \frac{\text{indicated value} - \text{true value}}{\text{true value}} \times 100\% \tag{1.3}
\]

However, it is a common practice to express the error as a percentage of the measuring range of the equipment:

\[
\text{percentage error} = \frac{\text{indicated value} - \text{true value}}{\text{maximum scale value}} \times 100\% \tag{1.4}
\]
“Precision” is a term often confused with accuracy, but a precise measurement may not be an accurate measurement. If the measuring device is subjected to the same input for several times and the indicated results lie closely together, then the instrument is said to be of high precision.

3). Possible and probable errors

Consider a measuring system consisting of three elements in series with maximum possible errors of $\pm a\%$, $\pm b\%$, and $\pm c\%$ respectively. It is unlikely that these maximum errors occur at the same time. Therefore a more practical way of expressing the overall system error is to take the square root of the sum of squares of the individual errors, which is known as the probable error.

$$\text{probable errors} = \pm\sqrt{(a^2 + b^2 + c^2)}\% \quad (1.5)$$

**Example 1.2** For a general measuring system where the errors in the transducer, signal conditioner, and recorder are $\pm 2\%$, $\pm 3\%$, and $\pm 4\%$ respectively, calculate the maximum possible error and the probable error.

Maximum possible error $= \pm(2 + 3 + 4)\% = \pm 9\%$

Probable error $= \pm\sqrt{(2^2 + 3^2 + 4^2)}\% = \pm\sqrt{29}\% = \pm 5.4\%

Table 1.1 summaries some other static performance terms (Haslam et al., 1981).

<table>
<thead>
<tr>
<th><strong>Reproducibility</strong></th>
<th>The ability of an instrument to display the same reading for a given input applied on a number of occasions.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Repeatability</strong></td>
<td>The reproducibility when a constant input is applied repeatedly at short intervals of time under fixed condition of use.</td>
</tr>
<tr>
<td><strong>Stability</strong></td>
<td>The reproducibility when a constant input is applied over long periods of time compared with the time of taking reading, under fixed condition of use.</td>
</tr>
<tr>
<td><strong>Constancy</strong></td>
<td>The reproducibility when a constant input is presented continuously and the conditions of test are allowed to vary within specified limits, due to some external effect such as temperature variation.</td>
</tr>
<tr>
<td><strong>Range</strong></td>
<td>The total range of values with an instrument is capable of measuring.</td>
</tr>
<tr>
<td><strong>Span</strong></td>
<td>The range of input signals corresponding to the desired working range of the output signal.</td>
</tr>
<tr>
<td><strong>Tolerance</strong></td>
<td>The maximum error.</td>
</tr>
<tr>
<td><strong>Linearity</strong></td>
<td>The maximum deviation from a linear relationship between input and output, i.e. from a constant sensitivity, expressed as percentage of full scale.</td>
</tr>
<tr>
<td><strong>Resolution</strong></td>
<td>The smallest change of input to an instrument which can be detected with certainty, expressed as a percentage of full scale.</td>
</tr>
<tr>
<td><strong>Dead-band</strong></td>
<td>The largest change of input to which the instrument does not respond due to friction or backlash effects, expressed as a percentage of full scale.</td>
</tr>
<tr>
<td><strong>Hysteresis</strong></td>
<td>The maximum difference between readings for the same input when approached from opposite directions, i.e. when increasing and decreasing the input, expressed as a percentage of full scale.</td>
</tr>
</tbody>
</table>

1.2.2 Dynamic performance

The static characteristics of measuring instruments are concerned only with the steady-state reading that the instrument settles down to, such as the accuracy of the reading, etc.
The dynamic characteristics of a measuring instrument describe its behaviour between the time a measured quantity changes value and the time when the instrument output attains a steady value in response. As with static characteristics, any values for dynamic characteristics quoted in instrument data sheets only apply when the instrument is used under specified environmental conditions. Outside these calibration conditions, some variations in the dynamic parameters can be expected.

The dynamic performance of both process measuring and control systems is very important and is specified by responses to certain standard test inputs: the step input, the ramp input, and the sine-wave input.

For a general linear, time-invariant measuring system, the following relation can be written between input and output for time \( t \) greater than zero:

\[
\sum_{i=0}^{n} a_i \frac{d^i q_o}{dt^i} + \sum_{i=0}^{m} b_i \frac{d^i q_o}{dt^i} + a_0 q_o = b_0 q_i + \sum_{i=0}^{m-1} d^{i+1} q_o + \sum_{i=0}^{m-1} b_i \frac{d^i q_o}{dt^i} + b_0 q_i
\]

(1.6)

where \( q_i \) is the measured quantity, \( q_o \) is the output reading and \( a_0 \ldots a_n, b_0 \ldots b_m \) are constants.

If we limit consideration to that of step changes in the measured quantity only, then Eq(1.6) reduces to:

\[
\sum_{i=0}^{n} a_i \frac{d^i q_o}{dt^i} + \sum_{i=0}^{m} b_i \frac{d^i q_o}{dt^i} + a_0 q_o = b_0 q_i
\]

(1.7)

Further simplification can be made by making certain special cases of Eq(1.7) which collectively apply to nearly all measurement systems.

**Zero-order instrument**

If all the coefficients \( a_1 \ldots a_n \) other than \( a_0 \) in Eq(1.7) are assumed zero, then:

\[
a_0 q_o = b_0 q_i \quad \text{or} \quad q_o = b_0 q_i / a_0 = K q_i
\]

(1.8)

where \( K \) is a constant known as the instrument sensitivity as defined earlier.

Any instrument which behaves according to Eq(1.8) is said to be of zero-order type. A potentiometer, which measures motion, is a good example of such an instrument, where the output voltage changes instantaneously as the slider is displaced along the potentiometer track.

**First-order instrument**

If all the coefficients \( a_2 \ldots a_n \) except for \( a_0 \) and \( a_1 \) are assumed zero in Eq(1.7) then:

\[
a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i
\]

(1.9)

Any instrument which behaves according to Eq(1.9) is known as a first-order instrument.

Applying Laplace transform to Eq(1.9) with zero initial condition, we get:
\[ a_1 S q_o (S) + a_0 q_o (S) = b_0 q_i (S) \]

and rearranging gives:

\[ q_o (S) = \frac{(b_0 / a_0) q_i (S)}{1 + (a_1 / a_0) S} \]

Defining \( K = b_0 / a_0 \) as the static sensitivity and \( \tau = a_1 / a_0 \) as the time constant of the system, Eq(1.10) becomes:

\[ q_o (S) = \frac{K q_i (S)}{1 + \tau S} \]  \hspace{1cm} (1.11)

If Eq(1.11) is solved analytically, the output quantity \( q_0 \) in response to a step change in \( q_i \) varies with time in the manner shown in Figure 1.4. The time constant \( \tau \) of the step response is the time taken for the output quantity \( q_0 \) to reach 63% of its final value.

The thermocouple is a good example of a first-order instrument. It is well known that, if a thermocouple at room temperature is plunged into boiling water, the output emf does not rise instantaneously to a level indicating 100°C, but instead approaches a reading indicating 100°C in a manner similar to that shown in Figure 1.4.

![Figure 1.4 Response of a first-order instrument to a step change](image)

A large number of other instruments also belong to this first-order class. This is of particular importance in control systems where it is necessary to take account of the time lag that occurs between a measured quantity changing in value and the measuring instrument indicating the change. Fortunately, the time constant of many first-order instruments is small relative to the dynamics of the process being measured, and so no serious problems are created.

**Second-order instrument**

If all coefficients \( a_1 \ldots a_6 \) other than \( a_0, a_1 \) and \( a_2 \) in Eq(1.7) are assumed zero, then we get:

\[ a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \]  \hspace{1cm} (1.12)

Applying Laplace transform to Eq(1.12) with zero initial condition gives:
\[ a_2 S^2 q_o(S) + a_1 S q_o(S) + a_0 q_o(S) = b_0 q_i(S) \]

and rearranging:

\[ q_o(S) = \frac{b_0 q_i(S)}{a_0 + a_1 S + a_2 S^2} \quad (1.13) \]

It is convenient to re-express the variables \(a_0, a_1, a_2\) and \(b_0\) in Eq(1.13) in terms of three parameters \(K\) (static sensitivity), \(\omega\) (undamped natural frequency) and \(\varepsilon\) (damping ratio), where:

\[
\begin{align*}
    k &= \frac{b_0}{a_0} \\
    \omega &= \sqrt{a_0/a_2} \\
    \varepsilon &= \frac{a_1}{2\sqrt{a_0 a_2}}
\end{align*}
\]

Re-expressing Eq(1.13) in terms of \(K\), \(\omega\) and \(\varepsilon\) we get:

\[
\frac{q_o(S)}{q_i(S)} = \frac{K}{S^2 / \omega^2 + 2\varepsilon S / \omega + 1} \quad (1.14)
\]

Any instrument whose response can be described by Eq(1.14) is known as a second-order instrument.

