Linear Systems and Signals 1

Professor O R Hinton

Overall Course Aims

Students will become familiar with a number of operational amplifier circuits, and techniques for their analysis. The broad principles of feedback in bandlimited circuits will also be introduced by considering the effect of typical operational amplifier characteristics.

In addition, fundamental signal processing concepts will be introduced, particularly those of linearity, complex frequency, negative frequency, Fourier analysis, and digital (sampled) signals.

Detailed Course Aims

1. You should know the characteristics of an ideal op-amp.
2. **You should know how to analyse** inverting amplifier configurations using an ideal op-amp.
3. **You should be familiar** with the structure and analysis of the following applications of the inverting configuration: the "simple" amplifier, a summing amplifier, an integrator, a differentiator, a "simple lag" (or lowpass) circuit, a simple highpass circuit, a simple bandpass circuit.
4. **You should know how to analyse** non-inverting amplifier configurations, and be familiar with the structure and analysis of a "simple" non-inverting buffer amplifier.
5. **You should know** the structure of a difference amplifier and a differential amplifier, and **know how to analyse** them. You should be familiar with the “balanced input” application of a differential amplifier.
6. **You should be able to construct** Bode plots of simple frequency dependent circuits, including: simple lowpass, simple highpass, simple bandpass.
7. **You should be able to interpret** the Bode plot of a resonant system, and **understand the significance** of the resonant frequency, the damping coefficient, the Q-factor, and the bandwidth of a resonant system, and **be able to relate** these to the 2nd order function in $j\omega$.
8. **You should know how to analyse** an op-amp circuit and **take into account** the true, finite gain of the op-amp.
9. **You should understand the significance** of the gain-bandwidth product (or unity gain bandwidth) of an op-amp and **be able to use** this to determine the bandwidth of an op-amp circuit from its gain.
10. **You should be able to explain** in broad terms how instability (spontaneous oscillations) can occur in an op-amp circuit with feedback.
11. **You should know the definition** of mean, mean-square-value, and root-mean-square-value, and **be able to evaluate** these for a sine wave, the sum of several sine waves, and other simple waveforms.
12. **You should be very familiar** with the use of dB.
13. **You should be familiar** with the concept of “superposition” in the context of linear systems.
14. **You should know** the key properties of power and energy signals, periodic signals, and discrete-time signals (digital signals).
15. **You should be familiar** with the complex phasor, and **how** the representation of real (co)sinusoids leads to the concept of -ve frequency.
16. **You should understand** how the discrete-time complex phasor leads to the concept of aliasing.
17. **You should be able to interpret** the complex frequency response function $H(\omega)$, and **be able to calculate** from it the gain and phase at a particular frequency.
18. **You should be familiar** with the general form of a digital filter (i.e. weighted sum of past inputs and outputs), and **know the distinction** between FIR and IIR types.
19. **You should know how to derive** the frequency response and unit impulse response of a digital filter from its *finite difference equation*.
20. **You should know what is meant** by convolution, for discrete-time signals only, and be able to calculate the output of a digital filter by using the principle of *convolution*.
21. **You should know the definition** of the exponential Fourier series expansion of a periodic signal, and **understand** its significance.

Course Contents

Part I: Operational Amplifiers

The Ideal Op Amp

Terminals; **Ideal op amp**: input & output impedance, bandwidth, gain, common-mode rejection

(*Sedra* sections 2.1 & 2.2)

The Inverting Amplifier Configuration

Circuit configuration and method of analysis: for simple amplifier.

Applications: summer, integrator, differentiator, low-pass, high-pass, band-pass, Sallen Key "biquadratic" (resonant) circuits. Time and frequency domain analysis, including Bode plots.

(*Sedra* sections 2.3 & 2.4)

The Non-Inverting Amplifier Configuration

Configuration; analysis; voltage follower circuit

(*Sedra* section 2.5)

Difference amplifier

Analysis: using superposition, analysis using node equations, use as differential amplifier, applications of differential amplifier, input impedance of differential amplifier, design examples.

(*Sedra* section 11.8)

Real Op-Amp Characteristics and Their Effects (Linear Effects Only)

Finite Op-Amp Gain: Effect on closed loop gain.

Finite Op-Amp Gain and Bandwidth: Frequency response of open-loop amplifier. Frequency response of closed-loop amplifiers: inverting, non-inverting; example using the 'single-pole' model; 'signal flow' model of op-amp circuit; stability in 'multiple-pole' model of op-amp; gain & phase margins

(*Sedra* sections 2.7 & 8.10)

Part II: Signals and Processes

Signal Properties and Parameters

Parameters: mean value, mean square value, dB, dynamic range, frequency content and spectrum.

Signal classification: power/energy, periodic/aperiodic, discrete/continuous time, random/deterministic

Signal Models - Single frequency

The phasor: continuous and discrete time; concept of +ve and -ve frequency; phasors and sine waves; phasor addition;

Frequency response: of simple circuit and of simple finite difference equation.

Linear Filtering: Time & Frequency

Introduction: linearity, time invariance;

Discrete-time systems: unit-impulse response, causality, convolution, frequency response.

Fourier Series

Definition using complex exponential series; example of square wave and rectangular wave; signal power from Fourier series coefficients.

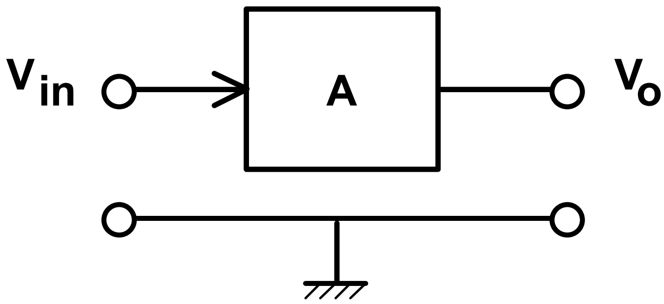
Books

Sedra: *Microelectronic Circuits*, A S Sedra & K C Smith, HRW Saunders College Printing, 1991, ISBN 0-03-051648-X.

Martin: *Signals and Processes - A Foundation Course*, J D Martin, Pitman, 1991, ISBN 0-273-03256-9. (Currently out of print).

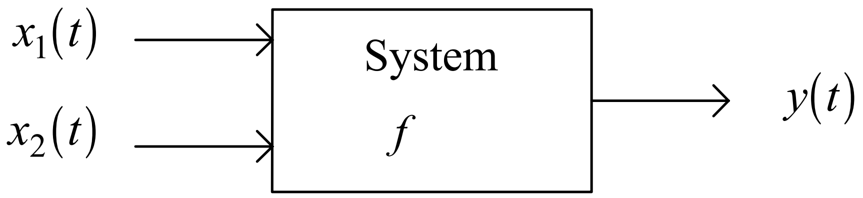
What is a "System"?

- It is common to hide the details of a "process" or "device" inside a "box".
- The function of the box is then described in "simple" terms that approximate the behaviour of the original process or device.
- The advantage is that the inner details of the device can then be ignored.
- For example, the following represents an amplifier with a gain of A:



$$V_o = A.V_{in}$$

What is a Linear System?



SUPERPOSITION

SINE WAVES

If $x_1(t)$ is a sine wave [with $x_2(t)=0$], then $y(t)$ is also a sine wave

EXAMPLES

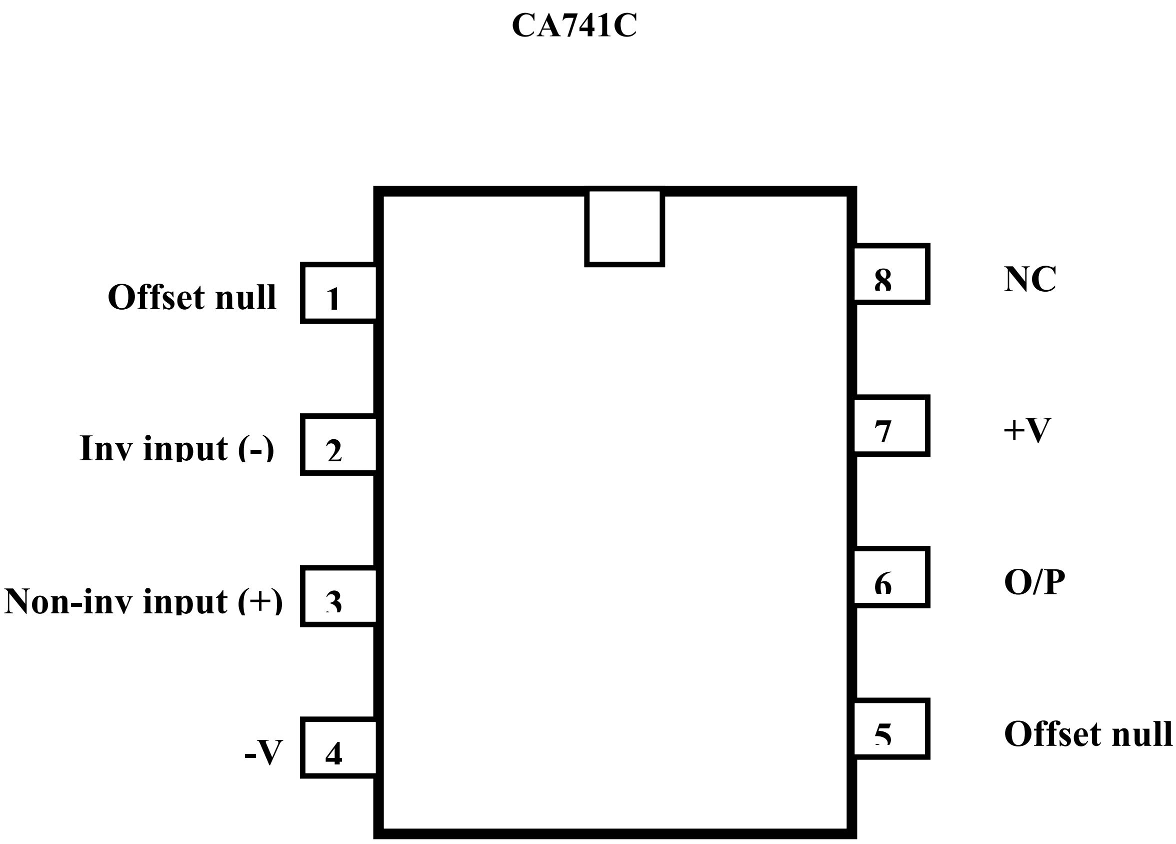
Linear functions	**Nonlinear functions**
multiplication	clipping
integration	squaring (or some other power)
differentiation	log
	sin, cos, tan

Operational Amplifiers

Introduction

What is an Operational Amplifier?