If Eq(1.14) is solved analytically, the shape of the step response obtained depends on the value of the damping ratio parameter \(\varepsilon\). The output responses of a second-order instrument for various value of \(\varepsilon\) are shown in Figure 1.5. For the case where \(\varepsilon=0\), there is no damping and the instrument output exhibits constant amplitude oscillations when disturbed by any change in the physical quantity measured. For light damping of \(\varepsilon=0.5\), the response to a step change in input is still oscillatory but the oscillations gradually die down. A further increase in the value of \(\varepsilon\) reduces oscillations and overshoot still more, as shown by the case \(\varepsilon=1\), and finally the response becomes very over-damped as shown by the case \(\varepsilon=1.5\) where the output reading creeps up slowly towards the correct reading. Clearly, the extreme response curves for cases \(\varepsilon=0\) and \(\varepsilon=1.5\) are unsuitable for any measuring instrument. Commercial second-order instruments are generally designed to have a damping ratio (\(\varepsilon\)) somewhere in the range of 0.6-0.8.

![Figure 1.5 Response of a second-order instrument to a step change](image)
Example 1.3 The differential equation describing a mercury-in-glass thermometer is

\[ 4 \frac{dy}{dt} + 2y = 2 \times 10^{-3} u \]

where \( y \) is the height of the mercury column in meters and \( u \) is the input temperature in °C. Find out the time constant and static sensitivity of the thermometer.

Applying Laplace transform to the above equation with zero initial condition, we have

\[ 4S y(S) + 2 y(S) = 2 \times 10^{-3} u(S) \]

Re-arranging the above equation gives

\[ y(S) = \frac{2 \times 10^{-3} u(S)}{4S + 2} = \frac{10^{-3} u(S)}{2S + 1} \]

Compare the above equation with Eq(1.11),

- time constant \( \tau = 2 \) s
- static sensitivity \( K = 10^{-3} \) m/°C

2. Signal conditioning
2.1 What is signal conditioning?

The transduced signal is rarely in a form ready for display or recording and may need to be increased in magnitude or modified in some way before display. The process of preparing the signal before display or recording is referred to as signal conditioning.

A signal conditioning device may have one or all of the following functions:

1). Amplification. The weak signal from the transducer is increased in magnitude by a device called as an amplifier, such as levers, gears, and electronic, pneumatic, and hydraulic amplifiers. The amount by which the signal is increased in called the gain or amplification.

2). Signal modification. The form of the signal or amplified signal is changed by a signal modifier such as bridge circuits and analogue-to-digital converters.

3). Impedance matching. The signal conditioner acts as a buffer stage between the transducing and recording elements, the input and output impedances of the matching device being arranged to prevent loading of the transducer and maintain a high signal level at the recorder.

2.2 Amplifiers

An amplifier is a device which increases the magnitude of its input signal. Its gain, \( G \), input, \( u \), and the corresponding output, \( y \), are related as

\[ y = Gu \]

(2.1)

Since the input and output of an amplifier have the same unit, the amplifier gain, \( G \), is dimensionless.
For amplifiers in series, the overall gain is the product of the individual amplifier gains.

**Example 2.1** A displacement amplifier has an amplification of 15000. If the output displacement is 3cm, find out the corresponding input displacement.

The input displacement, \( u = \frac{y}{G} = 3 \times 10^{-2} \text{m}/15000 = 2 \times 10^{-6} \text{m} \)

### 2.3 Operational amplifier circuit

An operational amplifier is shown in Figure 2.1 and it operates upon a direct voltage or current in some mathematical way. It is widely used in instrumentation and control engineering due to its following properties:

1). High gain, 200000 to \( 10^6 \);
2). Phase reversal, i.e. the output voltage is of opposite sign to the input;
3). High input impedance.

![Figure 2.1 An operational amplifier](image1)

![Figure 2.2 An operational amplifier circuit](image2)

These properties lead to the following very important implications:

1). With high voltage gain, for any sensible output voltage the input voltage will be so small that it can be assumed virtually zero. Thus the input point B is referred to as a *virtual earth* ("virtual" because it is not directly connected to the earth).
2). Since the input impedance is high and the input point is a virtual earth, the amplifier takes negligible current which is therefore assumed to be zero for a simplified analysis.

For the operational amplifier circuit with two resistors shown in Figure 2.2, we have

\[ i_1 = i_f \quad \text{(since the input current to the amplifier is negligible)} \]

That is

\[ \frac{v_1 - 0}{R_1} = \frac{0 - v_o}{R_f} \]

Therefore

\[ \frac{v_o}{v_1} = \frac{R_f}{R_1} \]
The operational amplifier circuit shown in Figure 2.2 is also known as an inverting amplifier since the output is out of phase by 180° (π radians) with respect to the input. This is indicated by the negative sign in the above equation.

### 2.3 The summer amplifier

![Figure 2.3 A summer amplifier](image)

Figure shows a summer amplifier. Since the input current to the amplifier is zero, we have

\[
\begin{align*}
i_f &= i_1 + i_2 + i_3 \\
-\frac{v_o}{R_f} &= \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} \\
v_o &= -R_f \left( \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} \right)
\end{align*}
\]

The output voltage is a weighted sum of the three input voltages.

If \( R_1 = R_2 = R_3 = R_f \), then \( v_o = -(v_1 + v_2 + v_3) \)

**Example 2.2** If in Figure 2.3, \( R_1 = 1 \, \text{k}\Omega, R_2 = 2 \, \text{k}\Omega, R_3 = 1.5 \, \text{k}\Omega, R_f = 10 \, \text{k}\Omega, v_1 = 1.5 \, \text{V}, v_2 = 2 \, \text{V}, \) and \( v_3 = 3 \, \text{V} \), determine the output voltage \( v_o \).

\[
v_o = -10 \, \text{k}\Omega \left( \frac{1.5 \, \text{V}}{1 \, \text{k}\Omega} + \frac{2 \, \text{V}}{2 \, \text{k}\Omega} + \frac{3 \, \text{V}}{1.5 \, \text{k}\Omega} \right)
= -45 \, \text{V}
\]

**Example 2.3** You have two operational amplifiers, two resistors of 35 k\(\Omega\), and three resistors of 10 k\(\Omega\). Build an instrumentation circuit which provides an output voltage \( V_{\text{out}} \) related to the input voltage \( V_{\text{in}} \) by

\[
V_{\text{out}} = 3.5V_{\text{in}} + 5
\]
This can be achieved using a summer amplifier and an inverting amplifier shown in Figure 2.4.

![Figure 2.4 A circuit for Example 2.3](image)

### 2.4 The non-inverting amplifier

A non-inverting amplifier is shown in Figure 2.5. Following the properties of an operational amplifier, the voltage at the input point “−” must be the same as that at the input point “+” and the operational amplifier takes negligible current. Therefore the current passing through $R_1$ is the same as that passing through $R_2$ and can be calculated as

$$i = \frac{v_{in}}{R_1} = \frac{v_{out} - v_{in}}{R_2}$$

The amplifier gain can therefore be calculated as

$$\frac{v_{out}}{v_{in}} = 1 + \frac{R_2}{R_1}$$

![Figure 2.5 A non-inverting amplifier](image)

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10
2.5 Bridge circuit

Electrical bridge circuits are widely used in industrial instrumentation.

Consider the resistance Wheatstone-bridge circuit shown in Figure 2.6. For zero voltage output, referred to as balanced condition, the following relationship holds.

\[ v_{AB} = v_{AD} \quad \text{i.e. } i_1R_1 = i_2R_4 \]

and

\[ v_{BC} = v_{DC} \quad \text{i.e. } i_1R_2 = i_2R_3 \]

Hence

\[ \frac{i_1R_1}{i_2R_2} = \frac{i_1R_4}{i_2R_3} \]

\[ \frac{R_1}{R_2} = \frac{R_4}{R_3} \]

If one of the resistor varies with the measured variable, e.g. temperature, then the output voltage \( v_o \) will also vary with the measured variable. Thus the bridge output can be used to indicate the measured variable.

In Figure 2.6, let the Wheatstone bridge be excited with a voltage \( V \), assume that the bridge is initially balanced and let

\[ R_1 = R_2 = R_3 = R_4 = R \]

then

\[ v_{AD} = v_{AB} = V/2 \]

and

\[ v_o = v_{AB} - v_{AD} = 0 \]

Let \( R_1 \) change by an amount \( \Delta R \) to \( (R_1 + \Delta R) \), then
\[
\Delta v_{AB} = \frac{R_1 + \Delta R}{R_1 + \Delta R + R_2} \times V = \frac{R + \Delta R}{2R + \Delta R} \times V
\]

\[
v_o = v_{AB} - v_{AO} = V \left( \frac{R + \Delta R}{2R + \Delta R} - \frac{1}{2} \right) = \frac{\Delta R}{4R + 2\Delta R} \times V
\]

If \( \Delta R \ll R \), then

\[
v_o = \frac{\Delta R}{R} \times \frac{V}{4}
\]

By a similar analysis, it can be shown that, if two resistors \( R_1 \) and \( R_2 \) vary, the expression becomes

\[
v_o = \frac{V}{4} \times \left( \frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} \right)
\]

Thus if the resistance changes are of the same sign and magnitude, they cancel each other out and the output voltage is zero.

**Example 2.4** A resistance Wheatstone bridge circuit made up of four resistors each of 120Ω has an excitation voltage of 5V. Determine the output voltage change when one resistor’s value changes by 1.2Ω.

\[
v_o = \frac{\Delta R}{R} \times \frac{V}{4} = \frac{1.2\Omega}{120\Omega} \times \frac{5V}{4} = 12.5\text{mV}
\]

### 2.6 Load effects

Since most measuring devices require energy to operate, they absorb energy from the source. The presence of the measuring device thus changes the characteristics of the quantity being measured. Figure 2.7 illustrates this load effect. In Figure 2.7, the voltage source (e.g. a battery) has an internal resistance \( R_i \) and a voltage meter has a resistance \( R_L \). When terminals A and B are open, i.e. no load,

\[
v_{AB} = V
\]

![Figure 2.7 Load effect](image-url)
When a meter having a resistance \( R_L \) is connected across terminals A and B, current \( I \) flows, therefore

\[
v_{AB} = V - IR_i
\]

Since

\[
I = \frac{V}{R_i + R_L}
\]

We have

\[
v_{AB} = V \left( \frac{R_L}{R_L + R_i} \right)
\]

If \( R_L \) is much larger than \( R_i \), then \( v_{AB} = V \). To minimise the loading effects, the following conditions must be satisfied:

a). the source resistance must be small, or
b). the resistance of the measuring device must be high.

### 3. Temperature sensors

#### 3.1 Introduction

Temperature is an important parameter in engineering, especially in the chemical process industry, and is very apparent to our senses. The International Practical Temperature Scale (IPTS) defines six primary fixed points for reference temperatures which are given in Table 3.1.

<table>
<thead>
<tr>
<th>Primary Fixed Points for Reference Temperatures</th>
</tr>
</thead>
<tbody>
<tr>
<td>the triple point of equilibrium hydrogen</td>
</tr>
<tr>
<td>the boiling point of oxygen</td>
</tr>
<tr>
<td>the boiling point of water</td>
</tr>
<tr>
<td>the freezing point of zinc</td>
</tr>
<tr>
<td>the freezing point of silver</td>
</tr>
<tr>
<td>the freezing point of gold</td>
</tr>
</tbody>
</table>

The freezing points of certain other metals are also used as **secondary fixed points** to provide additional reference points for the purpose of calibration, especially for calibrating instrument measuring high temperature. Table 3.2 gives some secondary fixed points.

<table>
<thead>
<tr>
<th>Secondary Fixed Points for Reference Temperatures</th>
</tr>
</thead>
<tbody>
<tr>
<td>the freezing point of tin</td>
</tr>
<tr>
<td>the freezing point of lead</td>
</tr>
<tr>
<td>the freezing point of antimony</td>
</tr>
<tr>
<td>the freezing point of aluminium</td>
</tr>
<tr>
<td>the freezing point of copper</td>
</tr>
<tr>
<td>the freezing point of nickel</td>
</tr>
<tr>
<td>the freezing point of palladium</td>
</tr>
<tr>
<td>the freezing point of platinum</td>
</tr>
<tr>
<td>the freezing point of rhodium</td>
</tr>
<tr>
<td>the freezing point of iridium</td>
</tr>
<tr>
<td>the freezing point of tungsten</td>
</tr>
</tbody>
</table>
3.2 Resistive temperature transducers

Resistive temperature sensors are probably the most common type in use. They are based on metals or semiconductors. The semiconductor versions are the most common and probably the cheapest. This type of temperature sensor is also known as the resistance temperature detector (RTD).

3.2.1 Metallic RTD

These transducers are similar in appearance to wire-wound resistors, and often take the form of a non-inductively wound coil of suitable metal wire such as platinum, copper, or nickel. They may be encapsulated within a glass rod to form a temperature probe, which can be very small in size.

The variation of resistance $R$ with temperature $T$ for most metallic materials can be represented by an equation of the form:

$$R = R_0 (1 + a_1 T + a_2 T^2 + ... + a_n T^n)$$

(3.1)

where $R_0$ is the resistance at temperature $T = 0$. The number of terms in the summation depends on the material, the accuracy required, and the temperature range to be covered. Platinum, nickel, and copper are the most commonly used metals and they generally require summation containing at least $a_1$ and $a_2$ for accurate representation. In engineering applications, it is often acceptable to model a metallic RTD using only constant $a_1$. For example, $a_1 = 0.004 (R$ in $\Omega$ and $T$ in K).

The normal resistance ($R_0$) of a metallic RTD can vary from a few ohms to several k$\Omega$. Generally speaking, 100 $\Omega$ is a fairly standard value. The resistance change of a metallic RTD can be quite large, typically up to 20% of the normal resistance over the design temperature range.

3.2.2 Thermistors

Thermistors are small semiconducting transducers, usually manufactured in the shape of beads, disc, or rods. They are made by combining two or more metal oxides. If oxides of cobalt, copper, iron, magnesium, manganese, nickel, tin, titanium, vanadium, or zinc are used, then the resulting semiconductor is said to have a negative temperature coefficient (NTC) of resistance, which means that as the temperature rises, the electrical resistance of the device falls. Most of the thermistors used in engineering are of this type, and they can exhibit large resistance variations, for example, 10 k$\Omega$ at 0°C and 200 $\Omega$ at 100°C. This high sensitivity allows quite small temperature changes to be detected. However, the accuracy of a thermistor is not as good as that of a metallic RTD, due to the unavoidable variations in the composition of the semiconductor which occur during manufacture. Most thermistors are manufactured and sold with tolerances of 10 to 20%. Any circuit using semiconductor thermistors must therefore include some arrangement for adjusting out errors.

Thermistors are markedly nonlinear (unlike metallic RTDs). The resistance-temperature relation is usually of the form:

$$R = R_0 e^{R_1 T_0^{-1/T N}}$$

(3.2)

where $R$ is the resistance (in $\Omega$) at temperature $T$ (in K), $R_0$ is the normal resistance at temperature $T_0$, and $\beta$ is a constant which depends on the thermistor material. The reference temperature $T_0$ is usually taken to be 298 K (25°C) and $\beta$ is of the order of 4000.

Thermistors can be used within the temperature range –60 °C to 150 °C. Their accuracy can be as high as ±0.1%. The main problem associated with thermistors is their nonlinearity.

Positive temperature coefficient (PTC) thermistors can also be made using compounds of barium,
lead, or strontium. PTC thermistors are usually used to provide thermal protection for wound equipment such as transformers and motor. The resistance of a PTC thermistor is low and reasonably constant below the switching temperature $T_R$. Above this temperature, the thermistor resistance increases spectacularly as shown in Figure 3.1. A PTC thermistor can be connected in series with the power supply. If the temperature becomes too high, the resulting high resistance effectively cut off the power supply.