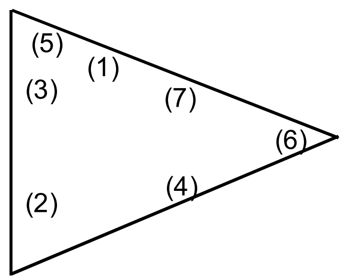
- Usually a single integrated circuit:



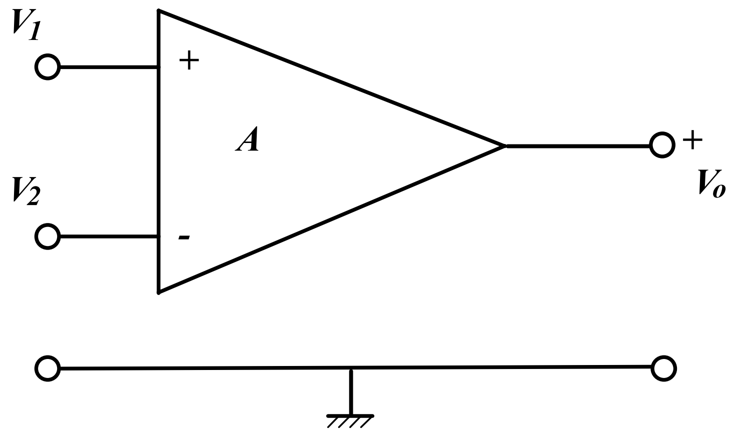
Circuit Diagram of an OA741 - a typical Op-amp

Connecting up the Terminals of an Op-Amp

- V_1 and V_2 are the input signals
- V_o is the output signal
- Passive component networks between V_1, V_2 and V_o are required to determine the function of the circuit



The Ideal Op Amp



Voltages measured relative to ground

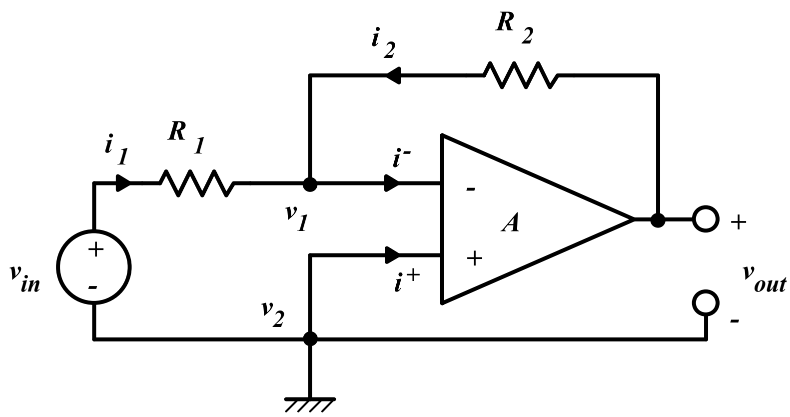
1. $V_0 = A(V_1 - V_2)$ (Voltages measured between terminal and ground)
2. Gain (A):
3. Common mode rejection:
4. Input impedance:
5. Output impedance:
6. Bandwidth (B Hz):

Typical Values for OA741

Gain (A):
Bandwidth (B):
Input impedance:
Output impedance:

Inverting Amplifier

Configuration



Voltage Gain (assuming ideal op-amp)

- Since the gain $A = \infty$ (ideal op-amp), assume that $v_1 = v_2$.
- Since the input impedance on each terminal = ∞ (ideal op-amp), assume $i^- = i^+ = 0$.
- The node equation at v_1 is then:

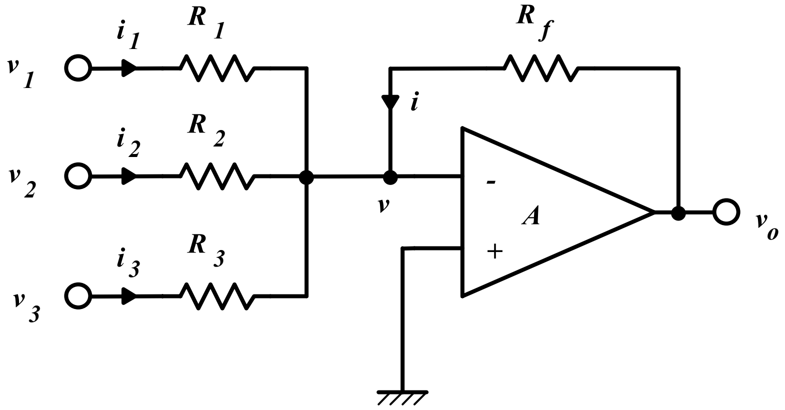
- The gain is therefore entirely determined by the value of the passive components.

Input Impedance (assuming ideal op-amp)

- The input impedance can be determined from:

Summing Amplifier

Configuration



- All voltages are measured relative to ground

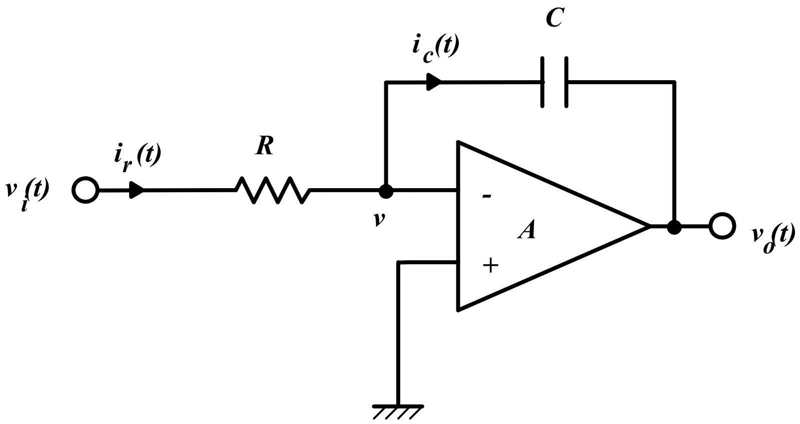
Analysis (assuming ideal op-amp)

- Node equation at v:

- Since $v = 0$,

Integrator

Configuration



Time Domain Analysis (assuming ideal op-amp)

- The capacitor will be charged by the current such that $v_o(t)$ grows more negative relative to v:

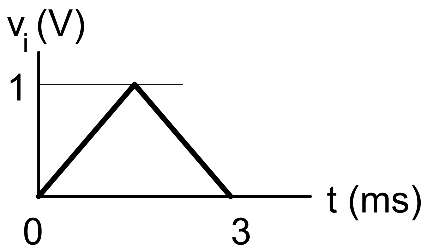
$$v_o(t) = v_o(0) - \frac{1}{C}\int_0^t i_c(t)dt$$

and since $i_c(t) = i_r(t) = \frac{v_i(t)}{R}$

$$v_o(t) =$$

- This is an integrator, where the output voltage is proportional to the integral of the input voltage.

- Example: suppose R=10kΩ, C=1μF, $v_o(0)=0$, and the following voltage waveform is applied:



Frequency Domain Analysis (assuming ideal op-amp)

- The node equation at *v* is:

$$I_r = I_c$$

$$\frac{V_i}{R} = -\frac{V_o}{1/j\omega C}$$

$$\frac{V_o}{V_i} = -\frac{1}{j\omega RC}$$

where I_r, I_c, V_o and V_i are the complex phasors representing the magnitude and phase of the sinusoidal waveforms of current and voltage.

As an example, suppose we have the following parameters:

ω = 12 kHz
R = 15 kΩ
C = 49 nF
V_i = 10 exp(j20°) volts

then:

$$\frac{V_0}{V_i} =$$

- Hence:

$$V_o =$$

Bode Frequency Plot

- To plot the frequency response, the magnitude and phase can be obtained from:

$$\text{Magnitude} = \left|\frac{V_0}{V_i}\right| = \left|-\frac{1}{j\omega RC}\right| = \frac{1}{\omega RC}$$

$$\text{Phase} = Arg\left[-\frac{1}{j\omega RC}\right] = Arg\left[\frac{j}{\omega RC}\right] = \tan^{-1}\left(\frac{1/\omega RC}{0}\right) = 90^0$$

- In the Bode plot it is common to plot these using:

  logarithmic scale for ω
  dB scale for magnitude (i.e. logarithmic)
  linear scale for phase

Units of dB

- dB are units usually used to define a power ratio

$$\text{Power ratio} = 10\log_{10}\left(\frac{P_1}{P_2}\right) \quad \text{dB}$$

  where P_1 and P_2 are two different powers being dissipated somewhere

- If you now have two sine waves of magnitude V_1 and V_2, and they are applied to a resistor R, then the ratio of the powers that would be dissipated are:

$$\text{Power ratio} = 10\log_{10}\left(\frac{P_1}{P_2}\right) \quad \text{dB}$$

$$= 10\log_{10}\left(\frac{V_1^2}{2R}\Big/\frac{V_2^2}{2R}\right) \quad \text{dB}$$

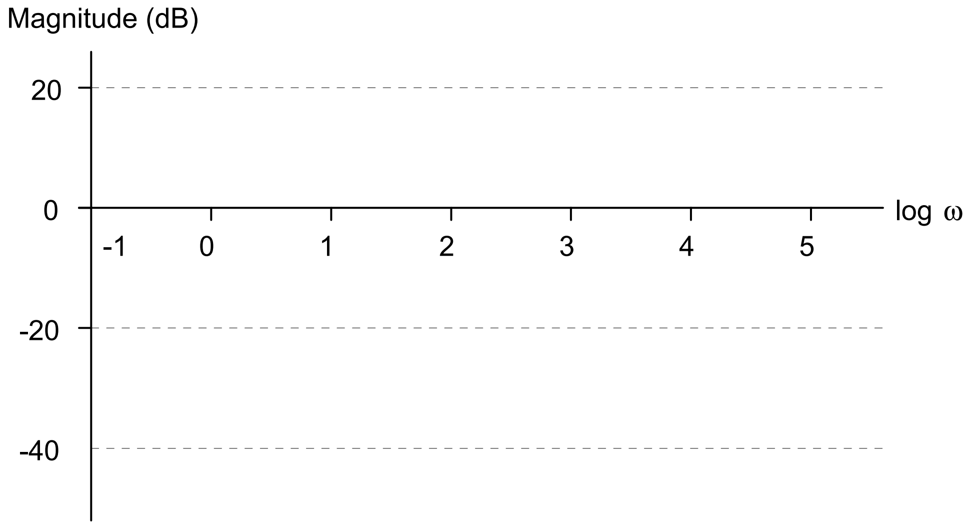
$$= 10\log_{10}\left(\frac{V_1^2}{V_2^2}\right) \quad \text{dB}$$

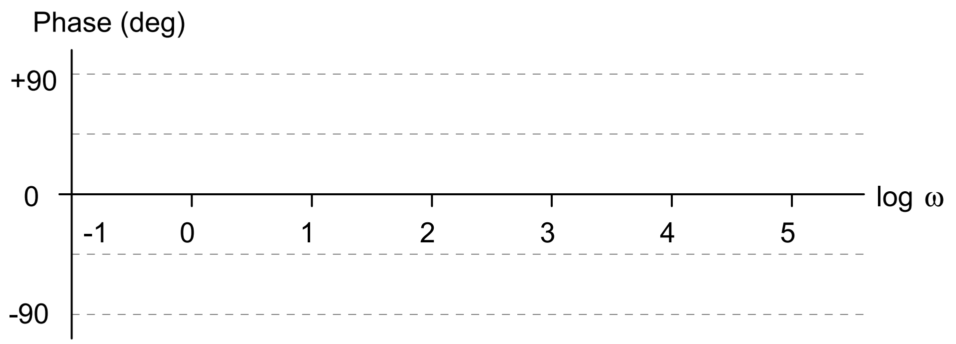
$$= 20\log_{10}\left(\frac{V_1}{V_2}\right) \quad \text{dB}$$

■ Hence

$$\text{Magnitude} = 20\log_{10}\left(\frac{1}{\omega RC}\right)$$
$$= -20\log_{10}(\omega RC)$$
$$= -20\log_{10}(\omega/\omega_c)$$

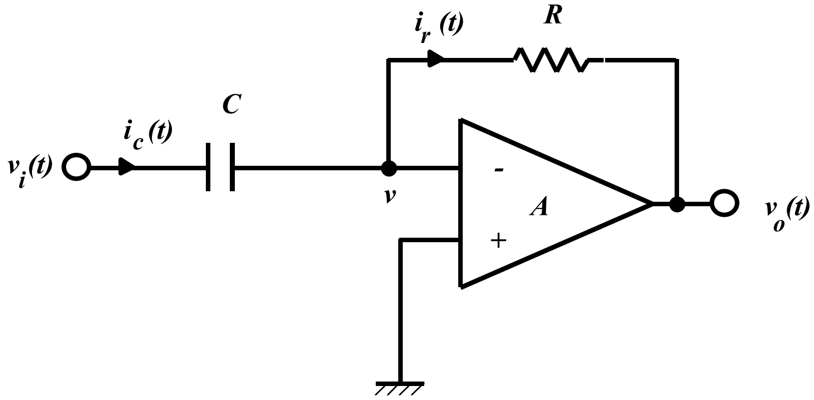
where $\omega_c = 1/RC$ rad/sec





Differentiator

Configuration



Time Domain Analysis (assuming ideal op-amp)

■

$$i_c(t) =$$

$$i_r(t) =$$

and since $i_c = i_r$,

$$-\frac{v_o}{R} =$$

$$\therefore v_o =$$

Frequency Domain Analysis (assuming ideal op-amp)

- The node equation at v is again:

$$\frac{V_o}{V_i} = -R\bigg/\frac{1}{j\omega C}$$

$$= -j\omega RC$$

- As an example, suppose we have the following parameters:

ω = 12 kHz
R = 15 kΩ
C = 49 nF
V_i = 0.1 exp(j20°) volts

then:

$$\frac{V_0}{V_i} =$$

- Hence:

$$V_o =$$

Bode Frequency Plot

- To plot the frequency response, the magnitude and phase can be obtained from:

$$\text{Magnitude} = \left|\frac{V_0}{V_i}\right| = \left|-j\omega RC\right| = \omega RC$$

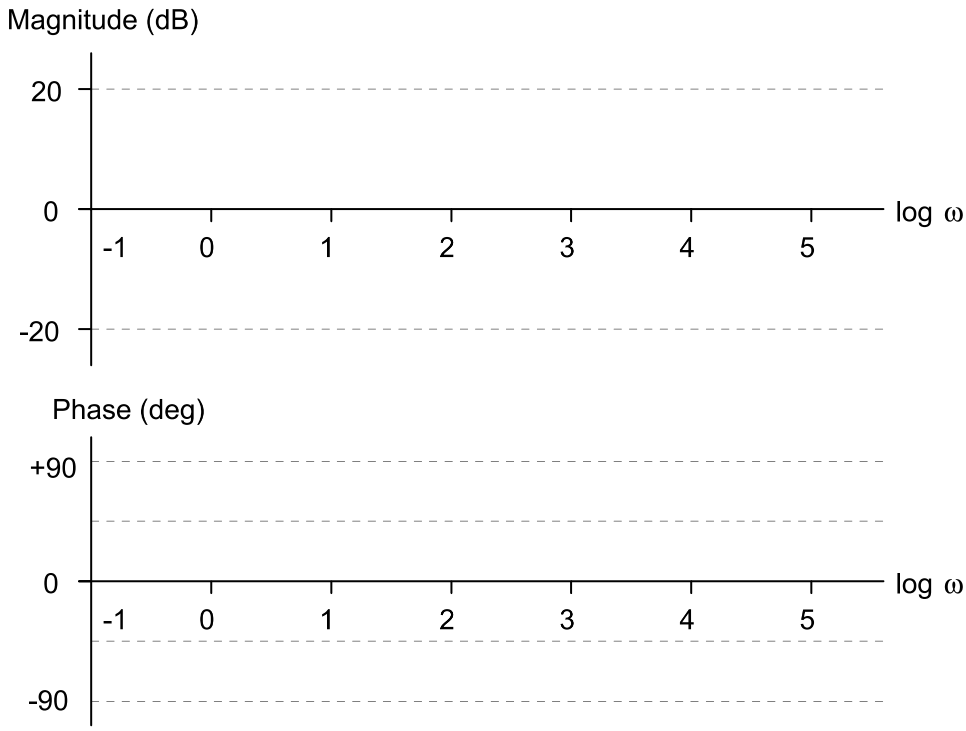
$$= 20\log_{10}(\omega RC) \quad \text{dB}$$

$$= 20\log_{10}(\omega/\omega_c) \quad \text{dB}$$

where $\omega_c = 1/RC$ rad/sec

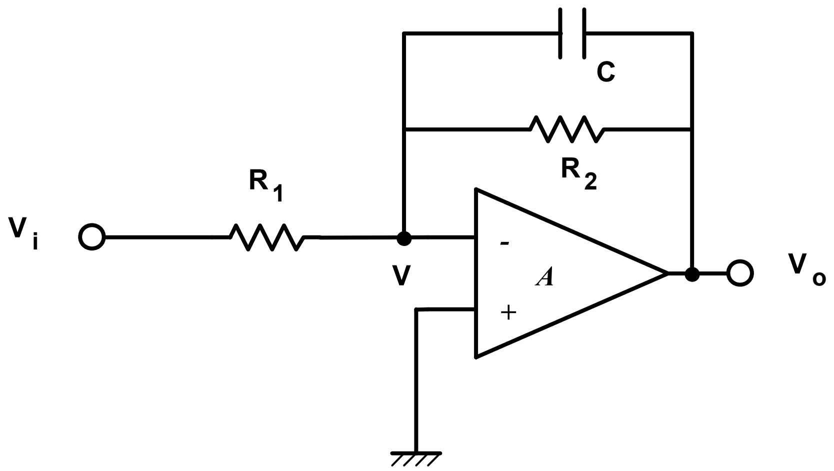
$$\text{Phase} = Arg[-j\omega RC] = \tan^{-1}\left(\frac{-\omega RC}{0}\right) = -90^0$$

- Hence Bode frequency plot is:



Simple lowpass circuit

Configuration



Frequency Domain Analysis (assuming ideal op-amp)

- The node equation at v is again:

$$\therefore V_o =$$

$$\therefore \frac{V_o}{V_i} =$$

where $K = R_2 / R_1$ and $\omega_c = 1/R_1C_1$

Bode Frequency Plot

Magnitude

- The magnitude can be obtained from:

$$\text{Magnitude} = \left|\frac{V_0}{V_i}\right| = \left|-K.\frac{1}{(1+j\omega/\omega_c)}\right| = \frac{|-K|}{|1+j\omega/\omega_c|} = \frac{K}{\left(1+\omega^2/\omega_c^2\right)^{1/2}}$$

$$= 20\log_{10}(K) - 20\log_{10}\left(1+\omega^2/\omega_c^2\right)^{1/2} \quad \text{dB}$$

- Plot the two terms seperately and then add them together

  For $20\log_{10}(K)$: This term is simply a constant (i.e. horizontal line)

- For $-20\log_{10}\left(1+\omega^2/\omega_c^2\right)^{1/2}$, consider the following 3 cases:

  (a) $\underline{\omega \ll \omega_c}$: Gain $\approx -20\log_{10}(1) = \underline{0\text{ dB}}$

  (b) $\underline{\omega \gg \omega_c}$: Gain $\approx -20\log_{10}\left(\omega^2/\omega_c^2\right)^{1/2} = \underline{-20\log_{10}(\omega/\omega_c)}$

  (c) $\underline{\omega = \omega_c}$: Gain $= -20\log_{10}\left(1+\omega_c^2/\omega_c^2\right)^{1/2}$

  $= -20\log_{10}\left(2^{1/2}\right)$

  $= \underline{-3}$ dB

- (a) and (b) are straight lines that intersect at $\omega = \omega_c$

Phase

- The phase can be obtained from:

$$\text{Phase} = Arg\left[\frac{V_0}{V_i}\right]$$

$$= Arg\left[-K.\frac{1}{(1+j\omega/\omega_c)}\right]$$

$$= Arg[-K] - Arg[1+j\omega/\omega_c]$$

$$= 180^{\circ} - \tan^{-1}(\omega/\omega_c)$$

- Term 1 is due to the inverting amplifier configuration
- For term 2, consider the following cases:

(a) $\underline{\omega << \omega_c:}$ Phase ≈ 0

(b) $\underline{\omega >> \omega_c:}$ Phase $\approx -90^{\circ}$

(c) $\underline{\omega = \omega_c:}$ Phase $= -\tan^{-1}(1) = -45^{o}$

(d) $\underline{\omega = \omega_c/10:}$ Phase $= -\tan^{-1}(0.1) = -5.7^{o}$

(e) $\underline{\omega = 10\omega_c:}$ Phase $= -\tan^{-1}(10) = -(90-5.7)^{o}$

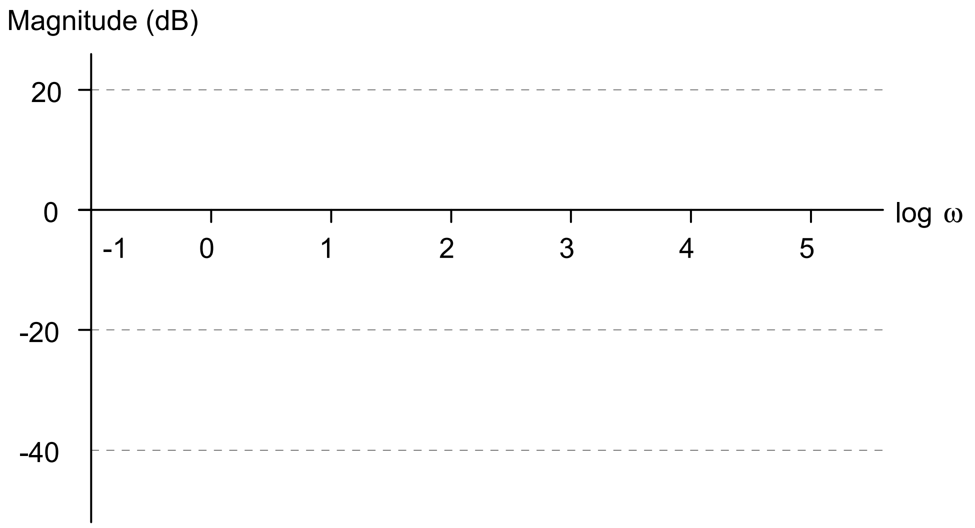
- The phase plot is therefore approximated by 3 straight lines