![Figure 3.1 The relationship between resistance and temperature in a PTC thermistor](image)

3.2.3 Resistance temperature sensor bridge circuit

Thermistors are modulating transducers and are normally used in a Wheatstone bridge circuit as shown in Figure 3.2. Since the resistances change in thermistors are quite large and nonlinear, special precautions must be taken in the bridge design to minimise nonlinearities. The fixed resistors $R_1$ and $R_2$ should be of considerable higher resistance (usually at least 10 times higher) than the sensing resistor $R_t$ in order to achieve reasonable linearity. If the bridge circuit is used remote from the sensor, then the effects of wire resistance and temperature gradient across the connecting wires should be considered using the four-wire arrangement shown in Figure 3.2.

![Figure 3.2 A bridge circuit for temperature measurement](image)
Resistance thermometer bridges may be excited with either AC or DC voltages. The current through the sensor is typically in the range from 1 to 25 mA. This current causes $I^2R$ heating to take place, rising the temperature of the thermometer above its surroundings. This leads to the self-heating error to occur. The magnitude of this error depends on the heat transfer conditions around the sensor.

### 3.3 Thermocouples

A thermocouple is a self-generating transducer comprising two or more junctions between dissimilar metals. The conventional arrangement is shown in Figure 3.3. One junction (the cold junction) has to be maintained at a known reference temperature, for example by surrounding it with melting ice. The other junction is attached to the object to be measured.

When a temperature difference is maintained across a given metal, the vibration and motion of electrons is affected so that a difference in potential exists across the metal. This potential difference is related to the fact that electrons in the hotter end of the material have more thermal energy than those in the cooler end and, thus, tend to drift toward the cooler end. This drift varies for different metals at the same temperature because of differences in their thermal conductivities.

![Figure 3.3 Standard thermocouple arrangement](image)

![Figure 3.4 Thermocouple arrangement with cold junction compensation](image)
Thermocouple materials are broadly divided into two arbitrary groups, base metal and precious metal thermocouples, based upon cost. The most commonly used industrial thermocouples are specified by type letters as shown in Table 3.3 (Turner and Hill, 1999).

The arrangement of Figure 3.3 is inconvenient because of the layout of leads and the need for a reference temperature. A more practical scheme is shown in Figure 3.4. The two wires are laid out side by side, and are connected to a voltage measuring circuit. The junctions between the two wires and the voltage do not cause any error signal to appear so long as they are at the same temperature. Since there is no proper reference junction with this approach, the system is liable to give an erroneous output if the temperature of the surrounding environment changes. This is avoided by the use of so-called cold junction compensation, in which the characteristics of the signal conditioning amplifier are modified by including a thermistor in the circuit.

The arrangement shown in Figure 3.4 is usually applied whenever thermocouples are used. The two wires are often enclosed within a tube or flexible sleeve of stainless steel or copper for protection, although this increases the time constant of the system.

To overcome the necessity for using long expensive lengths of thermocouple wire to connect the hot junction to a remote measuring instrument, extension leads are used. They should have thermo-electric properties similar to the thermocouple wires over the operating temperature range. Copper and copper-alloy leads are used with types S, R, and B thermocouples.

The main advantages of thermocouples are their wide temperature range, nominally from –180 to +1200°C for a Chromel/Alumel device, and their linearity.

<table>
<thead>
<tr>
<th>Type</th>
<th>Conductors (positive conductor first)</th>
<th>Accuracy</th>
<th>Service temperature</th>
</tr>
</thead>
</table>
| B    | Platinum: 30% rhodium alloy          | 0 – 1100°C: ±3 °C  
Platinum: 6% rhodium alloy  
1100 – 1550°C: ±4 °C | 0 – 1500°C |
| E    | Nickel: chromium/constantan          | 0 – 400°C: ±3 °C  
–200 – 850°C | |
| J    | Iron/constantan                       | 0 – 300°C: ±3 °C  
300 – 850°C: ±1 %  
–200 – 850°C | |
| K    | Nickel: chromium(Chromel)            | 0 – 400°C: ±3 °C  
Nickel: aluminium (Alumel)  
400 – 1100°C: ±1 %  
–200 – 1100°C | |
| R    | Platinum: 13% rhodium/platinum       | 0 – 1100°C: ±1 °C  
1100 – 1400°C: ±2 °C  
1400 – 1500°C: ±3 °C  
0 – 1500°C | |
| S    | Platinum: 10% rhodium/platinum       | as Type R  
0 – 1500°C | |
| T    | Copper/Constantan                     | 0 – 100°C: ±1 °C  
100 – 400°C: ±1 %  
–200 – 400°C | |

Thermocouple compensation

As mentioned earlier, it is not normally practical to have thermocouple cold junctions maintained at a controlled reference temperature. However, with the cold junctions at ambient temperature, which may change, some form of cold junction compensation is required. Consider the arrangement shown in Figure 3.5, which shows a thermocouple with its measuring junction at \( t \) °C and its cold junction at ambient temperature. The thermocouple output is \( E(0,t) \), but what is required is the output that would be obtained if the cold junction were at 0 °C, i.e. \( E(0,0) \). Thus a voltage \( E(0,a) \) must be added to \( E(0,t) \) to correct the output signal:
\[ E_{(0-t)} = E_{(a-t)} + E_{(0-a)} \]  

The voltage \( E_{(0-a)} \) is called the cold junction compensation voltage, and it is provided automatically by the circuit of Figure 3.5 which includes a thermistor \( R_t \), \( R_1 \), \( R_2 \) and \( R_3 \) are temperature-stable resistors. The bridge is first balanced with all the components at 0 °C an unbalance voltage will appear across AB. This voltage is scaled by selecting \( R_t \) such that the unbalance voltage across AB equals \( E_{(0-a)} \) in Eq(3.3).

![Figure 3.5 Bridge circuit with cold junction compensation](image)

4. Displacement sensors

Measurement of the displacement of an object are of fundamental importance in experimental science, and are the basis of measurements of velocity, acceleration, strain, and (by the use of elastic elements) force and pressure.

![Figure 4.1 Potentiometric displacement sensor](image)

4.1 Potentiometers

A potentiometer consists essentially of a resistive element which is provided with a movable contact as shown in Figure 4.1. The contact consists of a springy, conduction arm, which is arranged so that it
can be moved along the potentiometer track. A variable resistance is thus created between one end of the track and the movable contact. The contact motion can be linear, rotary, or a combination of the two, such as helical movement. Translational (also called linear) potentiometers are available with strokes form about 5 to 1000mm. Rotary versions are available with strokes from about 10⁶ to as much as 60 turns ( > 20000°).

\[ \frac{e_0}{e} = \frac{1}{(x_i / x_t) + (R_p / R_m)(1 - x_i / x_t)} \]  

Under ideal conditions \( R_p/R_m = 0 \) for an open circuit, and Eq(4.1) becomes:

\[ \frac{e_0}{e} = \frac{x_i}{x_t} \]  

Thus when no current is drawn the input-output relationship is a straight line. In practice \( R_m \neq \infty \), and there is a nonlinear relationship between \( e_o \) and \( x_t \). If \( R_p = R_m \), the maximum deviation from linearity is about 12%. If \( R_p = 10\% \) of \( R_m \), the error drops to about 1.5%. For values of \( R_p/R_m < 0.1 \) the position of maximum error is in the region where \( x_i/x_t = 0.67 \), and maximum error is approximately \( 15R_p/R_m \% \) of full scale.

To achieve good linearity therefore the input impedance \( R_m \) of any circuit connected to a potentiometer should be high compared with the potentiometer impedance \( R_p \), which should be kept as law as possible. Unfortunately this requirement conflicts with the almost invariable need for high sensitivity. Since the output \( e_o \) is directly proportional to the excitation voltage \( e \), at first sight it appears to be possible to get any desired output simply by increasing \( e \). However, potentiometers have a fixed power rating which is determined by their heat-dissipating capability. If the limiting heat dissipation is \( H \) watts, the maximum allowable excitation voltage is:
\[ e(\text{max}) = \sqrt{HR_p} \]

Thus, a low value of \( R_p \) allows only a small \( e \), and therefore a reduced sensitivity. The choice of \( R_p \) must be a compromise between considerations of loading and sensitivity.

### 4.2 Capacitive displacement sensors

The basic operation principle of a capacitive displacement sensors can be seen from the equation for a parallel-plate capacitor:

\[ C = K \varepsilon_0 \frac{A}{d} \]

where \( K \) is the dielectric constant, \( \varepsilon_0 = 8.85 \text{pF/m} \), \( A \) is the plate common area, and \( d \) is the plate separation.