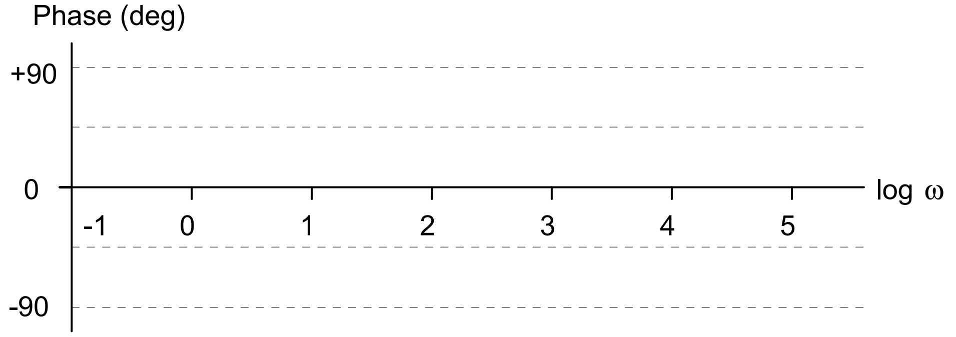
Example

- As an example, suppose:

$$\omega_c = 100 \text{ rad/sec}, \left(\text{i.e. } \log_{10}(\omega_c) = 2\right)$$

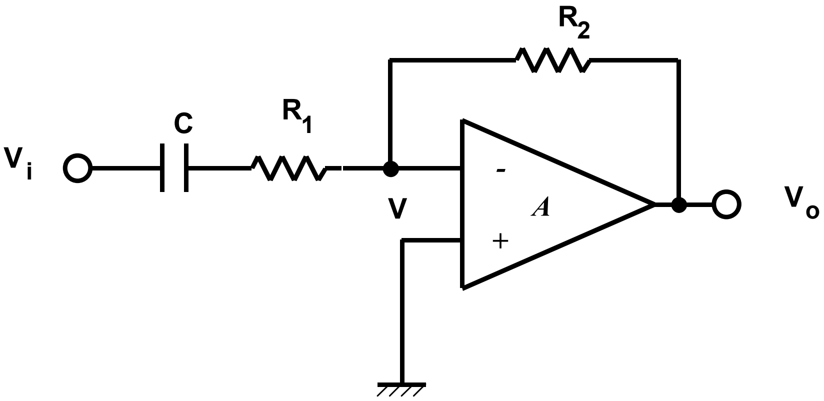
$$K = 10\ , \left(\text{i.e. } 20\log_{10}(10) = 20 \text{ dB}\right)$$





Simple highpass circuit

Configuration



Frequency Domain Analysis (assuming ideal op-amp)

$$\frac{V_i}{\left(R_1 + 1/j\omega C\right)} + \qquad\qquad\qquad\qquad = 0$$

- Rearranging we get:

$$\frac{V_o}{V_i} =$$

where $K = R_2/R_1$ and $\omega_c = 1/R_1C_1$

Bode Frequency Plot

Magnitude

- To plot the frequency response, the magnitude can be obtained from:

$$\text{Magnitude} = \left|\frac{V_0}{V_i}\right| = \left| - K.\frac{j\,\omega/\omega_c}{\left(1 + j\,\omega/\omega_c\right)}\right|$$

$$= \frac{\left|-K\right|.\left|\omega/\omega_c\right|}{\left|1 + j\,\omega/\omega_c\right|}$$

$$= \frac{\left(K\right).\left(\omega/\omega_c\right)}{\left(1 + \omega^2/\omega_c^2\right)^{1/2}}$$

$$= 20\log_{10}\left(K\right) - 20\log_{10}\left(1 + \omega^2/\omega_c^2\right)^{1/2} + 20\log_{10}\left(\omega/\omega_c\right)\ \text{dB}$$

- Each of these terms can be plotted separately, and then added together to obtain the overall plot.

- Term 1 is the constant term (as for the lowpass circuit).

  Term 2 is the lowpass characteristic (as for the lowpass circuit).

  Term 3 is the same as the differentiator circuit.

Phase

- To plot the phase response the phase can be obtained from:

$$\text{Phase} = Arg\left[\frac{V_0}{V_i}\right] = Arg\left[-K.\frac{j\,\omega/\omega_c}{\left(1+j\,\omega/\omega_c\right)}\right]$$

$$= Arg[-K] - Arg[1+j\,\omega/\omega_c] + Arg[j\,\omega/\omega_c]$$

- Each of these terms can be plotted separately, and then summed to give the overall plot

- Term 1 is identically 180° (i.e. phase inversion)

  Term 2 is the same as the lowpass filter

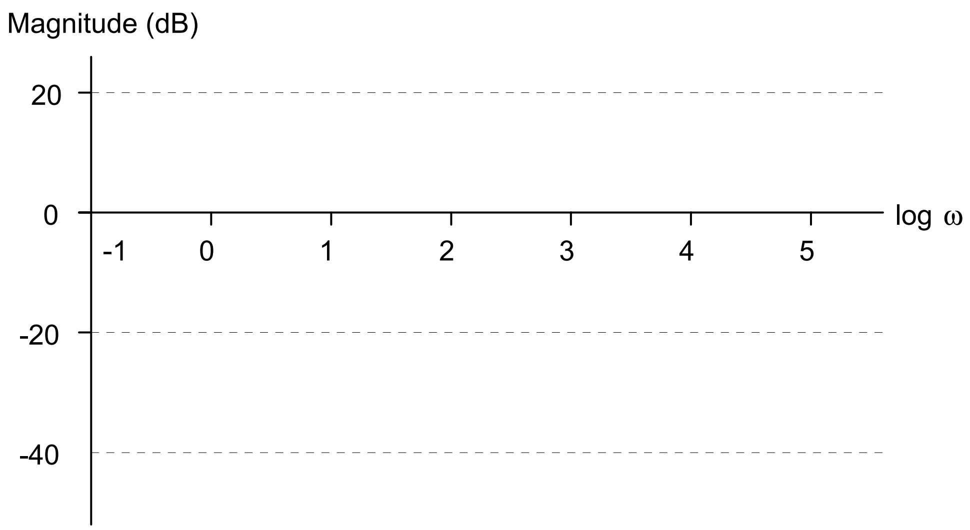
  Term 3 is the same as the differentiator

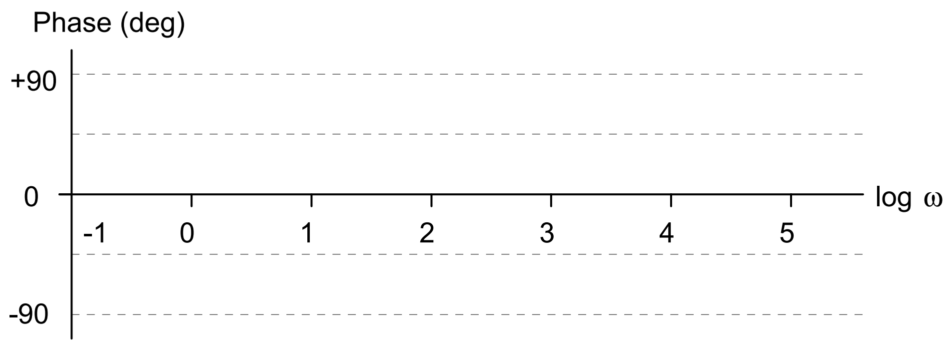
Example

- As an example, suppose:

$$\omega_c = 100 \text{ rad/sec}, \left(\text{i.e. } \log_{10}(\omega_c) = 2\right)$$

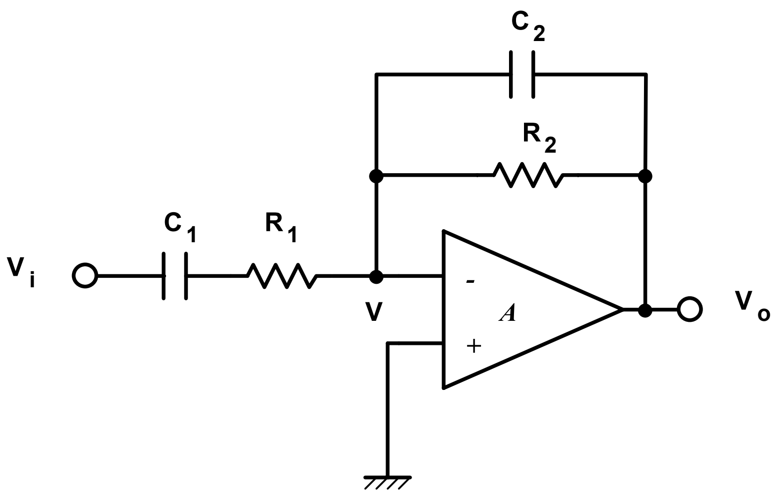
$$K = 10\ , \left(\text{i.e. } 20\log_{10}(10) = 20 \text{ dB}\right)$$





Bandpass circuit

Configuration



Frequency Domain Analysis (assuming ideal op-amp)

- The node equation at V is again:

- Rearranging we get:

$$\frac{V_o}{V_i} = ?$$

where $K = R_2 / R_1$ and $\omega_1 = 1/R_1 C_1$ and $\omega_2 = 1/R_2 C_2$

Bode Frequency Plot

Magnitude

- To plot the frequency response, the magnitude can be obtained from:

$$\text{Magnitude} = 20\log_{10}\left|\frac{V_0}{V_i}\right| \qquad \text{dB}$$

$$= 20\log_{10}K + 20\log_{10}\left(\frac{\omega}{\omega_1}\right)$$

$$- 20\log_{10}\left(1+\frac{\omega^2}{\omega_1^2}\right)^{\frac{1}{2}} - 20\log_{10}\left(1+\frac{\omega^2}{\omega_2^2}\right)^{\frac{1}{2}} \qquad \text{dB}$$

- Each of these terms can be plotted separately, and then added together to obtain the overall plot.

- Term 1 is the usual constant term.
  Term 2 is the same as the differentiator
  Terms 3 and 4 are the same as for the simple lowpass filter

Phase

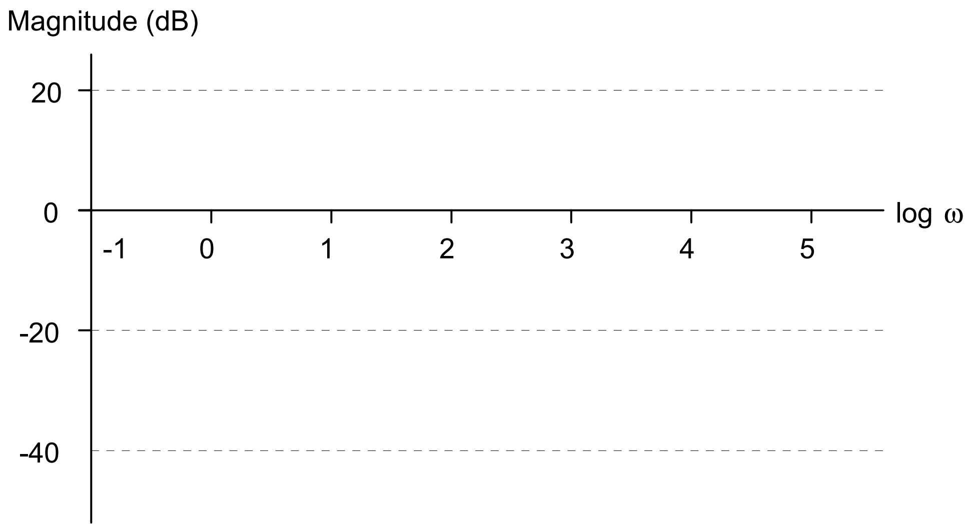
- To plot the phase response the phase can be obtained from:

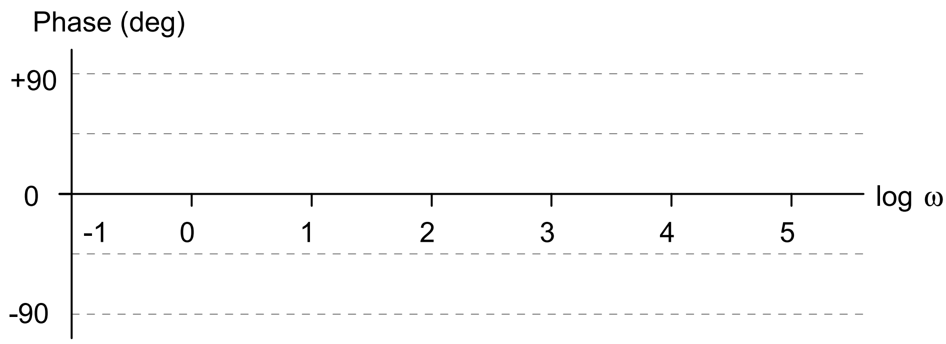
$$\text{Phase} = Arg\left[\frac{V_0}{V_i}\right] = \text{Arg}\left[-K\right] + Arg\left[\frac{j\omega}{\omega_1}\right]$$

$$- Arg\left(1+\frac{j\omega}{\omega_1}\right) - Arg\left(1+\frac{j\omega}{\omega_2}\right)$$

- Each of these terms can be plotted separately, and then summed to give the overall plot

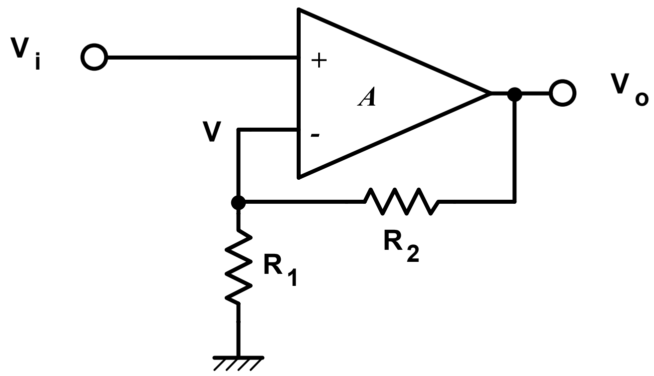
- Term 1 is identically 180° (i.e. phase inversion)
  Term 2 is the same as the differentiator
  Terms 3 and 4 are the same as the lowpass filter


Magnitude (dB)
20
0
-20
-40
-1
0
1
2
3
4
5
log ω


Phase (deg)
+90
0
-90
-1
0
1
2
3
4
5
log ω

Non-Inverting Amplifier

Configuration



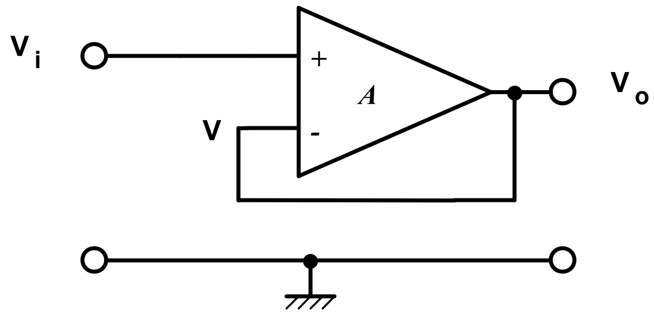
Analysis (assuming ideal op-amp)

- The node equation at V is again:

$$\frac{0-V}{R_1}+\frac{V_o-V}{R_2}=0$$

- Since $V_i = V$:

Unity Gain Buffer Amplifier



- Since V = V_o = V_i:

$$\frac{V_o}{V_i} = 1$$

- Can be used as a buffer amplifier
- Sometimes known as a voltage follower circuit

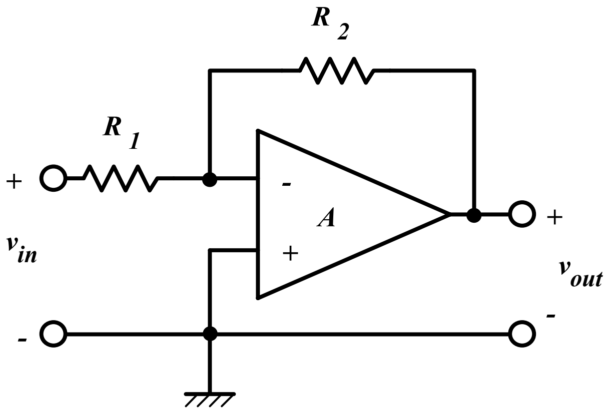
Characteristics of Non-Inverting Amplifier

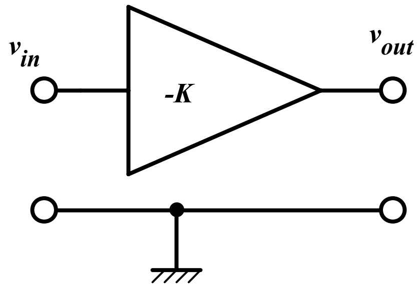
- Very high input impedance

  (e.g. ~1MΩ for OA741)

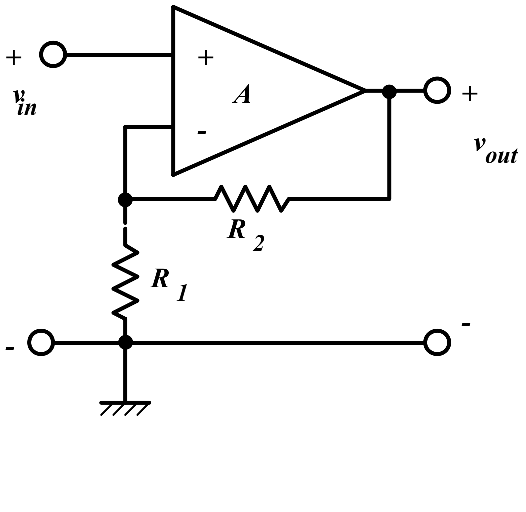
ACTIVE FILTERS
Circuits based on a VCVS
(Voltage Controlled Voltage Source)

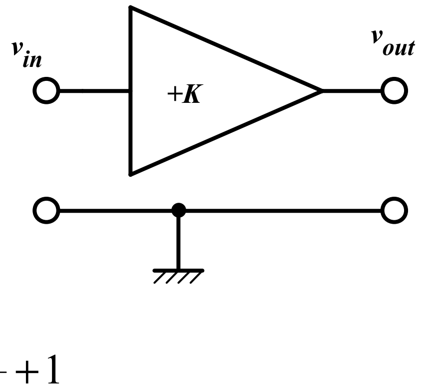
Basic Building Blocks





$$K = \frac{R_2}{R_1}$$

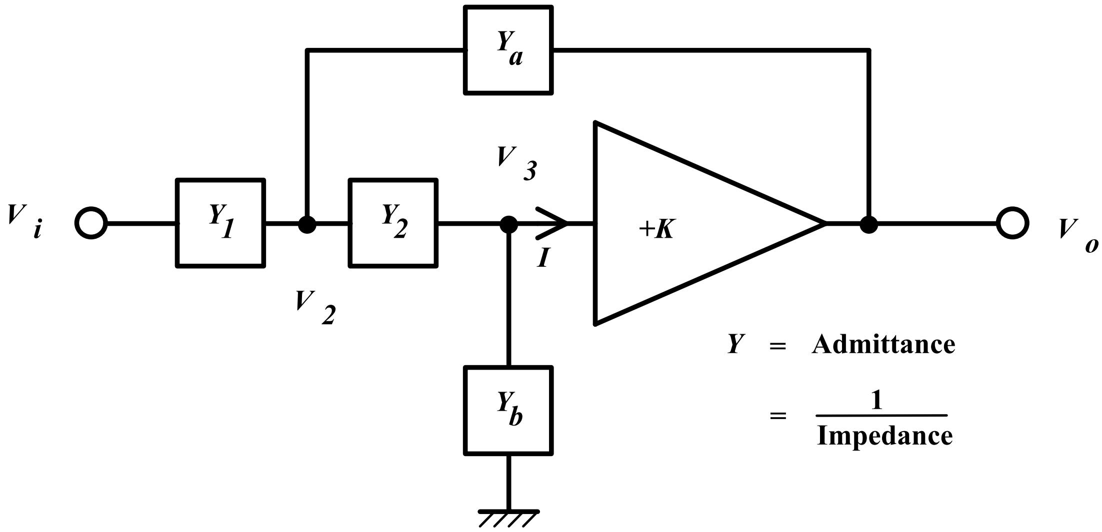




$$K = \frac{R_2}{R_1} + 1$$

Sallen Key Filter Circuit

- Now consider this circuit:



- There are two node equations at V_2 and V_3, together with the amplifier equation (assuming $I = 0$):

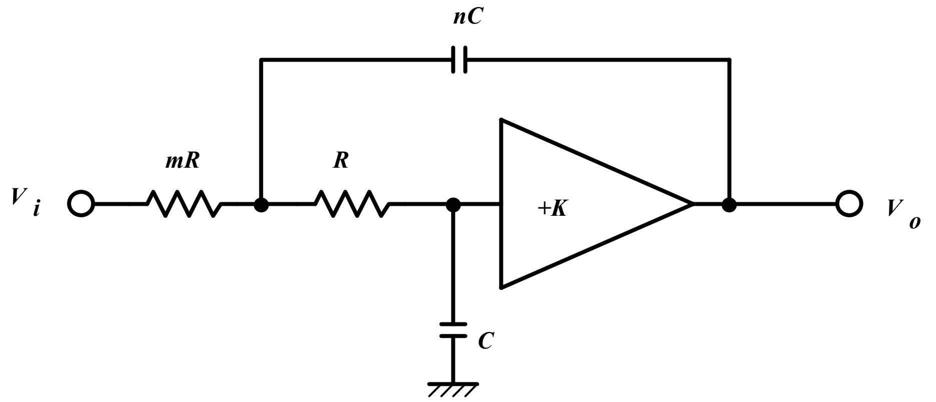
- Solving these simultaneous equations, we get:

$$\frac{V_o}{V_i} = \frac{KY_1Y_2}{(Y_1 + Y_2 + Y_a)(Y_2 + Y_b) - Y_2(Y_2 + KY_a)}$$

- By using Rs and Cs in various positions, low-pass, high-pass, bandpass, and bandstop filters can be constructed.