The capacity can be changed by a) variation of the distance between the plates; b) variation of the shared area of the plates; and c) variation of the dielectric constant. Figure 4.3 shows the variations in distance and common area for displacement sensing. An AC bridge or other active electronic circuit is employed to convert the capacity change into voltage or current signal.

![Figure 4.3 Capacity varies with plate distance and common area](image)

### 4.4 Inductive displacement transducers

Inductive position transducers do not suffer from the problems associated with a sliding contact, since they are inherently non-contact devices. The resolution available from a good-quality linear variable differential transformer (or LVDT) is equal to that obtained from a potentiometer. However, for many applications inductive sensors suffer from one inherent disadvantage: they are essentially AC devices, and cannot be run from DC battery supplies without extra complications.

An LVDT consists of a transformer with a single primary winding and two secondary windings connected in series opposing manor as shown in Figure 4.4. The object whose displacement is to be measured is physically attached to the centre iron core of the transformer. For an excitation voltage \( V_i = V_p \sin(\omega t) \), the emf induced in the secondary winding are given by

\[ V_o = K_o \sin(\omega t - \phi) \]
\[ V_b = K_b \sin(\omega t - \phi) \]

where the parameters \( K_a \) and \( K_b \) depend on the coupling between the respective secondary and primary windings, which is determined by the position of the iron core. When the core is in the centre position, \( K_a = K_b \), and we have \( V_b = V_a - V_b = 0 \).

Suppose now that the core is displaced upward by a distance \( x \) resulting \( K_a = K_1 \) and \( K_b = K_2 \), then we have

\[ V_o = V_a - V_b = (K_1 - K_2) \sin(\omega t - \phi) \]

If the core were displaced downward by a distance \( x \) resulting \( K_a = K_2 \) and \( K_b = K_1 \), then we have

\[ V_o = V_a - V_b = (K_2 - K_1) \sin(\omega t - \phi) = (K_1 - K_2) \sin(\omega t - \phi + \pi) \]

Therefore, for equal magnitude displacement \( x \) and \(-x\) of the core from the centre, the magnitude of the output voltage \( V_o \) is the same, but the phase of the output voltage is different. From the magnitude and phase of the output voltage the magnitude and direction of displacement can be measured. The relationship between the magnitude of the output voltage and the core position is approximately linear over a reasonable range of core movement and can be represented as

\[ V_o = Cx \]

where \( C \) is a constant.

---

**5. Pressure sensors**

**5.1 Introduction**

When a fluid (liquid or gas) comes into contact with a surface, it produces a force perpendicular to the surface. The force per unit area is called the pressure. The SI unit for pressure is the Pascal (Pa) and 1 Pa = 1 N/m².

Pressure measurements may be divided into three categories: absolute pressure, gauge pressure, and differential pressure. The absolute pressure is the difference between the pressure at a particular point in a fluid and the absolute zero of pressure, i.e. a complete vacuum. A mercury barometer is an
example of an absolute pressure sensor, since the height of the column of mercury measured the
difference between atmospheric pressure and the “zero” pressure of the vacuum that exists above the
column of mercury.

If a pressure sensor measures the difference between an unknown pressure and local atmospheric
pressure, the measurement is known as gauge pressure. If the pressure transducer measures the
difference between two unknown pressures, neither of which is atmospheric, then the measurement is
known as differential pressure.

There are three fundamental means by which a pressure may be measured. The simplest approach
involves balancing the unknown pressure against the pressure produced by a column of liquid of
known density. Instruments using this principle are called manometers. Figures 5.1 to 5.3 show the
manometers for measuring the absolute pressure, gauge pressure, and differential pressure
respectively. If the density of liquid is \( \rho \), then the absolute pressure in Figure 5.1 is \( \rho gh \), the gauge
pressure in Figure 5.2 is \( \rho gh \), and the differential pressure in Figure 5.3 is \( \rho gh \).

![Figure 5.1 Sealed U-tube measuring absolute pressure](image1)

![Figure 5.2 U-tube measuring gauge pressure](image2)
Figure 5.3 U-tube measuring differential pressure

The second method of pressure measurement involves allowing the unknown pressure to act on a known area. The resulting force is measured either directly or indirectly. Devices of this type are called dead-weight testers, and they are normally only used for calibrating other forms of pressure sensor.

In the third approach the unknown pressure is allowed to act on an elastic structure of known area and properties. Most commercial pressure sensors adopt this approach. The resulting stress, strain, or deflection is measured in a variety of ways.

5.2 Diaphragms

Diaphragms are probably the most popular elastic structure used in pressure sensors. They can be subdivided into two types; thin membranes under radial tension, which form part of an inductive or capacitive pressure sensor, and thicker diaphragms or plates, used in conjunction with resistive or piezoelectric transducers.

Figure 5.4 A diaphragm pressure sensor
Figure 5.4 shows a diaphragm pressure sensor. If pressure \( p_1 \) exists on one side of the diaphragm and \( p_2 \) on the other side, then the net force \( F \) on the diaphragm is given by

\[
F = (p_2 - p_1)A
\]

where \( A \) is the diaphragm area in \( \text{m}^2 \), \( p_1 \) and \( p_2 \) are pressures in \( \text{N/m}^2 \). A diaphragm is like a spring and extends or contracts until a Hooke’s force is developed balancing the pressure difference force. Thus the pressure difference is translated into diaphragm displacement, which can be measured by a displacement sensor.

5.3 Bourdon tubes

The Bourdon tube is the basis of many mechanical pressure sensors (particularly the familiar circular moving-pointer type). Bourdon tubes are also used in some electrical transducers, particularly those where the output displacement is to be sensed by potentiometers or differential transformers, both of which normally require rotary actuation. The basis of all forms of Bourdon tube is a tube of non-circular (and usually flat-sided or oval) cross-section. The resulting distortion tends to straighten the tube. The end of a C-type Bourdon tube undergoes a curved displacement as shown in Figure 5.5. Bourdon tube pressure sensors can be markedly non-linear, and often display an unwanted thermal sensitivity. They also suffer from hysteresis errors, which are usually of the order of 1-2% of full scale deflection.

![Figure 5.5 A Bourdon tube pressure sensor](image)

C-type Bourdon tubes have been used for pressures up to about \( 7 \times 10^8 \text{N/m}^2 \) (100 000psi). The spiral and twisted versions produce larger displacements, and are mainly used below \( 7 \times 10^6 \text{ N/m}^2 \) (1000psi). The best accuracy that can be achieved is usually around 0.1%.

5.4 Bellows

Figure 5.6 shows a bellows pressure sensor. The deflection of a bellows is usually more linear than that of a Bourdon tube. They are reversible with low hysteresis, and are often found in pneumatic systems where they act as pressure/displacement transducers. However, the most common application is undoubtedly in the production of low-cost aneroid barometers for atmospheric pressure measurement.

Bellows are manufactured in a variety of materials. The spring rate (modulus of compression) is proportional to the modulus of elasticity of the material from which the bellows is formed, and to the cube of the wall thickness.
Flow measurement is extremely important in process industries. The manner in which the flow rate is quantified depends on whether the quantity flowing is solid, liquid or gas. In the case of solids, it is appropriate to measure the rate of mass flow, whereas in the case of liquids and gases, flow is usually measured in terms of the volume flow rate.

**Mass flow rate**

Measurement of the mass flow rate of solids in the process industries is normally concerned with solids which are in the form of small particles produced by a crushing or grinding process. Such materials are usually transported by some form of conveyor, and this allows the mass flow rate to be calculated in terms of the mass of the material on a given length of conveyor multiplied by the speed of the conveyor. The mass flow rate of a fluid is usually determined by simultaneous measurement of the volume flow rate and the fluid density.