Low-Pass Sallen Key Circuit

- Now replace the boxes by a particular set of Rs and Cs:



- In other words the following substitutions have been made:

$$Y_1 = \frac{1}{mR} \qquad Y_a = j\omega nC$$

$$Y_2 = \frac{1}{R} \qquad Y_b = j\omega C$$

- Hence:

$$\frac{V_o}{V_i} = \frac{K\dfrac{1}{mR}\cdot\dfrac{1}{R}}{\left(\dfrac{1}{mR}+\dfrac{1}{R}+j\omega nC\right)\left(\dfrac{1}{R}+j\omega C\right)-\dfrac{1}{R}\left(\dfrac{1}{R}+j\omega nKC\right)}$$

$$= \frac{K}{(1+m+j\omega nmRC)(1+j\omega RC)-(m+j\omega nmKRC)}$$

$$= \frac{K}{(j\omega)^2 mnR^2C^2+(j\omega)\left[mnRC(1-K)+RC(1+m)\right]+1}$$

$$= \frac{K}{(j\omega)^2 mnR^2C^2+(j\omega)\sqrt{mn}RC\left[\dfrac{1+m+mn(1-K)}{\sqrt{mn}}\right]+1}$$

- This is usually written in the form:

$$\frac{V_o}{V_i} = \frac{K}{\left(j\omega/\omega_n\right)^2 + 2\xi\left(j\omega/\omega_n\right) + 1}$$

where

$\omega_n = ?$ resonant frequency

$\xi = ?$ damping coefficient

$Q = ?$ quality factor - or Q factor

$B = ?$ 3dB bandwidth of resonance

Frequency Response of Lowpass Sallen Key Circuit

- The value of ξ determines the form of the frequency response, as shown in the plot on the handout.

- When $\xi < 1$, the denominator of $\frac{V_o}{V_i}$ above is a quadratic with complex roots, and the response has a resonance.

- For $\underline{\omega << \omega_n}$: $20\log_{10}\left|\frac{V_o}{V_i}\right| \approx 20\log_{10}\left[\frac{K}{1}\right]$

$$= 20\log_{10} K \quad \text{dB}$$

- For $\underline{\omega >> \omega_n}$: $20\log_{10}\left|\frac{V_o}{V_i}\right| \approx 20\log_{10}\left[\frac{K}{(\omega/\omega_n)^2}\right]$

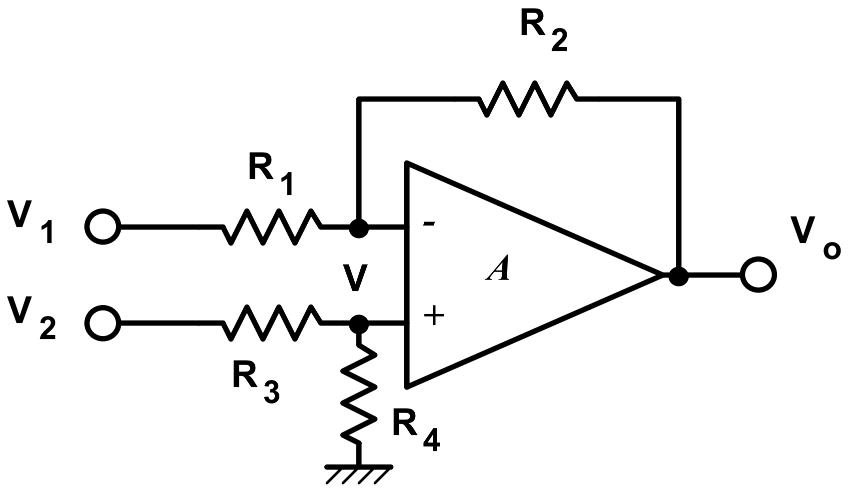
$$= 20\log_{10} K - 20\log_{10}(\omega/\omega_n)^2$$

$$= 20\log_{10} K - 40\log_{10}(\omega/\omega_n) \quad \text{dB}$$

(i.e. an asymptote of -40 dB/decade)

Difference Amplifer

Configuration



Analysis

- Node equations at V:

(1)

(2)

- Re-arranging equation (1):

(3)

- Re-arranging equation 2:

(4)

- Substitute for V from (4) into (3):

$$V_o = ?$$

(5)

Differential Amplifier

Analysis

- A special case of the difference amplifier is when

$$V_o = K(V_2 - V_1)$$

- Then from (5):

$$\frac{1 + R_2/R_1}{1 + R_3/R_4} = \frac{R_2}{R_1}$$

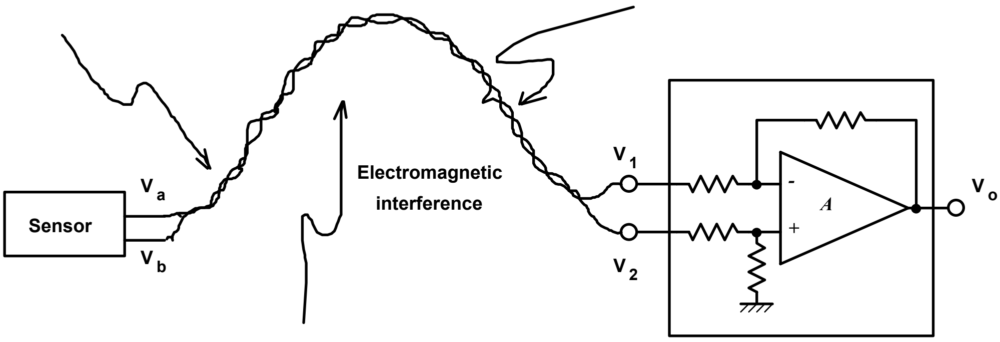
$$\frac{R_1}{R_2}\left(1 + \frac{R_2}{R_1}\right) = 1 + \frac{R_3}{R_4}$$

$$1 + \frac{R_1}{R_2} = 1 + \frac{R_3}{R_4}$$

$$\underline{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$$

- Then $V_o = ?$

Application for Differential Amplifier



- The sensor is connected to the amplifier with both wires of the “twisted pair” floating (i.e. neither is connected to ground).
- Suppose that the EM (electromagnetic) interference results in:

$$V_1 = V_a + N \qquad V_2 = V_b + N$$

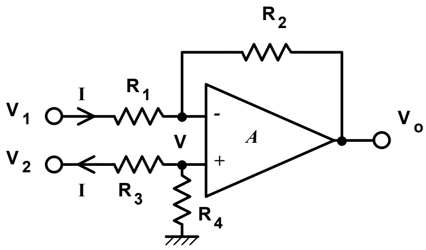
Then:

$$V_o = K\left[\left(V_2 - V_1\right)\right] = K\left[\left(V_b + N\right) - \left(V_a + N\right)\right]$$

$$\underline{V_o = K\left(V_b - V_a\right)}$$

- Since the interference is equal on both wires it is cancelled, which would not be the case if one of the inputs to the amplifier were earthed, such as for a standard inverting or non-inverting amplifier.
- This is often known as a “balanced” input.

Limitations of the Differential Amplifier



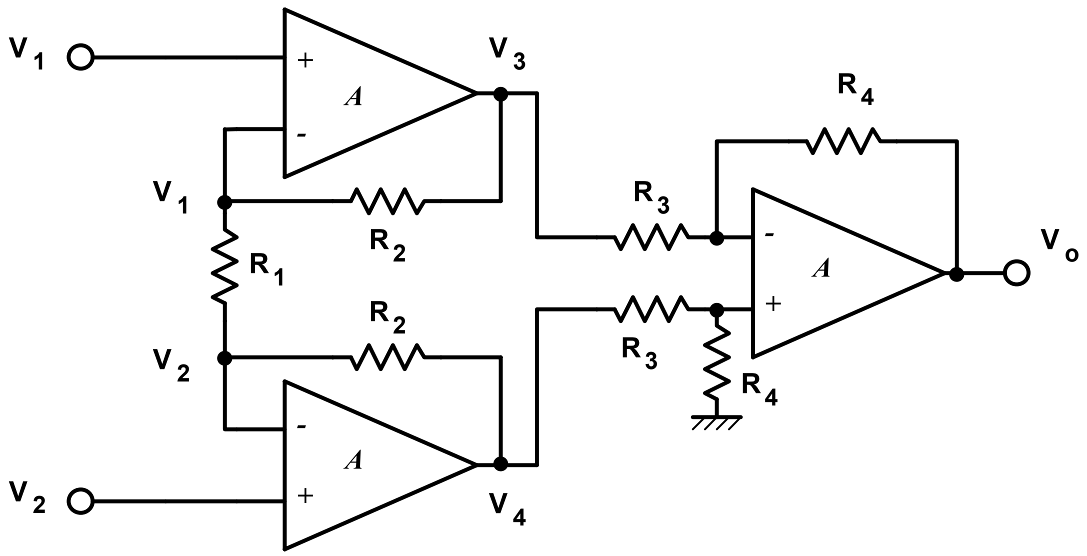
- Since the 2 input terminals to the op amp are almost at the same potential:

$$R_{in} = \frac{V_1 - V_2}{I} = R_1 + R_3$$

- This means that the input impedance R_{in} is relatively small

Instrumentation Amplifier

Configuration



Analysis (assuming ideal op-amp)

- The node equations at V_1 and V_2 are:

  ?

- Subtract the above equations to get:

$$2\frac{V_2}{R_1} - 2\frac{V_1}{R_1} + \frac{V_3}{R_2} - \frac{V_4}{R_2} + \frac{V_2}{R_2} - \frac{V_1}{R_2} = 0$$

Hence:

$$\left(V_2 - V_1\right)\left(\frac{2}{R_1} + \frac{1}{R_2}\right) + \frac{1}{R_2}\left(V_3 - V_4\right) = 0$$

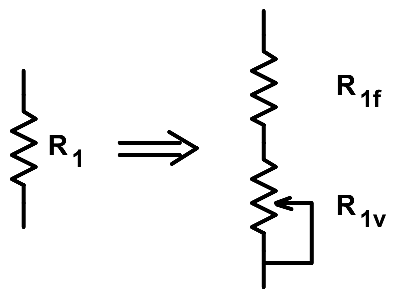
$$(V_3 - V_4) = (V_1 - V_2)\left(1 + 2\frac{R_2}{R_1}\right)$$

Hence:

$$V_o = ?$$

Gain

- Gain can be adjusted by:



- Usually put all the gain in the first stage. i.e. $R_4 = R_3 \approx 10k\Omega$
- Suppose R_{1v} is a 100 kΩ potentiometer, and we require a gain from 2 to 1000:

$$1 + \frac{2R_2}{R_{1f} + R_{1v}} = 2 \text{ to } 1000$$

$$1 + \frac{2R_2}{R_{1f}} = 1000$$

$$1 + \frac{2R_2}{R_{1f} + 100000} = 2$$

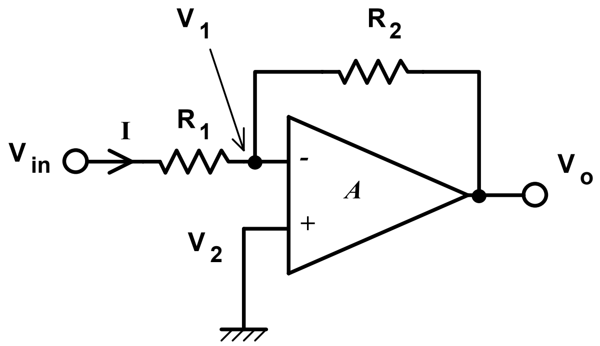
- Solving, we get:

$$R_{1f} = 100.2\,\Omega$$

$$R_2 = 50.05\,\Omega$$

Effect of Finite Op-Amp Gain: Inverting Amplifer

Configuration



- Now assume that A is not infinite (i.e. $V_1 \neq V_2$)

Analysis of Gain

- Node equation at V_1:

$$\frac{V_{in} - V_1}{R_1} + \frac{V_o - V_1}{R_2} = 0 \qquad (1)$$

- Also:

$$V_o = A(V_2 - V_1) = -AV_1$$

Hence

$$V_1 = -V_o / A \qquad (2)$$

- Substitute (2) into (1) to get:

Re-arranging, we get:

(3)

- Note that as $A \to \infty$, $\frac{V_o}{V_{in}} \to -\frac{R_2}{R_1}$

Analysis of Input Impedance

- From the circuit diagram:

$$
\begin{aligned}
I &= \frac{V_{in} - V_1}{R_1} \\
&= \frac{V_{in} - \left(-V_o / A\right)}{R_1} \\
&= \frac{V_{in}}{R_1}\left[1 + \frac{1}{A}\frac{V_o}{V_{in}}\right] \\
&= \frac{V_{in}}{R_1}\left[1 + \frac{1}{A}\left(-R_2/R_1\right)\left(\frac{1}{1 + \left(1 + R_2/R_1\right)/A}\right)\right]
\end{aligned}
$$

- Hence:

$$R_{in} = \frac{V_{in}}{I} = ?$$

(4)

Example for Typical Values

- Suppose that $R_1 = 1k\Omega$ and $R_2 = 100k\Omega$.

- Define:

$$\varepsilon = \frac{\text{Actual gain - } R_2/R_1}{R_2/R_1} \times 100 \tag{5}$$

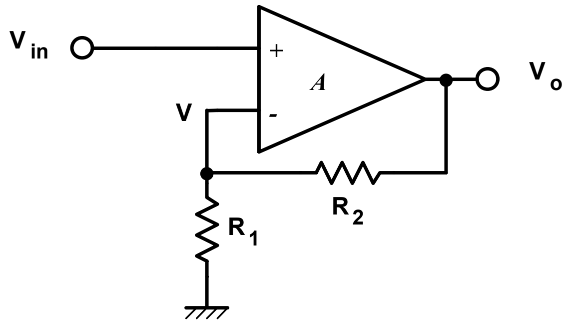
as a percentage error in the gain calculation.

- We can use equations (3), (4) and (5) to construct the following table:

A	Gain	ε	V	Input Impedance
10^5	99.99	-0.1 %	-0.1 mV	1.001 kΩ
10^4	99.90	-1.0 %	-0.99 mV	1.01 kΩ
10^3	90.83	-9.2 %	-9.08 mV	1.1 kΩ

Effect of Finite Op-Amp Gain: Non-Inverting Amplifer

Configuration



Analysis

- Node equation at V:

$$\frac{V_o - V}{R_2} + \frac{0 - V}{R_1} = 0$$

$$V = V_o \frac{R_1}{R_1 + R_2}$$

$$\underline{V = V_o \beta} \qquad \text{where} \qquad \beta = \frac{R_1}{R_1 + R_2} \qquad (6)$$

- Amplifier equation:

$$\left(V_{in} - V\right) A = V_o \qquad (7)$$

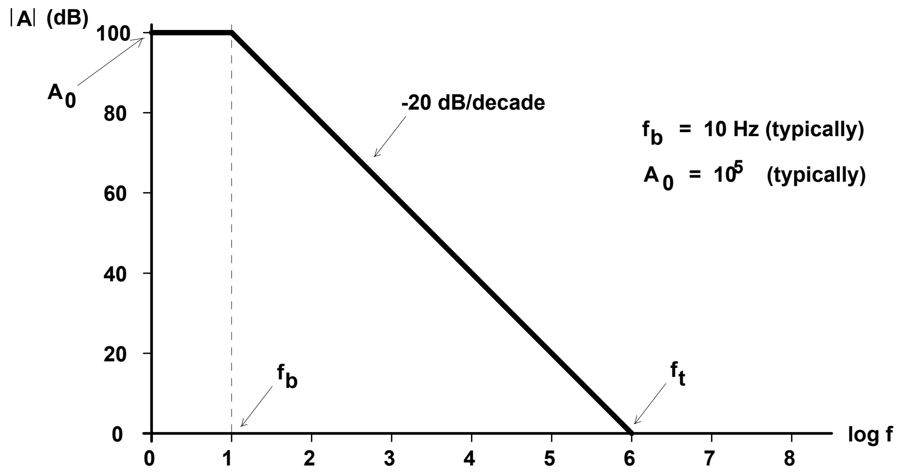
- Substitute (6) into (7):

$$\left(V_{in} - V_o \beta\right) A = V_o$$

$$\underline{\frac{V_o}{V_{in}} = ?} \qquad \text{where} \qquad \beta = \frac{R_1}{R_1 + R_2}$$

Finite Open Loop Gain and Bandwidth

Frequency Response of 741 Op-Amp



- Gain roll-off is a result of internal compensation
- Hence:

$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega_b}$$

- For $\omega >> \omega_b$:

$$A(j\omega) \cong \frac{A_0}{j\omega/\omega_b} = \frac{A_0\omega_b}{j\omega}$$

- Hence $|A|$ is unity at a frequency of

$$\omega_t = A_0\omega_b$$

This is called the "unity gain bandwidth", or the "gain bandwidth" product

- Hence

$$A(j\omega) = \omega_t / j\omega$$

Frequency Response of Inverting Amplifier

- $$\frac{V_o}{V_i} = \frac{-R_2/R_1}{1+(1+R_2/R_1)/A}$$

- Substitute for $\frac{1}{A} = \frac{j\omega}{\omega_t}$ from above

$$\frac{V_o}{V_i} = ?$$

- Hence this has a 1st order low-pass response with a 3dB bandwidth of:

$$\omega_{3dB} = \frac{\omega_t}{(1+R_2/R_1)}$$

Frequency Response of Non-Inverting Amplifier

- $$\frac{V_o}{V_i} = \frac{1+R_2/R_1}{1+(1+R_2/R_1)/A}$$

- Substitute for $\frac{1}{A} = \frac{j\omega}{\omega_t}$ from above

$$\frac{V_o}{V_i} = \frac{1+R_2/R_1}{1+\dfrac{j\omega}{\omega_t/(1+R_2/R_1)}} = \frac{G}{1+\dfrac{j\omega}{\omega_t/G}}$$

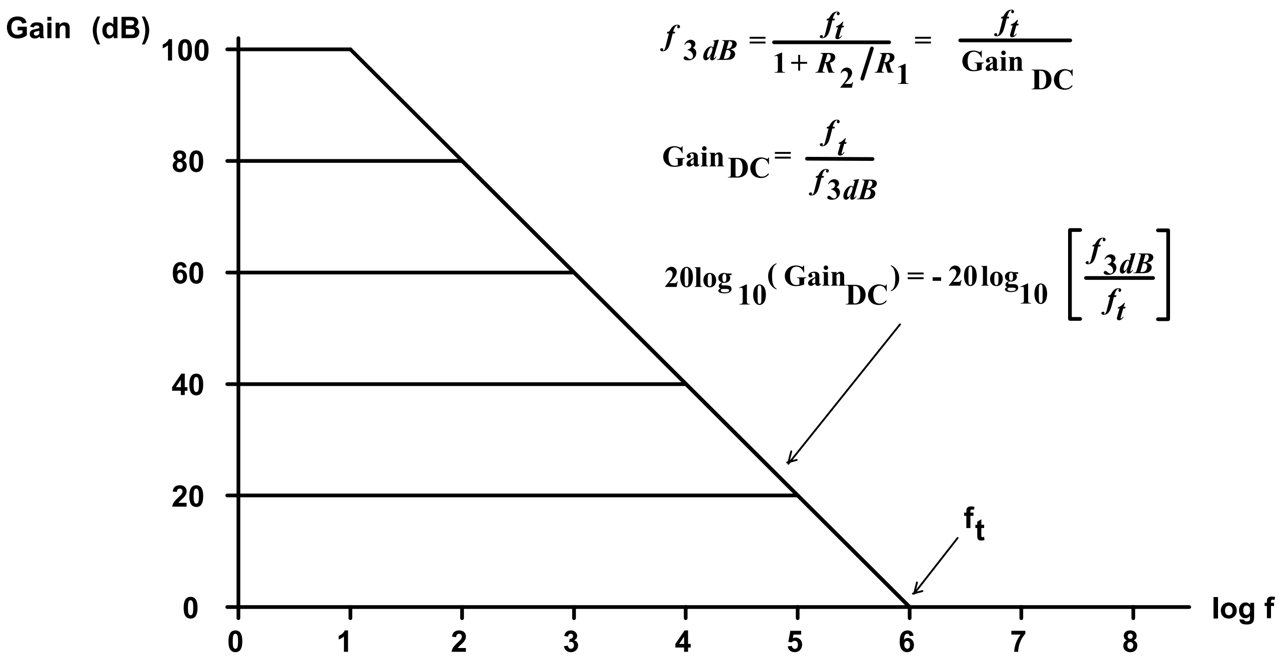
where G is the DC gain of the non-inverting amplifier circuit.