**Volume flow rate**

Volume flow rate is the appropriate way of quantifying the flow of all materials which are in a gaseous, liquid or semi-liquid slurry form (where solid particles are suspended in a liquid host). Materials in these forms are carried in pipes, and the common classes of instrument used for measuring the volume flow rate can be summarised as follows:

1. Differential pressure meters
2. Variable area meters
3. Positive-displacement meters
4. Turbine flowmeters
5. Electromagnetic flowmeters
6. Vortex-shedding flowmeters
7. Gate-type meters
8. Ultrasonic flowmeters
9. Cross-correlation flowmeters
10. Laser Doppler flowmeters
6.2 Differential pressure meters

Differential pressure meters involve the insertion of some device into a fluid-carrying pipe which causes an obstruction and creates a pressure difference on either side of the device. Such devices include the orifice plate, the venturi tube, the flow nozzle and the Dall flow tube. When such a restriction is placed in a pipe, the velocity of the fluid through the restriction increases and the pressure decreases. The volume flow rate is then proportional to the square root of the pressure difference across the obstruction.

\[ Q = K \sqrt{\Delta P} \]  

(6.1)

where \( Q \) is the volume flow rate, \( K \) is a constant for the pipe and liquid type, and \( \Delta P \) is the pressure drop across the restriction. The manner in which this pressure difference is measured is important. The normal procedure is to use a diaphragm-based differential pressure transducer.

All applications of this method of flow measurement assume that flow conditions upstream of the obstruction device are in steady state, and a certain minimum length of straight run of pipe ahead of the measurement point is specified to ensure this. The minimum lengths required for various pipe diameters are specified in British Standards tables, but a useful rule of thumb widely used in the process industries is to specify a length of ten times the pipe diameter. If physical restrictions make this impossible to achieve, special flow-smoothing vanes can be inserted immediately ahead of the measurement point.

Flow-restriction-type instruments are popular because they have no moving parts and are therefore robust, reliable and easy to maintain. One disadvantage of this method is that the obstruction causes a permanent loss of pressure in the flowing fluid. The magnitude and hence importance of this loss depends on the type of obstruction element used, but where the pressure loss is large, it is sometimes necessary to recover the lost pressure by an auxiliary pump further down the flow line. This class of device is not normally suitable for measuring the flow of slurries as the tappings into the pipe to measure the differential pressure are prone to blockage, although the venturi tube can be used to measure the flow of dilute slurries.

It is particularly important in applications of flow restriction methods to choose an instrument whose range is appropriate to the magnitudes of flow rate being measured. This requirement arises because of the square-root type of relationship between the pressure difference and the flow rate, which means that as the pressure difference decreases, the error in flow rate measurement can become very large. In consequence, restriction-type flowmeters are only suitable for measuring flow rates between 30% and 100% of the instrument range.

6.3 Orifice plate

The orifice plate, as shown in Figure 6.1, is a metal disk with a hole in it, which is inserted into the pipe carrying a flowing fluid. This hole is normally concentric with the disk. Over 50% of the instruments used in industry for measuring volume flow rate are of this type. The use of the orifice plate is so widespread because of its simplicity, cheapness and availability in a wide range of sizes. However, the best accuracy obtainable with this type of obstruction device is only \( \pm 2\% \) and the permanent pressure loss caused in the flow is very high, being between 50% and 90% of the pressure difference \( (P_1 - P_2) \) in magnitude. Other problems with the orifice plate are a gradual change in the discharge coefficient over a period of time as the sharp edges of the hole wear away, and a tendency for any particles in the flowing fluid to stick behind the hole and gradually build up and reduce its diameter. The latter problem can be minimised by using an orifice plate with an eccentric hole. If this hole is close to the bottom of the pipe, solids in the flowing fluid tend to be swept through, and the build-up of particles behind the plate is minimal.
A very similar problem arises if there are any bubbles of vapour or gas in the flowing fluid when liquid flow is involved. These also tend to build up behind an orifice plate and distort the pattern of flow. This difficulty can be avoided by mounting the orifice plate in a vertical run of pipe.

\[ P_1 \quad P_2 \]

**Figure 6.1 Orifice plate**

### 6.4 Flow nozzle

The form of a flow nozzle is shown in Figure 6.2. This is not prone to solid particles or bubbles of gas in a flowing fluid sticking in the flow restriction, and so in this respect it is superior to the orifice plate. Its useful working life is also greater because it does not get worn away in the same way as an orifice plate. These factors give the instrument a greater measurement accuracy. However, as the engineering effort involved in fabricating a flow nozzle is greater than that required to make an orifice plate, the instrument is somewhat more expensive. In terms of the permanent pressure loss imposed on the measured system, the flow nozzle is very similar to the orifice plate. A typical application of the flow nozzle is in measurement of steam flow.

\[ P_1 \quad P_2 \]

**Figure 6.2 Flow nozzle flow measurement**

### 6.5 Venturi

The venturi is a precision-engineered tube of a special shape, as shown in Figure 6.3. It is a very expensive instrument but offers very good accuracy (approximately ±1%) and imposes a permanent pressure loss on the measured system of only 10-15% of the pressure difference \((P_1 - P_2)\) across it. The smooth internal shape of this type of restriction means that it is unaffected by solid particles or
gaseous bubbles in the flowing fluid, and in fact can even cope with dilute slurries. It has almost no maintenance requirements and its working life is very long.

![Diagram of pressure measurement](image)

**Figure 6.3 Venturi flow measurement**

### 6.6 Pitot tube

The pitot tube is mainly used for making temporary measurements of flow. It measures the local velocity of flow at a particular point within a pipe rather than the average flow velocity as measured by other types of flowmeters. This is quite useful for situations where the local flow rates across the cross-section of a pipe need to be measured as in the case of non-uniform flow. For this purpose, multiple pitot tubes are normally used.

![Diagram of pitot tube](image)

**Figure 6.4 The pitot tube**

The instrument works on the principle that a tube placed with its open end in a stream of fluid, as shown in Figure 6.4 will bring to rest that part of the fluid which impinges on it, and the loss of kinetic energy will be converted to a measurable increase in pressure inside the tube. This pressure, $P_1$, and the static pressure of the undisturbed free stream of flow, $P_2$, are measured. The flow velocity can then be calculated as

$$v = C[2g(P_1 - P_2)]^{1/2} \quad (6.2)$$
where $C$ is the pitot-tube coefficient.

Once $v$ is calculated, the volume flow rate can then be calculated by multiplying $v$ by the cross-sectional area of the flow pipe.

Pitot tubes have the advantage that they cause negligible pressure loss in the flow. They are also cheap and easy to install. Their main disadvantage is that the measurement accuracy is normally only about $\pm 5\%$, and sensitive pressure measuring devices are needed to achieve even this level of accuracy, since the pressure difference created is very small.

6.8 Variable area flowmeters

In this class of flowmeter, the differential pressure across a variable aperture is used to adjust the area of the aperture. The aperture area is then a measure of the flow rate. This type of instrument normally only gives a visual indication of flow rate. Therefore, it is not useful in automatic control schemes. However, it is cheap and reliable and widely used in industry.

Figure 6.5 shows such a flowmeter. It consists of a tapered glass tube containing a float which takes up a stable position where it submerged weight is balanced by the upthrust due to the differential pressure across it. The position of the float is a measure of the effective annular area of the flow passage and hence of the flow rate. The accuracy of the cheapest instruments is only $\pm 3\%$, but more expensive versions offer measurement accuracies as high as $\pm 0.2\%$. The normal measurement range is between 10\% and 100\% of the full-scale reading for any particular instrument.

![Diagram of variable area flowmeter]

Figure 6.5 The variable area flowmeter

7. Level sensors

A wide variety of instruments are available for measuring the level of liquids. The simplest one is a dipstick. An ordinary dipstick is a metal bar with scales etched on it. By dipping it into a vessel, and then remove, the liquid level can be measured from how far up the scale the liquid has wetted. A human operator is required to remove and read the dipstick.

7.1 Float system

Measuring the position of a float on the surface of a liquid is another simple and cheap method of
liquid level measurement. Figure 7.1 shows a float level measuring system with a potentiometer.

![Figure 7.1](image)

**Figure 7.1 Float system**

### 7.2 Pressure measuring devices

The hydrostatic pressure due to liquid is directly proportional to its depth and hence to the level of its surface. In the case of open-topped vessels (or covered ones which are vented to the atmosphere), the level can be measured by inserting an appropriate pressure transducer at the bottom of the vessel as shown in Figure 7.2. The liquid level \( h \) is related to the measured pressure \( P \) by the following equation

\[
h = \frac{P}{\rho g} \quad (7.1)
\]

where \( \rho \) is the liquid density and \( g \) is the acceleration due to gravity.

One source of error in this method can be imprecise knowledge of the liquid density. This can be a particular problem in the case of liquid solutions and mixtures (especially hydrocarbons), and in some cases only an estimate of density is available. Even with single liquids, the density is subject to variation with temperature, and therefore temperature measurement may be required if very accurate level measurements are needed.

Where liquid-containing vessels are totally sealed, the liquid level can be calculated by measuring the differential pressure between the top and bottom of the tank, as shown in Figure 7.2(b). The liquid level is related to the differential pressure measured, \( \Delta P \), according to:

\[
h = \frac{\Delta P}{\rho g} \quad (7.2)
\]

The same comments as for the case of the open vessel apply regarding uncertainty in the value of \( \rho \). An additional problem which can occur is an accumulation of liquid on the side of the differential pressure transducer which is measuring the pressure at the top of the vessel. This can arise because of temperature fluctuations, which allow liquid alternately to vaporise from the liquid surface and then condense in the pressure tapping at the top of the vessel. The effect of this on the accuracy of the differential pressure measurement is severe, but the problem is easily avoided by placing a drain pot in the system. This should of course be drained regularly.
A final pressure-related system of level measurement is the bubbler unit shown in Figure 7.2(c). This uses a dip pipe which reaches to the bottom of the tank and is purged free of liquid by a steady flow of gas through it. The rate of flow is adjusted until gas bubbles are just seen to emerge from the end of the tube. The pressure in the tube, measured by a pressure transducer, is then equal to the liquid pressure at the bottom of the tank. It is important that the gas used is inert with respect to the liquid in the vessel. Nitrogen or sometimes just air is suitable in most cases. Gas consumption is low, and a cylinder of nitrogen may typically last for 6 months. The method is suitable for measuring the liquid pressure at the bottom of both open and sealed tanks. It is particularly advantageous in avoiding the large maintenance problem associated with leaks at the bottom of tanks at the site of the pressure tappings required by alternative methods.

### 7.3 Capacitive devices

Capacitive devices are now widely used for measuring the level of both liquids and solids in powdered or granular form. They are suitable for use in extreme conditions measuring liquid metals (high temperatures), liquid gases (low temperatures), corrosive liquids (acids, etc.) and high-pressure processes. Two versions are used according to whether the measured substance is conducting or not. For non-conducting (less than 0.1 $\mu$mho/cm$^2$) substances, two bare-metal capacitor plates in the form of concentric cylinders are immersed in the substance, as shown in Figure 7.3. The substance behaves as a dielectric between the plates according to the depth of the substance. For concentric cylinder plates of radius $a$ and $b$ ($b > a$), and total height $L$, the depth of the substance $h$ is related to the measured capacitance $C$ by:
\[ h = \frac{C \ln(b/a) - 2 \pi \varepsilon}{2 \pi \varepsilon_0 (\varepsilon - 1)} \]  \hspace{1cm} (7.3)

where \( \varepsilon \) is the relative permittivity of the measured substance and \( \varepsilon_0 \) is the permittivity of free space.

In the case of conducting substances, exactly the same measurement techniques are applied, but the capacitor plates are encapsulated in an insulating material. The relationship between \( C \) and \( h \) in Eq(7.3) then has to be modified to allow for the dielectric effect of the insulator.

![Figure 7.3 Capacitive level sensor](image)

Capacitive devices are useful in many applications, but become inaccurate if the measured substance is prone to contamination by agents which change its dielectric constant, for example, the ingress of moisture into powders.

![Figure 7.4 Ultrasonic level sensor](image)

**7.4 Ultrasonic level gauge**

The principle of the ultrasonic level gauge is illustrated in Figure 7.4. Energy from an ultrasonic source above the liquid is reflected back from the liquid surface into an ultrasonic energy detector.
Measurement of the time of flight allows the liquid level to be inferred. In alternative versions, the ultrasonic source is placed at the bottom of the vessel containing the liquid, and the time of flight between emission, reflection off the liquid surface and detection back at the bottom of the vessel is measured.

Ultrasonic techniques are especially useful in measuring the position of the interface between two immiscible liquids contained in the same vessel, or measuring the sludge or precipitate level at the bottom of a liquid-filled tank. In either case, the method employed is to fix the ultrasonic transmitter-receiver transducer at a known height in the upper liquid, as shown in Figure 7.5. This establishes the level of the liquid/liquid or liquid/sludge level in absolute terms.

![Figure 7.5 Measuring liquid/liquid interface](image)

When using ultrasonic instruments, it is essential that proper compensation is made for the working temperature if this differs from the calibration temperature. The speed of ultrasound through air varies with temperature at the rate of 0.607 m/s per °C. The speed of ultrasound also has a small sensitivity to humidity, air pressure and carbon dioxide concentration, but these factors are usually insignificant.

Temperature compensation can be achieved in two ways. First, the operating temperature can be measured and an appropriate correction made. Secondly, and preferably, a comparison method can be used in which the system is calibrated each time it is used by measuring the transit time of ultrasonic energy between two known reference points. This second method takes account of variations in humidity, pressure and carbon dioxide concentration as well as providing temperature compensation.

8. pH sensors
8.1 Introduction

pH is a parameter which quantifies the level of acidity or alkalinity in a chemical solution. It defines the concentration of hydrogen atoms in the solution in grams/litre and is expressed as:

\[ pH = \log_{10}\left[\frac{1}{H^+}\right] \]

where \( H^+ \) is the hydrogen ion concentration in the solution.
The value of pH can range from 0, which describes extreme acidity, to 14, which describes extreme alkalinity. Pure water has a pH of 7.

pH measurement is required in many process industries, especially those involving food and drink production. The most known method of measuring pH is to use litmus paper or some similar chemical indicator which changes colour according to the pH value. Unfortunately, this method gives only a very approximate indication of pH unless used under highly controlled laboratory conditions. Furthermore, this type of pH measurement can only be taken manually. On-line pH sensors are required in process automation. The glass electrode is at present the most common on-line pH sensor.

8.2 The glass electrode

The glass electrode consists of a glass probe containing two electrodes, a measuring one and a reference one, separated by a solid glass partition. Neither of the electrodes is in fact glass. The reference electrode is a screened electrode, immersed in a buffer solution, which provides a stable reference e.m.f. which is usually 0 V. The tip of the measuring electrode is surrounded by a pH-sensitive glass membrane at the end of the probe which permits the diffusion of ions according to the hydrogen ion concentration in the fluid outside the probe. The measuring electrode therefore generates an e.m.f. proportional to pH which is amplified and fed to a display meter. The characteristics of the glass electrode are very dependent on ambient temperature, with both zero drift and sensitivity drift occurring. Thus temperature compensation is essential. This is normally achieved through calibrating the system output before use by immersing the probe in solutions at reference pH values. Whilst the system is theoretically capable of measuring the full range of pH values between 0 and 14, the upper limit in practice is generally a pH value of about 12 because electrode contamination at very high-alkaline concentrations becomes a serious problem and also glass starts to dissolve at such high pH values. Glass also dissolves in acid solutions containing fluoride, and this represents a further limitation of use. If required, the latter problems can be overcome to some extent by using special types of glass.

Great care is necessary in the use of the glass electrode type of pH probe. First, the measuring probe has a very high resistance (typically $10^6 \, \Omega$) and a very low output. Hence, the output signal from the probes must be electrically screened to prevent any stray pick-up and electrical insulation of the assembly must be very high. The assembly must also be very efficiently sealed to prevent the ingress of moisture.

A second problem with the glass electrode is the deterioration in accuracy which occurs as the glass membrane becomes coated with the various substances it is exposed to in the measured solution. Cleaning at prescribed intervals is therefore necessary and this must be carried out carefully, using the correct procedures, to avoid damaging the delicate glass membrane at the end of the probe. The best cleaning procedure varies according to the nature of the contamination. In some cases careful brushing or wiping is adequate, whereas in other cases spraying with chemical solvents is necessary. Ultrasonic cleaning is often a useful technique, though it tends to be expensive. Steam cleaning should not be attempted, as this damages the pH-sensitive membrane. Mention must also be made about storage. The glass electrode must not be allowed to dry out during storage, as this would cause serious damage to the pH-sensitive layer.

Finally, caution must be taken with the response time of the instrument. The glass electrode has a relatively large time constant of 1 to 2 minutes, and so it must be left to settle for a long time before the reading is taken. If this causes serious difficulties, special forms of low-resistivity glass electrode are now available which have smaller time constants.
9. Signal conversion and data acquisition
9.1 Introduction

In industrial processes, historical plant operation information is stored in a database. Measurements taken over time form a ‘time history’. This time history may be analogue, such as the output from an accelerometer or may be inherently digital, such as the output from an optical shaft encoder, in which the signal is one of a finite number of discrete states. As digital data is highly immune to corruption and suitable for computer-based processing, many analogue signals must be transformed into a digital, or sampled data form, before storing and processing, even those which are continuously measured in time.