Example Using the Single Pole Model

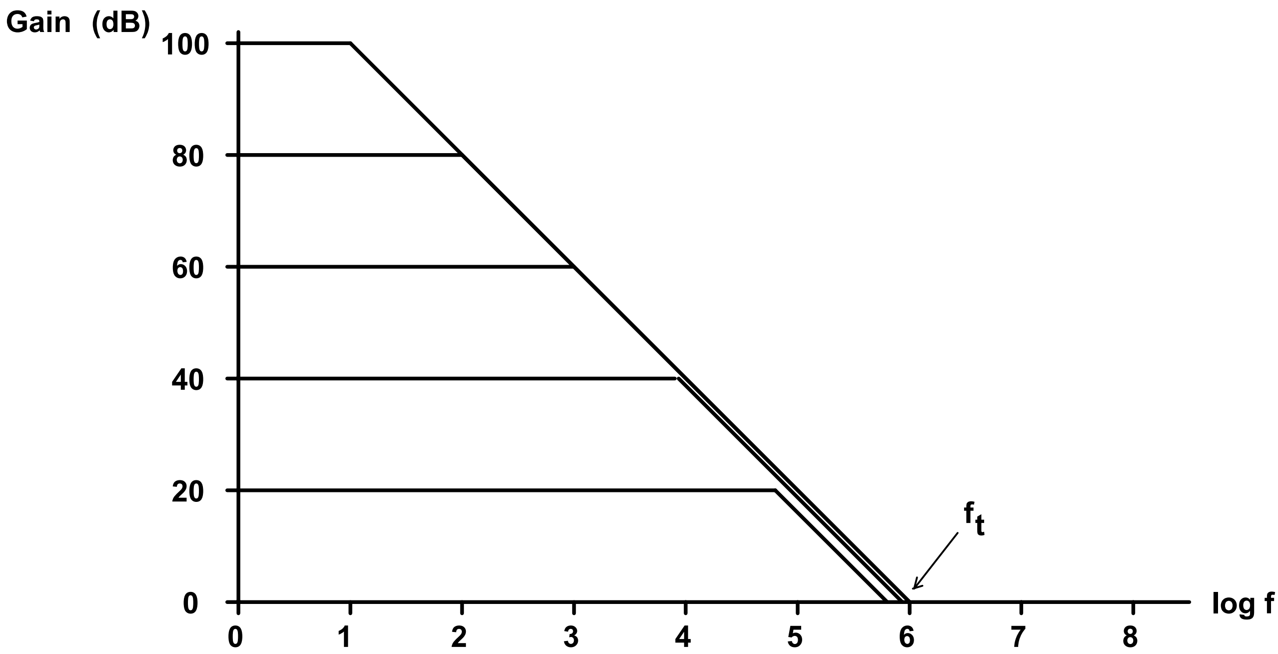
- Op-amp spec: $f_t = 1$ MHz

- Find bandwidth for gains of: +1000, +100, +10, +1, -1, -10, -100, -1000

Closed loop gain (G)	R_2/R_1	$f_{3dB} = f_t/(1+R_2/R_1)$
+1000 +100 +10 +1		
-1 -10 -100 -1000		



Non Inverting Amplifier



Inverting Amplifier

Op-amp Instability

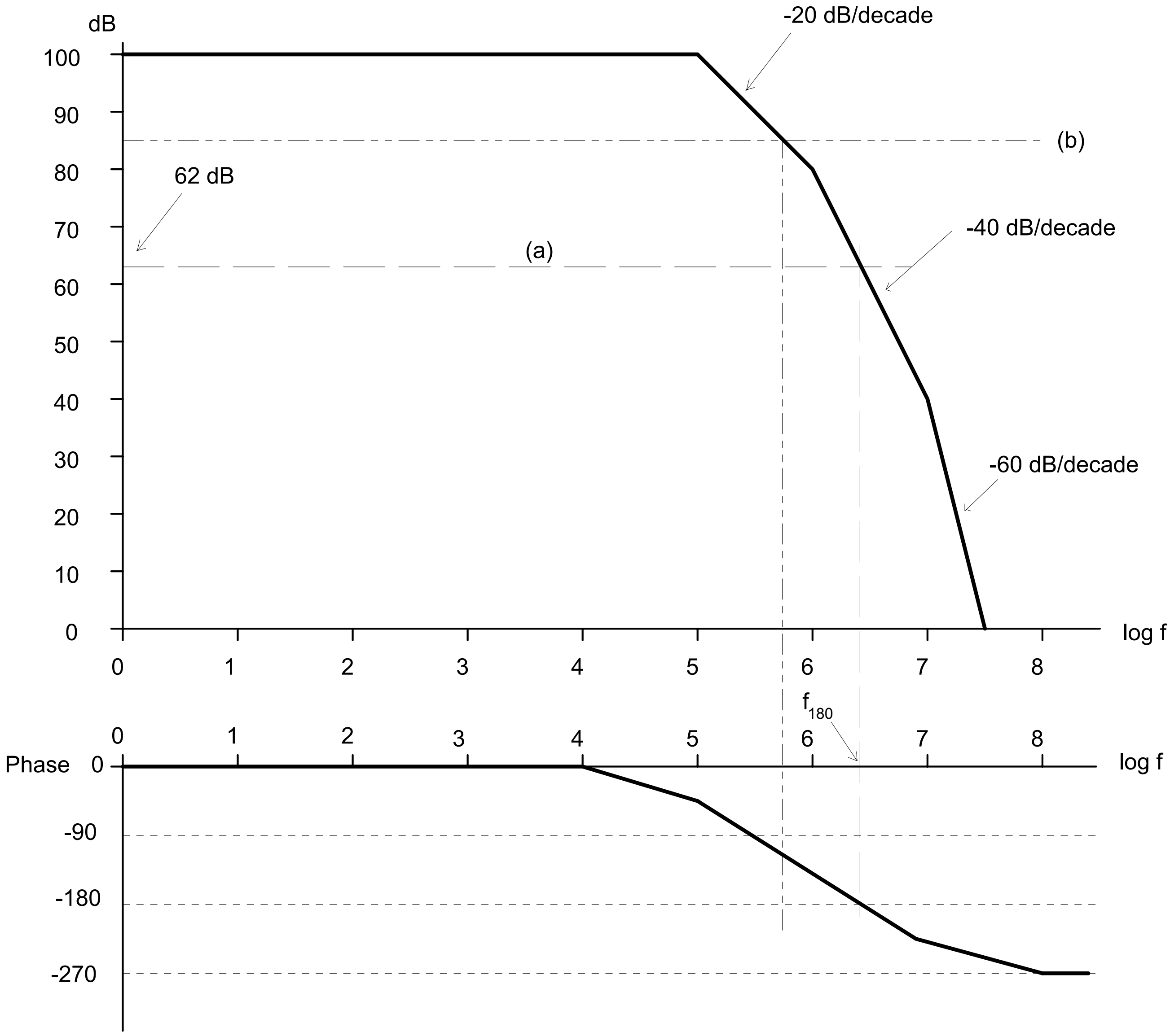
Suppose the op-amp can be modelled with 3 corner frequencies:

$$A(jf) = \frac{A_0}{\left(1 + j\frac{f}{f_1}\right)\left(1 + j\frac{f}{f_2}\right)\left(1 + j\frac{f}{f_3}\right)}$$

Choose $f_1 = 0.1\text{MHz}$
$f_2 = 1\ \text{MHz}$
$f_3 = 10\ \text{MHz}$

where the three corner frequencies are due to various frequency dependent components in the op-amp.

Bode plot of *A(jf) is:*



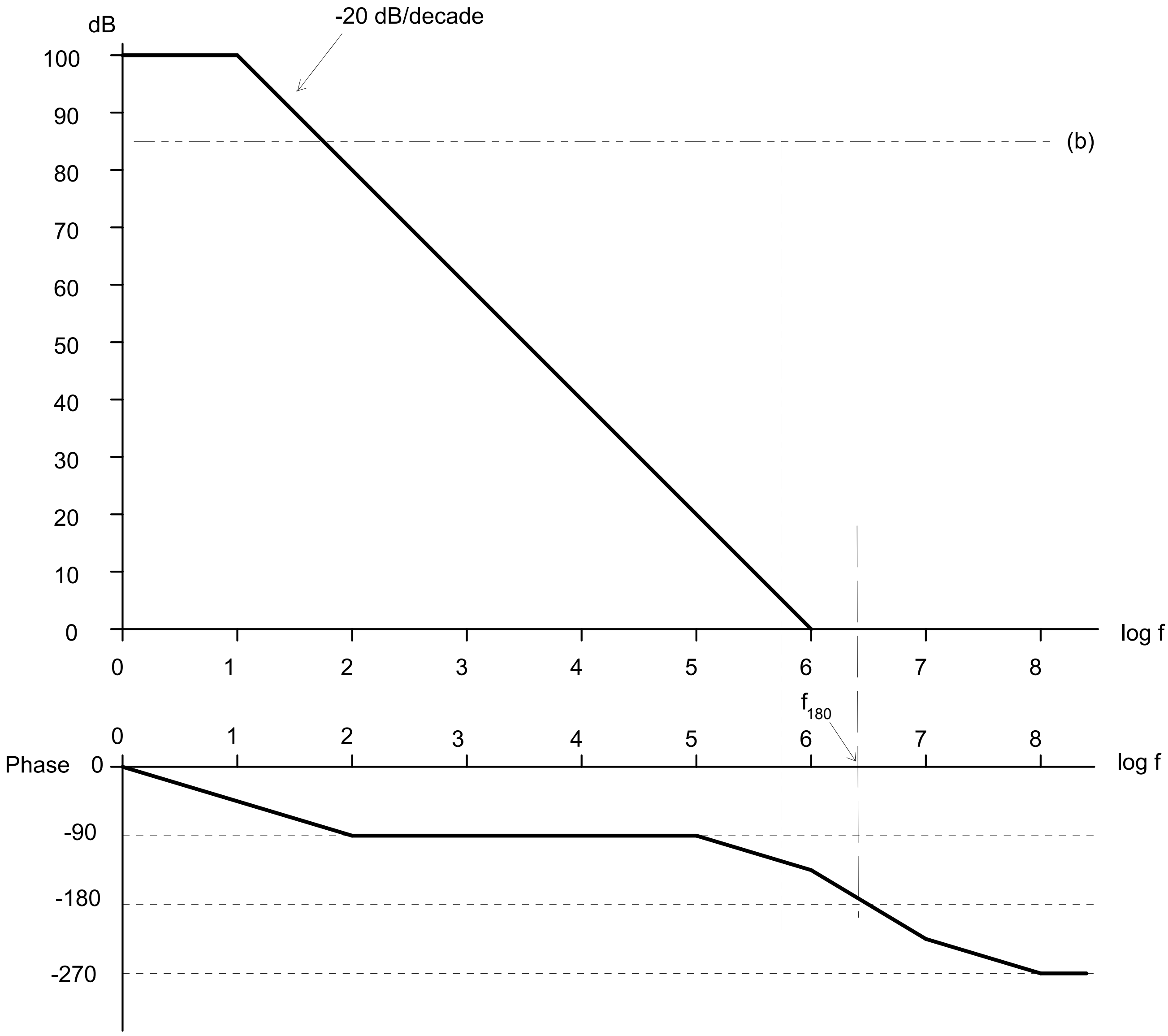
$|A(jf)|$ at f_{180} = 62 dB (= smallest gain possible for an amplifier using this device, whilst still being stable)

“Compensated” Op-Amp

Suppose that we now deliberately alter f_1 to be much smaller (by adding a capacitor in the circuit):

Choose $f_1 = 10$ Hz
$f_2 = 1$ MHz
$f_3 = 10$ MHz

The Bode plot then becomes:



$|A(jf)|$ at f_{180} is <0 dB (i.e. <1) (all amplifiers with a gain >1 are therefore stable using this device)