The transformation of continuous data to digital data is known as ‘analogue to digital (A/D) conversion’ and the opposite process as ‘digital to analogue (D/A) conversion’.

9.2 Analogue to digital conversion

It can be helpful to think of A/D conversion as consisting of two processes: sampling and quantization. When the continuous signal is sampled, a ‘snapshot’ is taken of the signal in time. This sample, which will probably be a voltage, must then be quantized: translated into a binary number.

While A/D converters do not generally use such distinct processes it is a useful way of picturing the process as it highlights the two types of approximation inherent in the conversion. The first is an approximation in time, in that a continuous signal is sampled at discrete intervals, and this, as we will see later, limits the frequency range of that data. The second approximation is due to forcing a signal with an infinite number of possible values into one with a limited number of binary digits. This is a quantization error and restricts the dynamic range of the signal after conversion.

The quantization error caused by A/D conversion can be illustrated by a simple example. An analogue signal \( v(t) \) which varies from 0 to 6V is to be converted to a 4-bit digital signal. The number of different output states into which the signal is divided is given by \( 2^4 = 16 \) and the relationship between the analogue input and the digital output is shown in Table 9.1. The effect of quantization can be seen in Figure 9.1 which shows an analogue signal and a four bit digital conversion of that signal (a four bit signal allows 16 different values).

<table>
<thead>
<tr>
<th>Analogue</th>
<th>Digital</th>
<th>Analogue</th>
<th>Digital</th>
</tr>
</thead>
<tbody>
<tr>
<td>([-0.2 \ldots 0.2)]</td>
<td>0000</td>
<td>([3.0 \ldots 3.4)]</td>
<td>1000</td>
</tr>
<tr>
<td>([0.2 \ldots 0.6)]</td>
<td>0001</td>
<td>([3.4 \ldots 3.8)]</td>
<td>1001</td>
</tr>
<tr>
<td>([0.6 \ldots 1.0)]</td>
<td>0010</td>
<td>([3.8 \ldots 4.2)]</td>
<td>1010</td>
</tr>
<tr>
<td>([1.0 \ldots 1.4)]</td>
<td>0011</td>
<td>([4.2 \ldots 4.6)]</td>
<td>1011</td>
</tr>
<tr>
<td>([1.4 \ldots 1.8)]</td>
<td>0100</td>
<td>([4.6 \ldots 5.0)]</td>
<td>1100</td>
</tr>
<tr>
<td>([1.8 \ldots 2.2)]</td>
<td>0101</td>
<td>([5.0 \ldots 5.4)]</td>
<td>1101</td>
</tr>
<tr>
<td>([2.2 \ldots 2.6)]</td>
<td>0110</td>
<td>([5.4 \ldots 5.8)]</td>
<td>1110</td>
</tr>
<tr>
<td>([2.6 \ldots 3.0)]</td>
<td>0111</td>
<td>([5.8 \ldots 6.2)]</td>
<td>1111</td>
</tr>
</tbody>
</table>
This quantization error can be made small by suitable choice of and correct use of modern equipment. Once a signal is in digital form it is highly immune to noise and degradation. A practical point to avoid serious problems with quantization errors is the correct matching of the analogue range to the input range of the A/D Converter. The first graph in Figure 9.2 shows that an eight-bit digitization (allowing 256 discrete values) provides a good approximation to the signal in Figure 9.1. The second graph in Figure 9.2 shows the same signal, also digitised using eight bits, but without using the full range of the A/D converter. The A/D converter here is set to ±10V, while the signal itself is only ±1V. This demonstrates the importance of considering quantization errors, especially in signals with a wide dynamic range.

From Table 9.1 it can be seen that a single digitised number is associated with a range of analogue values, any of which could have been quantized to that number. The width of a ‘step’ in Table 9.1 represents the range of analogue values which could potentially lead to each digital output and this
will be equal to the full analogue input range of the device, divided by the number of discrete digital states. The device represented in Table 9.1 has a normal input range of 0-6V, and the step width will be (6.0+2(0.2)/16=0.4. The ‘0.2’s’ are due to the fact that we have chosen to place the maximum and minimum analogue values in the middle of a digital step. The uncertainty caused by this quantization means, for example, that the digitised number 0010 could have been caused by any value between 0.6 and 1.0. This uncertainty may be thought of as a possible error of magnitude ±δ/2, where δ is the analogue value equating to the least significant bit in the digital representation. For most signals, this error can be assumed to be a random process uncorrelated with the signal having a uniform probability distribution, p(e), about the true value. The number of bits required in an A/D converter is determined by the dynamic range (the ratio, in dBs, between the largest and smallest amplitudes) which has to be measured. This, in turn, is governed by the signal to noise ratio (SNR), where:

\[ \text{SNR} = 10 \log_{10} \left( \frac{\sigma_x^2}{\sigma_e^2} \right) \]  

(9.1)

and where \( \sigma_x^2 \) is the variance of the signal to be measured and \( \sigma_e^2 \) is the variance of the error, which may be written as

\[ \sigma_e^2 = \int_{-\delta/2}^{\delta/2} (e - \bar{e})^2 p(e) \, de = \int_{-\delta/2}^{\delta/2} \frac{\delta^2}{12} \, de \]  

(9.2)

The relationship between the variance of the signal to be measured and the number of bits used is a trade-off and will need careful consideration if an application is close to the limit of the equipment to be used. The greater the variance the higher the SNR due to quantization but also the higher the likelihood of clipping. To ensure a very low probability of clipping in a random signal we may choose to ensure that the maximum range of a system with \( b \) bits, given by \( 2^b \delta \), is greater than five times the rms value of the signal:

\[ \sigma_s \geq \frac{2^b \delta}{5} \]  

(9.3)

Substituting into Eq(9.1) gives:

\[ \text{SNR} = 10 \log_{10} \left( \frac{12 \left( 2^{2b} \right)}{5^2} \right) \]

\[ = 10b \log_{10} (4) + 10 \log_{10} \left( \frac{12}{5^2} \right) \]

\[ \approx 6b - 3.2 \text{ dB} \]

Other errors which may be present in an A/D converter include (Turner and Hill, 1999):

**Linearity** – this is usually specified by the manufacturer of the A/D device as the maximum deviation from a straight line drawn between the full scale output and zero.

**Gain error** – most devices have a nulling input to deal with this effect. However, it is often found that the null setting required is temperature dependent, and there is no easy solution to this problem.

**Offset error** – similar to the above. Once again temperature dependence can be a problem.

### 9.3 Computer based data acquisition

In many industrial process applications, it is most convenient to simply use a purpose built card or
external adapter designed to allow a computer to directly acquire data. In such cases, the users need not worry about the details of the A/D conversion method used. The users should concentrate on cost, user-friendliness, and the following issues (Turner and Hill, 1999):

**Sampling rate.** The sampling rate at each input channel should be well over twice the maximum frequency contained in the signal to be sampled at that channel.

**Signal type.** Signal types include current, voltage, charge, and impedance. The cards are usually designed to present a relatively high impedance to a voltage source.

**Signal amplitude.** The range of the input signal will have to be matched to that of the card. An additional amplification stage may need to be included.

**Signal dynamic range.** The dynamic range of the signal determines the number of bits required in the data encoding. In some cases data compression may be required to get the most out of a data acquisition system.

**Triggering.** If the signal to be measured is a transient one, it may be required to start acquiring the data automatically. This can be done using a trigger which starts data acquisition when a particular condition is met, for example when the input voltage is higher than the threshold voltage. There are commonly two types of triggers, software triggers and hardware triggers. A hardware trigger uses a pin on the data acquisition card as an external trigger. When the trigger pin sees a high input, it starts read in data from the input lines. The users need to design a circuit which feeds the correct signal to the trigger pin at the correct time.

**Signal length.** The amount of data that you can acquire in one go depends on computer memory and disk space.

**Environment.** A conventional PC, fitted with a data acquisition card, may well be an appropriate cost-effective way of logging data in the laboratory. If data acquisition task has to be carried out in open air, or over a long period time in an industrial environment, specialist data logger or industrial PC may be appropriate.

**References**


